Photonic-Assisted Tunable Microwave Pulse Fractional Hilbert Transformer Based on a Temporal Pulse Shaping System

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***Abstract*—A photonic-assisted fractional Hilbert transformer with tunable fractional order implemented based on temporal pulse shaping (TPS) is proposed and experimentally demonstrated. The proposed fractional Hilbert transformer consists of a phase modulator and two dispersive elements with complementary dispersion. The fractional Hilbert transform (FHT) is realized if a step function is applied to the phase modulator to introduce a phase jump. The proposed technique is investigated numerically and experimentally. The results show that a real-time FHT is achieved with a tunable fractional order by tuning the step function applied to the phase modulator.**

***Index Terms*—Fractional Hilbert transform (FHT), Fourier transform, temporal pulse shaping (TPS) system.**

I. Introduction

TEMPORAL pulse shaping (TPS) has been widely investigated in the past few years due to its important applications in areas such as frequency analysis [1] and arbitrary waveform generation (AWG) [2]–[5]. The concept of TPS was originally proposed by Heritage and Weiner [6]. In [1], a TPS system was employed for the analysis of the spectrum of a microwave signal. Since the output in the TPS system is in the time domain, a fast measurement of the spectrum could be implemented using a real-time oscilloscope. The most important application of the TPS technique is to achieve AWG [2]–[5]. Since the output waveform is determined by the modulation signal, the output waveform to be generated can be programmed in real time [2], [3]. In addition, the use of a TPS system incorporating a phase modulator can generate a pulse burst with tunable repetition rate [4]. Recently, we demonstrated that the use of a TPS system can generate a high-frequency and frequency-chirped microwave waveform [5].

Due to the wide applications in modern communications and image processing, the Hilbert transform (HT) is one of the most useful signal processing functions [7]. The HT can be implemented in the electrical domain using digital electronics. However, due to the limited sampling rate of the state-of-the-art digital electronics, the speed of an electronic Hilbert transformer is low. Thanks to the high frequency and large bandwidth provided by modern optics, the implementation of the HT in the optical domain would provide a solution for the processing of a high-frequency and broadband microwave signal. The HT can be implemented in the optical domain based on a phase-shifted fiber Bragg grating (PS-FBG) [8], with a bandwidth as large as a few hundred of gigahertz. Recently, we proposed to implement fractional HT (FHT) based on a directly designed FBG using the discrete layer peeling (DLP) method [9]. Since the order of the FHT can be an arbitrary number, it provides large flexibility in signal processing. The significance of using an FBG designed based on the DLP method [9] is that the strength of the FBG is high, which would lead to a significantly increased signal-to-noise ratio (SNR) at the output of the Hilbert transformer [9]. The major limitation of the HT using an FBG is the poor programmability. In addition, the nonflat magnitude response of the FBG will also impact the performance of the HT.

In this letter, we propose and experimentally demonstrate a fractional Hilbert transformer with a tunable fractional order. The technique is achieved by using a TPS system consisting of a phase modulator (PM) and two dispersive elements (DEs) with complementary dispersion. The FHT is realized if a step function is applied to the PM to introduce a phase jump to the spectrum of the microwave signal to be Hilbert transformed. The proposed technique is investigated numerically and experimentally. The results show that a real-time FHT can be achieved with a tunable fractional order by tuning the step function applied to the PM.

II. Principle

In order to bring in new degrees of freedom in signal analysis, the standard HT was generalized by defining a new transfer function [10]

$$H_{\mathrm{FHT}}(\omega) = \begin{cases} e^{j\varphi}, & \omega < 0 \\ e^{-j\varphi}, & \omega > 0 \end{cases} \tag{1}$$

where $\varphi = P \times \pi/2$ and P is the fractional order. The new transform is called FHT. Obviously, the standard HT is a special case of the FHT when the fractional order equals to 1.

