# Optimization of Semi-Dynamic Lightpath Rearrangements in a WDM Network 

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#### Abstract

In this paper, we study the Routing and Wavelength Assignment (RWA) problem in a semi-dynamic scenario where rearrangements are conducted in a series of sessions after traffic demands vary. Unlike pure static RWA problems, each rearrangement scheme must consider established lightpaths in the previous session. A novel formulation of the WDM network rearrangement problem is used to minimize rejected new demands and rerouted lightpaths. This is done by coordinating the re-routing of existing lightpaths with the adaptation to varying demands. The Lagrangean Relaxation and Subgradient Method (LRSM) has been successfully used to solve the problem along with fairness consideration. The superior performance and reduced computation complexity of our algorithm are demonstrated in sample networks. In addition, we evaluate the benefit of using wavelength converters in a WDM network rearrangement. In contrast to previous studies with conclusions that wavelength converters are of little value in the static RWA problem, we show that wavelength converters improve network performance in a WDM network rearrangement.


Index Terms-RWA, optimization, fairness, semi-dynamic, Lagrangean relaxation, subgradient.

## I. Introduction

IN wavelength-routed Wavelength Division Multiplexing (WDM) networks, lightpaths need to be re-routed in response to traffic pattern changes, network failures or new network resource installations. When compared with the pure static Routing and Wavelength Assignment (RWA) problem, the lightpath rearrangement problem features a strong correlation between the existing and the projected RWA settings. Taking such correlation into account, the WDM network rearrangement problem has been studied in different scenarios, e.g., survivable re-routing [1], pure RWA rearrangement [2], single-hop broadcast network rearrangement [3], virtual topology rearrangement in ring networks [4], dynamic virtual topology adaptation in response to measured load imbalance [5], heuristics based virtual topology migration [6, 7, 8], and virtual topology design to minimize average packet hop distance [3]. As an extension of the static RWA problem we study the network rearrangement problem at the all-optical layer by minimizing the number of transitions given the physical network capacity/topology matrix, RWA matrix of existing lightpaths and the new lightpath demand matrix.

[^0]A WDM network rearrangement scheme usually consists of three phases [9]: i) using a rearrangement policy to decide whether rearrangement should be conducted; ii) selecting a new RWA setting based on a certain optimization objective; and iii) migrating from the current setting to a new RWA setting. We focus on the second phase in this paper, i.e., an algorithm to choose a new RWA setting. We assume that the rearrangement is triggered when a traffic threshold is exceeded or at a fixed time period (e.g., monthly or weekly). Our approach assumes that the transition from an existing RWA setting to a new setting is accomplished in a centralized manner as opposed to a distributed approach of deciding the lightpaths one at a time.

We study an off-line optimization process (i.e., not realtime), where lightpath demands are processed in batches. For every batch of lightpath demands, a lightpath rearrangement session is conducted. A network is required to remain unchanged until the next rearrangement, i.e., no lightpath is established or removed before the next rearrangement. New lightpath demands are accumulated for processing in the next rearrangement session. In every rearrangement session, lightpath demands are accepted based on the overall network resource availability, the new traffic pattern and the optimization objectives. To provision for newly accepted lightpath demands, existing lightpaths may have to be re-routed. Note that the WDM network rearrangement problem is different from the static or dynamic RWA problems. In a static RWA problem, the existing lightpaths are free to be re-routed for each rearrangement. However, in real network operations, a WDM network rearrangement scheme has to maintain existing lightpaths as much as possible, or it will cause disruptions to upper-layer traffic. In a dynamic RWA problem, lightpath demands arrive one-by-one and leave after a service time period. When a lightpath demand arrives, a decision has to be made immediately on whether the lightpath demand is accepted or rejected (blocked). Dynamic RWA is an online process, which generally does not allow re-routing of existing lightpaths.

