(a) Is the system memoryless?
(b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.
(c) Is the system invertible?

1.17. Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

(a) Is this system causal?
(b) Is this system linear?

1.18. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.
(a) Is this system linear?
(a) Is this system time-invariant?
(c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0.

1.19. For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant or both.
(a) $y(t) = t^2 x(t-1)$ **(b)** $y[n] = x^2[n-2]$
(c) $y[n] = x[n+1] - x[n-1]$ **(d)** $y[n] = \mathcal{O}d\{x(t)\}$

1.20. A continuous-time linear system S with input $x(t)$ and output $y(t)$ yields the following input-output pairs:

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t},$$
$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}.$$

(a) If $x_1(t) = \cos(2t)$, determine the corresponding output $y_1(t)$ for system S.
(b) If $x_2(t) = \cos(2(t - \frac{1}{2}))$, determine the corresponding output $y_2(t)$ for system S.

BASIC PROBLEMS

1.21. A continuous-time signal $x(t)$ is shown in Figure P1.21. Sketch and label carefully each of the following signals:
(a) $x(t-1)$ **(b)** $x(2-t)$ **(c)** $x(2t+1)$
(d) $x(4 - \frac{t}{2})$ **(e)** $[x(t) + x(-t)]u(t)$ **(f)** $x(t)[\delta(t + \frac{3}{2}) - \delta(t - \frac{3}{2})]$

1.22. A discrete-time signal is shown in Figure P1.22. Sketch and label carefully each of the following signals:
(a) $x[n-4]$ **(b)** $x[3-n]$ **(c)** $x[3n]$
(d) $x[3n+1]$ **(e)** $x[n]u[3-n]$ **(f)** $x[n-2]\delta[n-2]$
(g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$ **(h)** $x[(n-1)^2]$

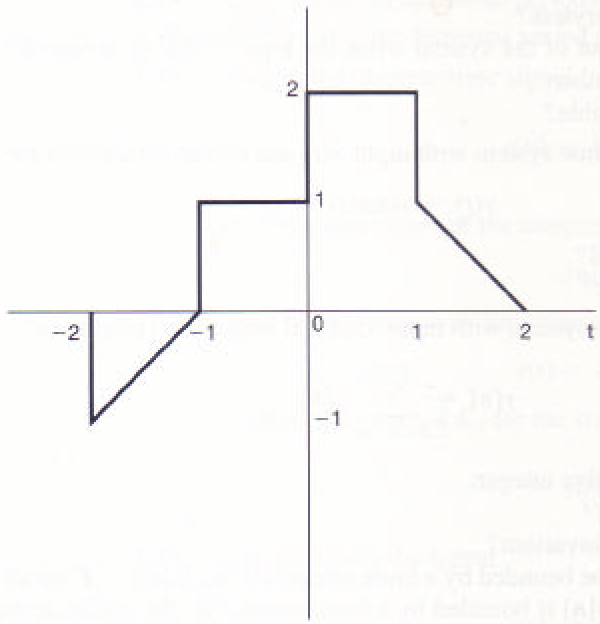

Figure P1.21

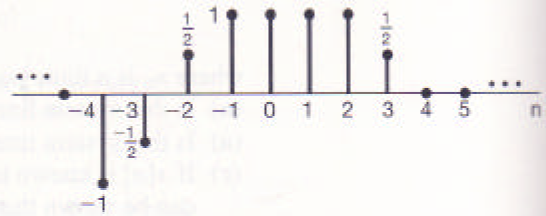

Figure P1.22

1.23. Determine and sketch the even and odd parts of the signals depicted in Figure P1.23. Label your sketches carefully.

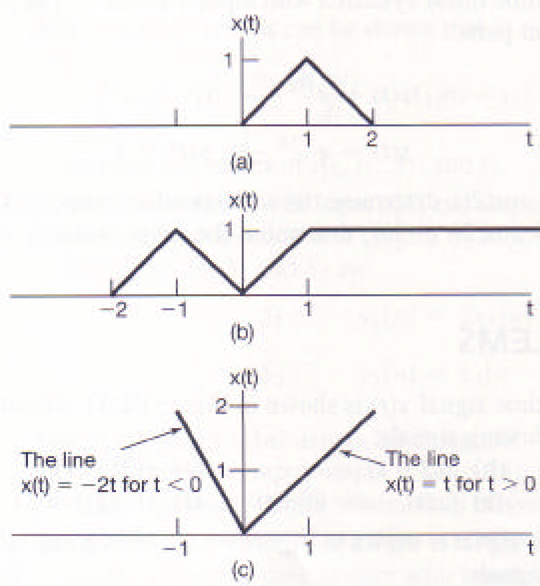

Figure P1.23

1.24. Determine and sketch the even and odd parts of the signals depicted in Figure P1.24. Label your sketches carefully.

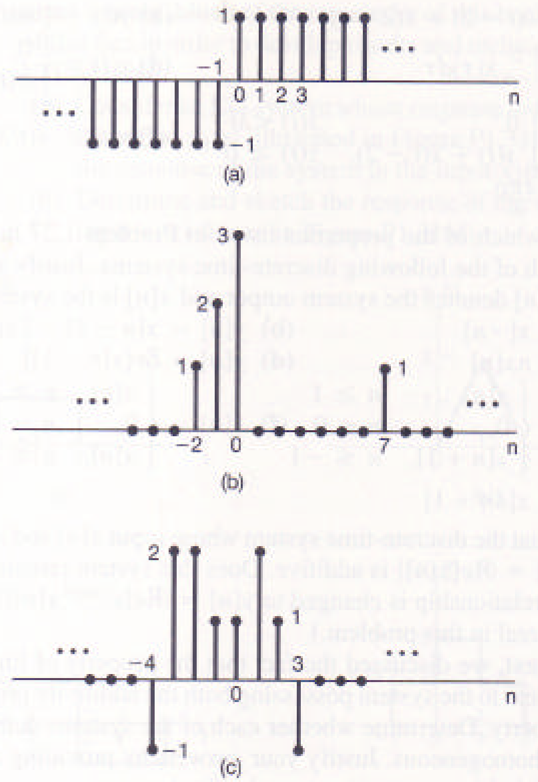

Figure P1.24

1.25. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$ **(b)** $x(t) = e^{j(\pi t - 1)}$

(c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$ **(d)** $x(t) = \mathcal{E}v\{\cos(4\pi t)u(t)\}$

(e) $x(t) = \mathcal{E}v\{\sin(4\pi t)u(t)\}$ **(f)** $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$ **(b)** $x[n] = \cos(\frac{n}{8} - \pi)$ **(c)** $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$ **(e)** $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

1.27. In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless

(2) Time invariant

(3) Linear

(4) Causal

(5) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.