The FHT can be implemented with a tunable fractional order using a system shown in Fig. 1. A microwave pulse to be processed is firstly modulated on an optical carrier at a Mach–Zehnder modulator (MZM). Since the MZM is biased at the minimum transmission point, the envelope of the modulated signal $x(t)$ is proportional to the waveform of the microwave signal $m(t)$. Then the modulated optical signal is sent to

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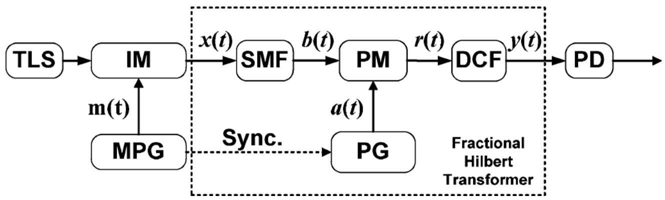

Fig. 1. Schematic of the TPS-based FHT system. TLS: Tunable laser source. IM: Intensity modulator. MPG: Microwave pulse generator. SMF: Single-mode fiber. PM: Phase modulator. PG: Pattern generator. DCF: Dispersion-compensating fiber. PD: Photodetector.

a TPS-based fractional Hilbert transformer. The fractional Hilbert transformer consists of a phase modulator and two dispersive elements with complementary dispersion, which are a single-mode fiber (SMF) and a dispersion-compensating fiber (DCF). A step function is applied to the phase modulator to introduce a phase jump.

Mathematically, the impulse responses of the two dispersive fibers are given by $h_{\pm\ddot{\Phi}}(t) = \exp(\mp j\pi t^2/\ddot{\Phi})$, where $\ddot{\Phi}$ is the group velocity dispersion (GVD).

Let $x(t)$ and $b(t)$ be the complex envelopes of the signal at the input and the output of the SMF, respectively. Then

$$b(t) = x(t) * h_{+\ddot{\Phi}}(t) = \int_{-\infty}^{+\infty} x(\tau)\cdot\exp\left(-\frac{j\pi(t-\tau)^2}{\ddot{\Phi}}\right)d\tau \quad (2)$$

If the time duration of $x(t)$ is sufficiently small or the GVD is sufficiently large to satisfy $|\Delta t^2/\ddot{\Phi}| \ll 1$, where Δt is the time duration of the input pulse. Equation (2) can be approximated by

$$b(t) \approx \exp\left(-\frac{j\pi t^2}{\ddot{\Phi}}\right) X(\omega)|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} \quad (3)$$

where $X(\omega)$ is the Fourier transform (FT) of $x(t)$.

If a step function $a(t) = \begin{cases} -V_a, & t<0 \\ V_a, & t>0 \end{cases}$, where V_a is a constant voltage, is applied to the PM, the modulated signal at the output of the PM is given by

$$\begin{aligned} r(t) &= b(t)\cdot\exp\left(j\pi\frac{a(t)}{V_\pi}\right) \\ &= \exp\left(-\frac{j\pi t^2}{\ddot{\Phi}}\right)\{X(\omega)\exp[j\varphi(\omega)]\}|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} \end{aligned} \quad (4)$$

where V_π is the half-was voltage of the PM and

$$\varphi(\omega)|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} = \begin{cases} -\pi(V_a/V_\pi), & \omega>0 \\ \pi(V_a/V_\pi), & \omega<0 \end{cases} \quad (5)$$

is the phase jump. The modulated signal is then sent to the second dispersive element. At the output of the fractional Hilbert transformer we have

$$\begin{aligned} y(t) &= r(t) * h_{-\ddot{\Phi}}(t) \\ &= \ddot{\Phi}\exp\left(\frac{j\pi t^2}{\ddot{\Phi}}\right)\Im^{-1}\{X(\omega)\cdot\exp[j\varphi(\omega)]\}|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} \end{aligned} \quad (6)$$

where $\Im^{-1}$ denotes the operation of the inverse FT.