One key issue of the WDM network rearrangement problem is to coordinate a new RWA setting with the existing one. A two-step rearrangement design approach was proposed in [10], where the first step is to find a new RWA setting that best matches a new traffic pattern, and the second step is to modify the obtained new RWA setting, so that it requires the least changes from the existing RWA setting. As a variation of this procedure, the second step was changed in [5] to choose the RWA setting that requires the least number of lightpaths. A multi-objective evolutionary algorithm was proposed by integrating the two steps into one dominant objective [11]
of achieving an optimum RWA performance (e.g., minimal blocking). However, the algorithm proposed [11] is heuristicbased. Since it is not a systematic optimization procedure, no bound can be obtained for the algorithmic performances, and thus there is no way to know how close the final value is to the optimum. Therefore the achievability of the overall objectives cannot be evaluated. For all the two-step approaches and their variations, a major challenge is to find the very limited number of RWA settings (out of many possibilities) to achieve the best RWA performance. As a result, the rearrangement options are very much limited when the first priority is set to the best RWA performance as in an independent static RWA by ignoring the correlations. Potentially, many rearrangement options that require much fewer changes (at a cost of a marginal increase in traffic blocking) are excluded right from the beginning because they cannot achieve the questionable objective of the best RWA performance. Moreover, within a two-step framework, it is difficult to study the tradeoffs between the performance of a new RWA setting and the changes from the existing RWA setting. A heuristic solution was proposed aiming at optimizing the two steps at the same time [12], but without considering the re-routing of existing lightpaths. Thus, the tradeoffs between the re-routing and the rejection of lightpath demands cannot be studied. The congestion relief and fairness for node pairs were not considered in [12] either. This challenge has motivated us to provide a solution to study their tradeoffs. Unlike other heuristic-based approaches, our algorithm provides a good theoretical bound to evaluate the optimality of the result. Our formulation is link-based, as opposed to the path-based formulation proposed in [12]. In the path-based formulation, the paths have to be pre-selected and the selection of the paths is an optimization problem by itself.

In one study of a broadcast WDM network rearrangement problem [3], the optimization objective was to maximize the rearrangement gain, which is the difference between a performance reward and the rearrangement cost. The performance bound was compared to the one derived from a greedy algorithm. In solving a predication-based on-line traffic rearrangement problem [6], the rearrangement gain of a wavelength-routed WDM network was used. However, it is still unknown how the network rearrangement schemes perform in a wavelength-routed WDM network, because no performance bound has been discovered yet. Although performance rewards and rearrangement costs were modeled as conflicting objectives in a wavelength-routed WDM network [2], its stochastic search-based algorithm cannot guarantee the convergence and hence the optimality of its result. Its informal formulation has prevented the development of formal mathematical solutions.

In this paper, we would like to quantitatively study the performance gain of using a limited number of wavelength converters with a limited conversion range in the WDM network rearrangement. To the best of our knowledge, the impact of a limited number and conversion range of wavelength converters has not been fully studied in the context of WDM network rearrangement. The previous studies on WDM network rearrangement assume either full wavelength conversion at all nodes $[1,2,5,7,10,11]$, or no wavelength conversion at all [4,

8, 12]. Unlike the static RWA problem where the wavelength conversion only demonstrates a marginal contribution [13, 14], the real value of wavelength conversion in the WDM network rearrangement will be demonstrated in this paper.

The inherent complexity of the WDM network rearrangement problem requires advanced optimization algorithms. The majority of previous studies use either heuristics [5, 8, 10, 12], or a branch-and-bound approach based on linear programming relaxation available from many commercial tools such as CPLEX [7], or other methods such as a stochastic searchingbased genetic algorithm [2]. Although useful observations are made from the study of small-scale networks, no general conclusions can be made about the optimality of the suboptimal solution for large networks [7]. Moreover, no performance bound is available for the previous approaches when applied to large-scale networks. Performance bounds have been derived only in special cases, e.g., for a network with limited tunability [15] and for a broadcast network [3]. Recently, the Lagrangean Relaxation and Subgradient Methods (LRSM) have demonstrated its potential in solving large-scale optimization problems such as the static RWA problem [13, 16, 17], the virtual path planning and packet routing problems in a layered network [18], and the two-layer routing problem [19, 20]. For example, the pure static RWA problem in a mesh network with limited wavelength conversion capability was studied in [13]. A link-based formulation was proposed, considering the fairness of demand acceptance among different node pairs. The LRSM framework was used to solve the proposed RWA problem. Great computational efficiency has been demonstrated when compared with other existing algorithms. Despite the advances made by the LRSM in the static RWA problems, the rearrangable network in the real world cannot directly adopt the same approach, because the static RWA assumes that all the existing lightpaths can be freely re-routed in the rearrangement.

