Frequency-Tunable Microwave and Sub-Terahertz Generation Based on Time-Delayed Optical Combs

Montasir Qasymeh, Wangzhe Li, Student Member, IEEE, and Jianping Yao, Senior Member, IEEE

Abstract—A novel approach to generating a frequency-tunable microwave or sub-terahertz wave based on time-delayed optical combs is proposed and demonstrated. The fundamental principle is to generate multiple optical combs with identical comb profile, but with each optical comb carried by an optical carrier at a different wavelength. If the optical carriers are spaced with an identical wavelength spacing, the optical combs will be time delayed with an identical time delay after passing through a dispersive fiber. By applying these optical combs to a photodetector (PD), microwave comb lines at the fundamental and higher-order harmonic frequencies will be generated. For a well-designed time-delay structure, however, the desired microwave harmonic will have the highest output due to constructive interference while the other harmonics will be suppressed. An analysis is performed, which is verified by a proof-of-concept experiment. A microwave signal that is tunable from 16.8 GHz to 27 GHz is generated. The performance of the generated signal in terms of stability and phase noise is also evaluated.

Index Terms—Microwave generation, microwave photonics, optical comb, terahertz generation.

I. INTRODUCTION

PHOTONIC microwave generation has several advantages over conventional electronic techniques [1], [2], such as high frequency and wide frequency tunability. In addition, thanks to the extremely low loss of the state-of-the-art fibers, a microwave signal can be distributed over an optical fiber over a long distance, which can find applications such as antenna remoting and broadband wireless access.

Numerous techniques for generating microwave signals by optical means have been reported. Typically, the schemes are implemented to ultimately producing two optical waves of different wavelengths, and a microwave is generated by beating the two optical waves at a photodetector (PD). These include utilizing two different light sources [3], employing nonlinear devices [4] and utilizing external optical modulation [5]. It is known, utilizing two independent light sources will produce a microwave signal with a high phase noise. Several proposals were reported to solve this problem, including employing a dual-wavelength single-longitudinal-mode laser source and two phase-locked laser sources through an optical phase-locked loop [6] or optical injection locking [7]. However, these approaches are expensive and complicated [1]. Optical nonlinear process, on the other hand, could also be utilized to generate high quality microwave signals. For example, microwave generation based on four-wave-mixing (FWM) utilizing a highly nonlinear fiber [4] or a semiconductor optical amplifier (SOA) [8] was proposed. Nonetheless, the associated ultralow conversion efficiency imposes a severe limitation.

Alternatively, external optical modulation is a promising solution for generating high quality microwave signals, thanks to the high performance in terms of tunability, stability and simplicity. However, the achieved multiplication factor, defined as the ratio between the generated and the input microwave frequencies, is limited. For instance, a frequency octupling (a multiplication factor of 8) was achieved based on two cascaded Mach–Zehnder modulators (MZMs) [9]. Furthermore, a multiplication factor of 12 was recently reported based on a joint operation of polarization modulation, four-wave mixing, and stimulated-Brillouin-scattering-assisted filtering [10]. On the other hand, a much greater multiplication factor could be achieved utilizing an optical comb [11]-[13], in which two narrowband optical filters were utilized to select two of the optical comb lines. The multiplication factor can be as large as 50, only limited by the bandwidth of the PD. The major limitation of this approach is that the two optical filters must be extremely stable with an ultra-narrow bandwidth to ensure a correct wavelength selection, which would make the system very complicated and costly. In addition, the tuning of the microwave frequency is achieved by tuning of the optical filters, which again increases the system complexity and cost.

In this paper, we propose a novel approach to generating a microwave or sub-terahertz signal utilizing time-delayed optical combs without using optical or RF filters. The proposed approach is frequency tunable and can potentially achieve a very large multiplication factor. The fundamental principle is to generate multiple optical combs with identical comb profile, but each optical comb is carried by an optical carrier at a different wavelength. If the optical carriers are spaced with an identical wavelength spacing, the optical combs will be time delayed after passing through a dispersive fiber, with each microwave comb copy at the output of the PD.
having different phase shift. Consequently, the microwave-comb copies will interfere. For a well-designed time-delay structure, the desired microwave harmonic will have the highest output due to constructive interference while the other harmonics will be suppressed. An analysis is performed, which is verified by a proof-of-concept experiment. A microwave signal that is tunable from 16.8 GHz to 27 GHz is generated. The performance of the generated signal in terms of stability and phase noise is also evaluated.