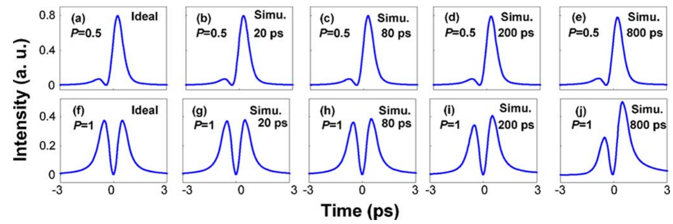

Fig. 2. Numerical results for ideal FHT and TPS-based FHT. (a) Ideal FHT with a fractional order of 0.5. The TPS-based FHT with a fractional order of 0.5 and a rise time of (b) 20 ps, (c) 80 ps, (d) 200 ps, and (e) 800 ps. (f) Ideal FHT with a fractional order of 1. The TPS-based FHT with a fractional order of 1 and a rise time of (g) 20 ps, (h) 80 ps, (i) 200 ps, and (j) 800 ps.

Again, if $|\Delta t^2/\ddot{\Phi}| \ll 1$, (6) can be approximated as

$$\begin{aligned} y(t) &\approx \ddot{\Phi}\Im^{-1}\{X(\omega)\cdot\exp[j\varphi(\omega)]\}|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} \\ &= \ddot{\Phi}\Im^{-1}\{X(\omega)\cdot H_{\mathrm{FHT}}(\omega)\}|_{\omega=-\frac{2\pi t}{\ddot{\Phi}}} \end{aligned} \quad (7)$$

As can be seen the output waveform is a Hilbert transformed version of the input waveform. By simply tuning the amplitude of the step function applied to the PM, we can easily adjust the fractional order of the fractional Hilbert transformer.

If the 3-dB bandwidth of the input signal $x(t)$ is $\Delta\omega$, then the temporal width of the pulse $b(t)$ at the output of the SMF is $\Delta\omega\cdot\ddot{\Phi}/2\pi$ [11]. Practically, the step function $a(t)$ generated by a pattern generator does not have an ideal jump at $t = 0$, and a nonzero rise time must be considered. To make the impact negligible, the rise time of the step function compared with the temporal width of the stretched pulse $b(t)$ should be sufficiently small, $\tau_r \ll \Delta\omega\cdot\ddot{\Phi}/2\pi$, where τ_r is the rise time of the step function. For example, if a dispersive medium with a value of GVD of 10000 ps^2 is used and the rise time of the step function is 20 ps, the bandwidth of the pulse to be transformed should be at least 20 GHz with 10 considered as a factor for being large enough. In other words, a transform-limited Gaussian-like pulse with temporal width up to 22 ps can be accurately Hilbert transformed when the rise time is 20 ps. Considering that 20 ps is the shortest rise time that an electronic device can provide, if we want to extend the time duration of the pulse under processing, we may need to increase the GVD of the DEs to keep $|\Delta t^2/\ddot{\Phi}| \ll 1$ valid. In such a case, longer SMF and DCF are needed. However, if the SMF and DCF are too long, other effects such as the third order dispersion (TOD) must be taken into consideration.

To further investigate the impact of the rise time on the performance of the proposed system, a simulation is performed. In the simulation, a 550-fs Gaussian pulse is applied to the fractional Hilbert transformer, with the rise time increased from 20 ps, the shortest rise time of an electronic device can reach as far as we know, to 800 ps, and the output waveforms are shown in Fig. 2. It can be seen the output waveforms become more distorted compared with the ideal waveforms with the increase of the rise time. To quantitatively evaluate the errors due to the nonzero rise time, we calculate the normalized root mean square errors (NRMSEs). For $P = 0.5$, the NRMSE is smaller than 2% for the rise times from 20 to 800 ps. For $P = 1$, the NRMSE is within 12%.