We would like to study the WDM network rearrangement problem in a semi-dynamic scenario. A special consideration is made to co-ordinate a new RWA setting with the existing one. We shall optimize the demand rejection, the rearrangement, and the network congestion, as well as the fairness among node pairs. We shall also address the loadbalancing, as a proactive traffic engineering technique, because it can preserve critical network resources for future lightpath demands. Fundamentally, a WDM network rearrangement scheme is a reactive traffic engineering technique. When the proactive traffic engineering technique is integrated with such reactive traffic engineering technique, better long-term performance can be achieved. By extending the work presented in [13], we propose a penalty-based Integer Linear Programming (ILP) formulation by incorporating the correlation of two consecutive rearrangement sessions. We use the same policy as [12, 21], which stipulates no bandwidth reduction for a given node pair, unless the lightpath demand for the node pair is reduced in the new session. Our formulation allows the application of our LRSM solution framework, which has been proved to be highly effective.

Specifically, the main contributions of this paper are the following:

1) A new formal model correlating the existing and the
projected new RWA settings: the rearrangement gain is modeled as minimizing the penalty for rejecting lightpath demands and re-routing existing lightpaths. When selecting lightpath demands to be established in a RWA rearrangement session, we guarantee that the number of lightpaths for a given node pair is not reduced unless the lightpath demand between the node pair is reduced.
2) The unification of the classical static RWA problem [13] (where existing lightpaths are freely re-routed) and the semi-dynamic RWA problem: this is done by introducing the re-routing penalty coefficient.
3) The improvement on the fairness of lightpath demands among different node pairs: we develop an approach to fairly treat lightpath demands when insufficient network resources result in the dropping of some lightpath demands. With our fairness improvement, less node pairs suffer from excessive bandwidth shortage, and network resources are fairly allocated to all node pairs.
4) The enhancement of network load-balancing in a lightlyloaded network: our approach minimizes the utilization of the most demanding link so that lightpath demands are spread into the network in a balanced manner.
5) An optimization algorithm based on the LRSM: unlike heuristic-based approaches, our algorithm provides the theoretical performance bound and a feasible nearoptimal solution at the same time. The duality gap, which indicates the optimality of a near-optimal solution, is well controlled. Our approach demonstrates excellent performance and high efficiency.
6) The demonstration of much greater benefit of using wavelength converters in the WDM network rearrangement (when comparing to the static RWA problems): we investigate the performance gain when using limited number of wavelength converters with a limited conversion range. To our best knowledge, we are the first to quantitatively demonstrate the advantages of having wavelength conversion in a semi-dynamic WDM networks for simplifying the network operations, administration and management.
This paper is organized as follows. In Section II, network model and assumptions are introduced. The WDM network rearrangement problem is formulated in Section III. An LRSM-based solution method is provided in Section IV. The optimization algorithm is detailed in Section V. The results of numerical tests are presented in Section VI, including a comparison with previous results and the new performance evaluation of fairness. We conclude in Section VII.