The remainder of the paper is arranged as follows. In Section II, a theoretical analysis is presented. The generation of a microwave signal with different numbers of optical carriers and comb lines are simulated. In Section III, a proof-of-concept experiment is presented. A tunable microwave signal from 16.8 GHz to 27 GHz is generated and its phase noise performance is evaluated. In Section IV, a conclusion is drawn.

II. PRINCIPLE

The proposed structure is shown in Fig. 1. As can be seen, multiple optical carriers, spaced with an identical wavelength spacing, are sent to an optical comb generator to generate multiple optical combs. Here, an intensity modulator (IM) and a phase modulator (PM) in cascade are used to form an optical comb generator [14]. A dispersive fiber is then used to provide a wavelength-dependent time delay. Consequently, the multiple time-delayed optical combs are sent to a PD to generate a microwave signal.

![Structure for microwave or terahertz wave generation based on time-delayed optical combs](image)

Fig. 1. The structure for microwave or terahertz wave generation based on time-delayed optical combs. TLS: Tunable laser source, OC: Optical coupler, IM: Intensity modulator, PM: Phase modulator, PD: Photodetector, PS: RF phase shifter.

Indeed, the proposed structure in Fig. 1 is similar to the structure of a photonic microwave delay-line filter [15]. It is known that a photonic microwave delay-line filter generates multiple time-delayed copies of the driving microwave signal [16][17]. Due to constructive and destructive interferences, a periodic spectral response is generated. For the same filter structure, if the microwave signal is multiple time-delayed microwave combs, only the comb line with its frequency corresponding to the constructive interference will be obtained at the output of the system and the other comb lines will be suppressed. The key advantage of this system is that the photonic microwave delay-line filter is tunable which enables the generation of a frequency tunable microwave or sub-terahertz wave. This is the key motivation of using a photonic microwave delay-line filter structure for frequency-tunable microwave generation.

The optical field $E_1(t)$ at the output of the optical coupler is given by

$$E_1(t) = A_p e^{j\omega_p t} + A_2 e^{j\omega_2 t} + \ldots + A_p e^{j\omega_p t}$$  \hspace{1cm} (1)

where $A_p$ and $\omega_p$ are the amplitude and frequency of the $p$-th optical carrier, respectively, and $P$ is the total number of optical carriers. For a flat comb with $2N+1$ optical comb lines, the optical field $E_2(t)$ at the output of the optical comb is given by

$$E_2(t) = A_1 \sum_{n=-N}^{N} e^{j(\omega_1 + n\omega_m) t} + A_2 \sum_{n=-N}^{N} e^{j(\omega_2 + n\omega_m) t}$$

$$+ \ldots + A_p \sum_{n=-N}^{N} e^{j(\omega_p + n\omega_m) t},$$  \hspace{1cm} (2)

The transfer function of the dispersive fiber is given by

$$H(j\omega) = \exp\left(j \frac{1}{2} \beta_2 L \omega^2\right),$$

where $\beta_2$ (ps/nm/km) and $L$ are the dispersion coefficient and the length of the fiber, respectively. The optical field $E_3(t)$ at the output of the dispersive fiber is thus given by

$$E_3(t) = A_1 e^{j\omega_1 t} \sum_{n=-N}^{N} e^{j n\omega_m t} e^{j \frac{1}{2} \beta_2 L (\omega_1 + n\omega_m)^2}$$

$$+ A_2 e^{j\omega_2 t} \sum_{n=-N}^{N} e^{j n\omega_m t} e^{j \frac{1}{2} \beta_2 L (\omega_2 + n\omega_m)^2}$$

$$+ \ldots + A_p e^{j\omega_p t} \sum_{n=-N}^{N} e^{j n\omega_m t} e^{j \frac{1}{2} \beta_2 L (\omega_p + n\omega_m)^2}.$$  \hspace{1cm} (3)