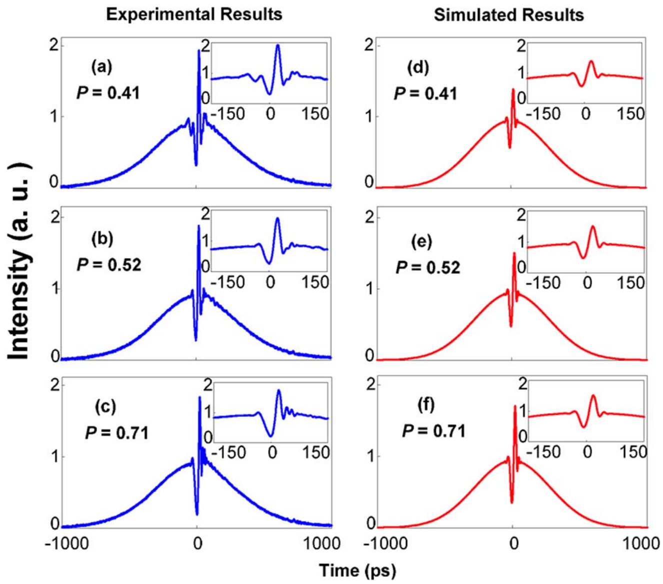

Fig. 3. Experimental results for the fractional orders of (a) 0.41, (b) 0.52, and (c) 0.71. The experimental results are compared with the simulation results in (d), (e), and (f). A zoom-in view of the central portion is given as an inset.

In the above analysis, only the GVD is considered. The impact of higher-order dispersion, mainly the TOD, is also evaluated. Based on our calculations, if the GVD and the TOD are both matched, the NRMSE due to the TOD is within 1%. When there is a mismatch in TOD, then the NRMSE will increase. For example, for a mismatch of 5% in TOD, the total NRMSE is within 13% for $P = 1$.

III. Experiment

A proof-of-concept experiment is then carried out based on the setup shown in Fig. 1. Since a microwave pulse with a 3-dB bandwidth greater than 12.5 GHz is hard to generate, in the experiment an optical pulse from a mode-locked laser (MLL) is directly employed and is applied as an input to the fractional Hilbert transformer. The 3-dB temporal width of the pulse from the MLL is 550 fs. A 36.91-km SMF (SMF1) with a value of dispersion of -4872 ps^2 is employed to stretch the pulse from the MLL, and the stretched pulse is sent to a PM. The PM has a half-wave voltage of 10.9 V and the 3-dB temporal pulse width of the stretched pulse is 4.9 ns. A DCF with a value of dispersion of 4872 ps^2 is connected after the PM. A step function generated by an arbitrary waveform generator (Tektronix AWG7102) and amplified by a microwave amplifier is applied to the PM via the RF port. A Hilbert transformed pulse is thus obtained at the output of the DCF.

Since the pulse at the output of the DCF has a temporal width of only several picoseconds, which is too fast to be detected by a photodetector (PD). To monitor the output pulse, another 6.74-km SMF (SMF2) with a value of dispersion of -890 ps^2 is used to stretch the output pulse, to make it wide enough to be detected by the PD. In addition, the transformed pulse would also be distorted during the stretching process. The distortion is also considered in the simulation shown in Fig. 3.

The experimental results for the FHT with different fractional orders of 0.41, 0.52 and 0.71 are shown in Fig. 3. Simulation results by taking into consideration of the additional 6.74-km SMF are also shown as a comparison. The experimental results agree well with the results by the simulations. Note that a 20-GHz electrooptic PM with an ultralow half-wave voltage of 3 V at 1 GHz is commercially available. If such a PM is employed, a FHT with a fractional order from 0 to 2 can be easily achieved.

The rise time of the step functions for the three different fractional orders is 63.3 ps, which is small and has negligible impact on the FHT. The NRMSEs of the three experimentally obtained pulses in Fig. 3(a), (b) and (c) are 4.24%, 3.98% and 3.87%, respectively.

IV. Conclusion

The implementation of a photonic-assisted microwave pulse fractional Hilbert transformer with tunable fractional order based on TPS was proposed and experimentally demonstrated. The FHT was realized by introducing a phase jump to the spectrum of the input pulse via phase modulation and the tunability of the fractional order was achieved by changing the amplitude of the step function applied to the PM. The key advantage of this technique is its flexibility in changing the fractional order, which may find applications where a Hilbert transform with a tunable order is needed.

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