For the remainder of this paper, the following notations and variables are used:
$e_{i j} \quad$ the physical fiber between node $i$ and node $j ; e_{i j} \in$ $\mathcal{E}$;
$f$ the number of 'dummy' demands, which is equal to $u\left(X_{s d}^{\prime}-N_{s d}\right)$;
$n_{i j} \quad$ an integer representing the number of wavelengths in the wavelength set $W_{i j}$. Note that $n_{i j}=n_{j i}$;
$s, d$ the source and the destination, respectively, of a lightpath;
$s_{s d n}$ the $n^{\text {th }}$ new lightpath demand between $(s, d)$;
$t_{s d n}^{\prime}$ the $n^{\text {th }}$ existing lightpath between $(s, d)$ from the previous session;
$v \quad$ the degree of the wavelength conversion;
$w_{i j c}$ the $c$ th wavelength channel on physical fiber $e_{i j} \quad\left(0<c \leq n_{i j}\right) ;$
$A$ the set of admission status of all the lightpath demand matrix, i.e., $\left\{\alpha_{s d n}\right\}$;
$A_{s d}$ the variable set $\left\{\alpha_{s d n}\right\}_{s d}$, the set of admission statuses of all the lightpath demands of $(s, d)$;
$B \quad$ the variable set $\left\{\beta_{s d n}\right\}$;
$D$ the number of source-destination pairs that have lightpath demands, but are not assigned any lightpath;
$D_{s d n}$ the total routing dual cost of $s_{s d n}$;
$E \quad$ the set representing all fiber links in the network;
$F_{i c} \quad$ the number of wavelength converters that convert a signal from wavelength $c$ to other wavelengths on node $i$;
$G \quad$ the penalty coefficient of congestion;
$H_{s d}$ the integer equal to $\max \left(N_{s d}, X_{s d}^{\prime}\right)$;
$I_{i}(c)$ the set of wavelengths, to which the traffic from wavelength $c$ can be converted on node $i$;
$N$ the number of nodes in the network;
$N_{s d}$ the number of lightpath demands between $(s, d)$;
$P_{s d k}$ defined as $\left(P-\left(H_{s d}-k\right) S\right),(k \geq 1)$, which means the penalty for rejecting one more lightpath demand of $(s, d)$ when there are already $k-1$ lightpaths demands rejected. If $f>0, P_{s d 1}=P_{s d 2}=\ldots=$ $P_{s d f}=0$;
$P_{s d}(k)$ the penalty coefficient for rejecting $k$ lightpath demands of $(s, d)$, which can be represented by $\sum_{h=1}^{k} P_{s d h} . P_{s d}(0)=0 ;$
$Q \quad$ the penalty coefficient for re-routing one existing lightpath;
$S \quad$ the step size of rejection penalty coefficient for fairness consideration;
$T$ the overall number of source-destination pairs that have the lightpath demands;
$\mathcal{V}$ the set representing all the nodes in the network;
$(\mathcal{V}, \mathcal{E})$ an undirected graph representing the DWDM network;
$W$ the number of wavelengths used in the network;
$W_{i j}$ the wavelength set $\left\{w_{i j c}, 0<c \leq n_{i j}\right\}$ available in the physical link $e_{i j}$;
$X_{s d}^{\prime}$ the number of existing lightpaths between $(s, d)$ from the previous session;
$Z \quad$ the overall number of $s_{s d n}$ 's in the network;
$* \alpha_{s d n}$ a 0-1 integer variable indicating the admission status of $s_{s d n}$, equal to zero, if $s_{s d n}$ is rejected; one otherwise;
$\beta_{s d n}$ a 0-1 integer variable indicating the re-routing status of $t_{s d n}^{\prime}$, equal to one, if $t_{s d n}^{\prime}$ is re-routed; zero otherwise;
$\gamma \quad$ a variable representing the congestion on all $e_{i j}$ ( $0 \leq \gamma \leq 1$ );
$\delta_{i j c}^{s d n}$ a 0-1 integer variable, representing the use of wavelength channel $w_{i j c}$ by lightpath $s_{s d n}$; (Note)
$\phi_{j, a b}^{s d n}$ a 0-1 integer variable, representing the use of wavelength converters; (Note)
$\Delta_{s d n}$ the variable set $\left\{\delta_{i j c}^{s d n}\right\}_{s d n}$, representing the wavelength assignment for $s_{s d n}$; (Note)
$\Delta \quad$ the variable set $\left\{\delta_{i j c}^{s d n}\right\}=\left\{\Delta_{\text {sdn }}\right\}$; (Note)
$\Phi_{s d n}$ the variable set $\left\{\phi_{j, a b}^{s d n}\right\}_{s d n}$, representing the wavelength converter assignment for $s_{s d n}$; (Note)
$\Phi \quad$ the variable set $\left\{\phi_{j, a b}^{s d n}\right\}=\left\{\Phi_{s d n}\right\}$. (Note)
Note: We also use an apostrophe to denote the same variable/set for the lightpaths from the previous rearrangement session. For example, the notation $\Delta_{s d n}^{\prime}$ represents the set $\Delta_{s d n}$ of the existing RWA of $t_{s d n}^{\prime}$ from the previous session.

## II. Network Operations, Modeling, and Assumptions

We consider a general WDM mesh network of $N$ nodes interconnected by $E$ fibers. Each fiber has $W$ non-interfering Wavelength Channels (WCs). Two nodes can be logically connected through a lightpath (optical channels) defined to be of a concatenated sequence of WCs of the same wavelength color [23]. Since there could be multiple lightpath demands between any source-destination pair, we allow more than one lightpath being set up between them. To achieve further flexibility, we shall allow several lightpaths of different wavelengths (colors) to be chained together by wavelength converters installed on the nodes to form a semi-lightpath (denoted by $s_{s d n}$ ) [24]. For simplicity, we shall use 'lightpath' in this paper to mean the semi-lightpath from now on.

Some of the most promising all-optical wavelength translation techniques, such as four-wave mixing in Semiconductor Optical Amplifiers (SOAs), have strong relations between the input and output wavelengths [22, 32]. The wavelength converters based on these technologies have some degree of wavelength dependency too, and thus we use the limited wavelength converter structure as that in [13, 14] to model these relationships. Without loss of generality, we assign the same index $c$ to all the WCs of the same wavelength color. The lightpath from a wavelength $c$ can be only converted to wavelengths in set $I_{i}(c)=\{c, c+1, \ldots,(c+(v-1))\}$ mod $W$, where $v$ is the degree of wavelength conversion and $W$ is the number of wavelengths supported by the fiber. Note that other wavelength converter architecture can be formulated in similar way by defining different $I_{i}(c)$. We use the current architecture in order to make a fair comparison with [13, 14].