The current at the output of the PD is

$$i(t) = \Re\{E_3(t) E_3^*(t)\},$$  \hspace{1cm} (4)

where $\Re$ is the responsivity of the PD. Here, we consider a case where the frequency spacing between any two optical carriers is larger than the PD bandwidth. Thus, the beating between any two optical carriers will not be detected by the PD. We further consider that this condition is also applied to the comb lines of the optical combs carried by different optical carriers. Mathematically, this condition is expressed as

$$(\omega_x - N\omega_m) - (\omega_y + N\omega_m) > B$$  \hspace{1cm} (5)

where $x, y \in \{1, 2, \ldots, P\}, P$ is the number of the optical carriers.
carriers, and \( B \) is the bandwidth of the PD. It then follows that the PD current is given by

\[
i(t) = |A|^2 \sum_{n=-N}^{N} e^{j\omega_n t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)} 
\sum_{m=-N}^{N} e^{-j\omega_m t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)} \]

\[
+ |A|^2 \sum_{n=-N}^{N} e^{j\omega_n t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)} 
\sum_{m=-N}^{N} e^{-j\omega_m t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)} 
\]

\[
+ \cdots
\]

\[
+ |A|^2 \sum_{n=-N}^{N} e^{j\omega_n t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)} 
\sum_{m=-N}^{N} e^{-j\omega_m t} e^{-j\frac{1}{2} \beta_L (n + m \alpha_p \omega_m)}
\]

(6)

As can be seen from (6), each optical carrier generates precisely the same microwave components, but with different phase shifts. The microwave components at the same frequency will interfere either constructively or destructively, depending on the relative phase shifts, which can be controlled by adjusting the wavelength spacing of the optical carriers or the dispersion of the dispersive fiber.

Note that the structure shown in Fig. 1 is in fact a photonic microwave delay-line filter if the optical comb generator is replaced by an IM that is biased at the quadrature point (while holding all parameters, as in Fig. 1). The transfer function of the associated photonic microwave filter is given by

\[
T(j\omega_p) = \sum_{p=1}^{P} A_p e^{j\frac{1}{2} \beta_L (2\omega_p, \alpha_m \omega_m)}
\]

(7)

where \( P \) is again the total number of the optical carriers, and \( \omega_p \) and \( \omega_m \) are the \( p \)-th optical carrier and input microwave frequencies, respectively.

A simulation of the proposed system is performed, in which the number of optical comb lines is 17 (\( N = 8 \)), and the number of optical carriers are 1, 3 and 10. In the simulation, the frequency of the input microwave drive signal is 6 GHz, the optical wavelength spacing is 0.171 nm, and the length of a dispersive fiber is 16 km and the dispersion coefficient is \( \beta_2 = 20 \text{ ps}^2/\text{km} \). The amplitude distributions of the generated microwave harmonics are shown in Fig. 2. Assume that the microwave signal to be generated is the \( J \)-th harmonic, the electrical spurious suppression ratio (SSR), defined as the ratio between the amplitude of the \( J \)-th harmonic and the highest amplitude of the other harmonics, is the largest for the case where 10 optical carriers are employed. The reason of the largest SSR is that the employment of 10 optical carriers makes the photonic microwave delay-line filter have the best frequency selectivity. The frequency responses of the associated photonic microwave delay-line filter with 3 and 10 optical carriers are shown in Fig. 3. It is clearly seen that the filter with 10 optical carriers has better frequency selectivity that with 3 optical carriers.

Note that the multiplication factor can be further increased by increasing the number of the optical comb lines (which is readily achievable). Thus, the multiplication factor is limited only by the bandwidth of the PD.
following condition
\[
D \cdot FSR = M \cdot f_m, \tag{8}
\]
where \(D\) is the number of the passband, and \(M\) is the desired multiplication factor. For example, in the simulations presented in Figs. 2 and 3, where \(FSR = 11.25 \text{ GHz}\), it can be easily shown that the only peak number and multiplication factor which satisfy condition (8) are \(D = 8\) and \(M = 15\). This means that the 15\(^{th}\) harmonic is selected by the 8\(^{th}\) passband and the other harmonics are not fall in any of the passbands and will be suppressed.

III. EXPERIMENT

A proof-of-concept experiment is performed based on the setup shown in Fig. 1. In the experiment, the photonic microwave delay-line filter is implemented with 3 taps by using three optical carriers. The wavelengths of the optical carriers are \(\lambda_1 = 1545.2 \text{ nm}\), \(\lambda_2 = 1548.9 \text{ nm}\), and \(\lambda_3 = 1552.6 \text{ nm}\) with a wavelength spacing of 3.7 nm. An IM and a PM (both with a bandwidth of 20 GHz) are cascaded to form an optical comb generator. A tunable RF phase shifter (PS) is utilized to ensure the generation of a flat optical comb. A single-mode fiber (SMF) of a length of \(L = 16 \text{ km}\) and \(\beta_2 = 20 \text{ ps}^2/\text{km}\) is used as a dispersive element. A PD with a bandwidth of 30 GHz is utilized to detect the microwave signal.