We shall study dynamic rearrangements of a WDM network with lightpath demands scheduled in batches. For every scheduled batch of lightpath demands, an RWA rearrangement session is carried out. The network does not establish or remove any lightpaths for a period of time until the next rearrangement session.

We make the following assumptions:

1) All scheduled RWAs of the established lightpaths are fixed until the next rearrangement session. In the next session, the demand matrix might change and the network traffic can be re-routed accordingly, by taking into consideration of the existing network arrangement (i.e., the RWAs of the existing lightpaths). This is similar to other classic static RWA problems like [26].
2) All wavelength converters have a limited (but the same) conversion degree and a limited capacity.
3) Wavelength converters with the same index in a node have a share-per-node structure: The signals from the same wavelength use the same type of converter as in [27]. The wavelength converters that can convert wavelength $c$ to other wavelengths have the same index $c$.
4) The number of converters with the same index in one node is limited: This will be translated into Constraint (e) later to make our formulation more practical (although harder).
5) The WCs are bi-directional: The lights of the same wavelength can travel in both directions simultaneously.
6) One WC allows at most one lightpath being routed through in each direction: This will be stipulated by Constraint (c) later.

## III. Problem Formulation

The relationship between the number of demands $\left(N_{s d}\right)$ and the number of existing lightpaths $\left(X_{s d}^{\prime}\right)$ between $(s, d)$ is of great importance to the problem formulation. Therefore we propose the following Session Coordination Processing procedure, by relating each new lightpath demand with one existing lightpath, to formulate the network rearrangement and to facilitate a decompositional solution approach.

## A. Session Coordination Processing

We define new lightpath demands as the demands to be established (might as well be rejected) in the new session and existing lightpaths as the lightpaths already established in the previous session (including those to be abandoned). To facilitate the illustration, we introduce the variable $s_{s d n}$ to represent the $n$th new lightpath demand of source-destination pair $(s, d)$. Let $t_{s d n}^{\prime}$ represent the $n$th existing lightpath of ( $s, d$ ). Let $\alpha_{s d n}$ be a 0-1 integer variable, representing the rejection status of $s_{s d n}$, i.e., it has a value of 0 , if $s_{s d n}$ is rejected, and 1 if admitted. Note that all existing lightpaths are the lightpath demands that have been accepted in the previous session $\left(\alpha_{s d n}^{\prime}=1\right)$. For every $(s, d)$, we denote the number of new lightpath demands as $N_{s d}$ and the number of existing lightpaths as $X_{s d}^{\prime}$.

Before formulating the problem, we should first understand the relationship between the capacities of the two sessions. To ensure that the 'promised' capacity from the previous session does not suddenly 'disappear' in the new session, we should use the following rules:

Rule 1: If the number of new lightpath demands between ( $s, d$ ) is more than or the same as the number of existing lightpaths (i.e., $N_{s d} \geq X_{s d}^{\prime}$ ), the lightpath demands accepted in the new session must not be less than the previous session.

Rule 2: If there is less capacity demand in the new session between $(s, d)$, i.e., $N_{s d}<X_{s d}^{\prime}$, the lightpath demands accepted in the new session have to be equal to $N_{s d}$.

These two rules (formulated in Constraint (h) later) avoid the drastic rearrangement in the optical layer, resulting in less disturbance in the upper layer traffic and more stable network performances. Note that when $N_{s d}=0$, all existing lightpaths between $(s, d)$ will be disconnected in the new session (according to Rule 2).

One major difficulty of the formulation lies in the fact that for the same $(s, d), X_{s d}^{\prime}$ is generally different from $N_{s d}$, and we want to accommodate every new lightpath demand with an existing lightpath in order to relate the two consecutive sessions. At the same time, the rejection penalty assigning scheme ${ }^{1}$ in [13] cannot be used any more. The reason is that all new demands $s_{s d n}$ 's for the same $(s, d)$ in [13] are indistinguishable in that they cannot be differentiated from each other, while in this paper each $s_{s d n}$ must be identified by an associated existing lightpath $t_{s d n}^{\prime}$ taking on a different wavelength assignment.