The generated optical comb at the optical carrier \(\lambda_1\) is shown in Fig. 4. Here, the microwave drive signal with a frequency of \(f_m = 5.6 \text{ GHz}\) is applied to the IM and PM, as shown in Fig. 1. An RF amplifier is used to increase the power of the microwave drive signal to the IM to generate a higher number of optical comb lines.

The optical combs carried by the three optical carriers at \(\lambda_1, \lambda_2\) and \(\lambda_3\) are shown in Fig. 5. Since the wavelength spacing is 3.7 nm, which corresponds to a beat frequency of 461.6 GHz, it is too high to be detected by the PD. Therefore, the condition in (5) (i.e., the beating between any two different optical carriers or between any two optical comb lines from different optical carriers cannot be detected) is guaranteed.

We first show the operation of the system when only a single optical carrier is employed. Since only one single carrier is employed, the photonic microwave delay-line filter has a single tap; therefore, the frequency response is flat with not frequency selectivity. Fig. 6 shows the generated microwave signal when only a single optical carrier at \(\lambda_1\) is on while the other two carriers are off. Here, limited by the bandwidth of the PD, up to the fifth harmonic are detected. As can be seen, despite the flat optical comb, the microwave harmonics do have a shape. In addition to the third-order harmonic at 16.8 GHz is generated, other harmonics are also generated.

Then we switch on all the three optical carriers. The photonic microwave delay-line filter has three taps. Fig. 7 shows the detected microwave signal when all the three optical carriers are on. As can be seen, only the third harmonic at 16.8 GHz is detected. This is because the frequency response of the filter is designed such that the frequency corresponding to the third-order harmonic has a resonance peak and the other harmonics are suppressed.
To better explain the result shown in Fig. 7, we perform a measurement of the frequency response of the associated photonic microwave delay-line filter with the same parameters. As can be seen in Fig. 8, the frequency of the 3rd order harmonic is located at the peak of the frequency response of the associated photonic microwave delay-line filter while the frequencies of the first- and the second-order harmonics, are located at the nulls of the frequency response of the associated photonic microwave delay-line filter. This explains the suppression of the first- and the second-order harmonics, which is confirmed by the experimental result in Fig. 7.

The measurement in Fig. 8 is limited to 20 GHz because of the limited bandwidth of the used IM. We note also that the used network analyzer has a DC block, which explains the drop in the frequency response at DC, and that the connecting wires have a large attenuation at high frequencies, which explains the attenuation at high frequency.

The quality of the generated microwave signal is evaluated. Since the generated microwave signal is the beating of the optical comb lines within the same optical comb, the quality of the generated signal is determined by the quality of the microwave drive signal [18]. Fig. 9 shows a zoom-in view of the generated microwave signal. It can be seen that the 20-dB bandwidth is as small as 100 Hz, confirming the high quality of the generated microwave signal.

To further evaluate the quality of the generated microwave signal, the phase noise is measured, as shown in Fig. 10. Here, the phase noise of the microwave drive signal is also shown as a comparison. As can be seen the phase noise of the generated signal has about 9.5 dB degradation, which is consistent with the theoretical phase degradation due to the frequency multiplication. It is known that the phase noise degradation for a multiplication factor of $M$ is given by $10 \log_{10} M$. Since the multiplication factor is 3, the degradation is 9.5 dB, which is confirmed by the phase noise measurement in Fig. 10 [18]. Note that the phase noise degradation of the generated microwave signal at higher offset frequencies (>10^3 Hz) is more than 9.5 dB, which is resulted due to the additional phase noise from the generation system.

The tunability is also investigated. The frequency of the
generated signal can be tuned by tuning the frequency of the microwave drive signal. Fig. 11 shows the generated microwave signals when the frequency of the microwave drive signal is tuned at 7.8 GHz and 9 GHz. For the two cases, one optical carrier and three optical carriers are employed. As can be seen, when one optical carrier is employed, although a microwave signal at the third-order harmonic, 23.4 GHz and 27 GHz, is generated, the SRR is small since other harmonics are not sufficiently suppressed, as shown in Fig. 11(a) and (b). When the three optical carriers are all on, due to the selectivity provided by the photonic microwave delay-line filter, a microwave signal at the third-order harmonic is generated and other harmonics are suppressed, as shown in Fig. 11(c) and (d). A high SSR is maintained.

![Generated Microwave Signals](image)

Fig. 11. The generated microwave signals. (a) One optical carrier and the frequency of the microwave drive signal is 7.8 GHz. (b) One optical carrier and the frequency of the microwave drive signal is 9 GHz. (c) Three optical carriers and the frequency of the microwave drive signal is 7.8 GHz. (d) Three optical carriers and the frequency of the microwave drive signal is 9 GHz.

The stability of the operation is also studied. To do so, we allow the system to operate in a room environment for 3 hours. No visible changes in the microwave power are observed. The excellent stability is mainly due to the fact that the system has a structure with a finite impulse response (FIR), which enables a stable operation. In addition, the IM is biased at the quadrature point, which makes the system less sensitive to bias drift.

**IV. CONCLUSION**

We have proposed and experimentally demonstrated a novel method to generate a frequency-tunable microwave and sub-terahertz wave based on time-delayed optical combs. The fundamental operation of the system is to employ an optical delay-line structure to form a photonic microwave delay-line filter to select one of the beat frequencies without the need of optical or microwave filters. The key contribution of the work is that a tunable microwave signal with a large multiplication factor and high SSR can be generated with a greatly simplified structure. The frequency tunability was achieved by simply tuning the frequency of the microwave drive signal. The generation of a microwave signal at 24.3 and 27 GHz by applying a microwave drive signal at 7.8 GHz was demonstrated. The quality of the generated microwave signal was also evaluated. It was shown that the phase noise of the generated microwave signal was determined by the phase noise of the microwave drive signal for an offset frequency smaller than $10^3$ Hz. When the offset frequency is greater than $10^3$ Hz, the phase noise introduced by the generation system will play a more important role. The use of a low-noise PD will reduce the phase noise at higher frequency offset.

**REFERENCES**

Montasir Qasymeh received the BSc degree in electrical engineering from Mutah University, Mutah, Jordan, in 2003, the MSc degree in optical communication and photonic technologies from Politecnico di Torino, Turin, Italy, in 2005, and a PhD degree in electrical engineering from Dalhousie University, Halifax, NS, Canada, in 2010.

In 2010 he was awarded a MITACS Elevate Fellowship, he is currently a postdoctoral fellow at the Microwave Photonics Research Laboratory, School of Information Technology and Engineering, University of Ottawa, Ottawa, ON, Canada. His current research interests include microwave and terahertz photonics. He is also active in the field of nonlinear optics and electro-optic devices.

Wangzhe Li (S'08) received the BE degree in electronic science and technology from Xi'an Jiaotong University, Xi'an, China, in 2004, and the MSc degree in optoelectronics and electronic science from Tsinghua University, Beijing, China, in 2007. He is currently working toward the PhD degree in electrical and computer engineering at the Microwave Photonics Research Laboratory, School of Information Technology and Engineering, University of Ottawa, Ottawa, ON, Canada.

His current research interests include photonic generation of microwave and terahertz signals.

Jianping Yao (M’99-SM’01) received his PhD degree in Electrical Engineering in 1997 from the Université de Toulon, Toulon, France.

He joined the School of Information Technology and Engineering, University of Ottawa, Ontario, Canada, in 2001, where he is currently a Professor, Director of the Microwave Photonics Research Laboratory, and Director of the Ottawa-Carleton Institute for Electrical and Computer Engineering. From 1999 to 2001, he held a faculty position with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He holds a Yongqian Endowed Visiting Chair Professorship with Zhejiang University, China. He spent three months as an invited professor in the Institut National Polytechnique de Grenoble, France, in 2005.

His research has focused on microwave photonics, which includes all-optical microwave signal processing, photonic generation of microwave, mm-wave and THz, radio over fiber, UWB over fiber, fiber Bragg gratings for microwave photonics applications, and optically controlled phased array antenna. His research interests also include fiber lasers, fiber-optic sensors and bio-photonics. He is an Associate Editor of the International Journal of Microwave and Optical Technology. He is on the Editorial Board of IEEE Transactions on Microwave Theory and Techniques. Dr. Yao received the 2005 International Creative Research Award of the University of Ottawa. He was the recipient of the 2007 George S. Glinski Award for Excellence in Research. He was named University Research Chair in Microwave Photonics in 2007. He was a recipient of an NSERC Discovery Accelerator Supplements award in 2008. Dr. Yao has authored or co-authored over 300 papers, including over 170 papers in refereed journals and over 130 papers in conference proceedings.

Dr. Yao is a registered professional engineer of Ontario. He is a Fellow of the Optical Society of America and a Senior Member of the IEEE Photonics Society and IEEE Microwave Theory and Techniques Society.