To resolve the above difficulty, we introduce a variable $H_{s d}=\max \left(N_{s d}, X_{s d}^{\prime}\right)$ to relate the two adjacent sessions and to simplify the description. Because the new lightpath demands ( $s_{s d n}$ 's) between $(s, d)$ are all indistinguishable before they are associated with any existing lightpath, without losing generality, when $N_{s d} \geq X_{s d}^{\prime}$, we relate the new lightpath demand $s_{s d n}$ to an existing lightpath $t_{s d n}^{\prime}$, by using the same index $n$. Specifically, each new lightpath demand $\left(s_{s d n}\right)$, with the index $0<n \leq X^{\prime}{ }_{s d}$, corresponds to the existing lightpath $t_{s d n}^{\prime}$; the $s_{s d n}^{\prime} \mathrm{s}$ with $X_{s d}^{\prime}<n \leq N_{s d}$ are not associated with any existing lightpath. $\left(H_{s d}=\max \left(N_{s d}, X_{s d}^{\prime}\right)=N_{s d}\right.$ new lightpath demands in total) However, if $N_{s d}<X_{s d}^{\prime}$, the choice of which existing lightpaths to disconnect has to be optimized. In this case, we can create $H_{s d}=\max \left(N_{s d}, X_{s d}^{\prime}\right)=X_{s d}^{\prime}$ new lightpath demands between $(s, d)$ in the formulation $(f=$ $X_{s d}^{\prime}-N_{s d}$ new lightpath demands are 'dummy' demands), and since Constraint (h) requires the eventual number of accepted lightpath to be equal to $N_{s d}$, the optimization result will not be influenced by introducing more 'dummy' demands, and we can further make the formulation much easier by just associating every existing lightpath with a new lightpath demand. Based on the procedure above, we can simply use $H_{s d}=\max \left(N_{s d}, X_{s d}^{\prime}\right)$ new lightpath demands for every $(s, d)$ in the formulation.

## B. Objective and Constraints

Our formulation is penalty-based by penalizing the rejection of demands, the re-routing and the congestion. We adopt the same rejection penalty system introduced in [13] to consider the fairness, i.e., assigning increasing rejection penalties to the $s_{s d n}$ 's of the same $(s, d)$. Let $P_{s d}(k)$ be the penalty for rejecting $k$ lightpath demands of $(s, d), Q$ be the penalty for re-routing an existing lightpath (in other words, a lightpath is re-routed), and $G$ be the penalty for the congestion. They are used to formulate our objective function in the following.

1) Objective Function: Our objective function is to obtain $\min _{B, \Delta, \Phi}\{J\}$, where
$A, B, \Delta, \Phi$
$\mathbf{J} \equiv \sum_{(s, d)}\left[P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)+\sum_{0<n \leq H_{s d}} Q \cdot \beta_{s d n}\right]$
$+G \cdot \gamma$
The function $J$ is the overall penalty consisting of the rejection penalty, the re-routing penalty and the congestion

[^1]

Fig. 1. The illustration graph of the rejection penalty for $\mathrm{s}, \mathrm{d}$.
penalty. The summation of $P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)$ is the total rejection penalty for the overall lightpath demand in the network (See the following paragraph for more detailed explanations). The summation of $Q \cdot \beta_{s d n}$ penalizes the overall rearrangement, where the variable $\beta_{s d n}$ represents the rerouting status of an existing lightpath $t_{s d n}^{\prime}$ corresponding to an $s_{s d n}$ (see Constraint (g), where $\beta_{s d n}$ equals one if $t_{s d n}^{\prime}$ is re-routed, and 0 otherwise). The third term $G \cdot \gamma$ in the objective function represents the penalty for the congestion, where the variable $\gamma(0 \leq \gamma \leq 1)$ is defined to be congestion (similar to [25]) of the network. As seen in Constraint (f) later, it represents the largest percentage of the WCs used on all links.

Fig. 1 shows a typical rejection penalty assigning scheme for an example of an $(s, d)$ with 3 lightpath demands, where a rejection is penalized increasingly. Specifically, rejecting the first lightpath carries the least penalty of $P_{s d 1}=40$. Rejecting the second lightpath demand is penalized more with $P_{s d 2}=70$. Rejecting the last lightpath demand carries the highest penalty of $P_{s d 3}=100$. Our optimization algorithm always tries to obtain the lowest overall penalty in order to balance the rejection of the demands among all the $(s, d)$ 's. In order to reflect the fact that one cannot penalize every $s_{s d n}$ in the procedure of associating $s_{s d n}$ with the non-identical $t_{s d n}^{\prime}$, we denote $P_{s d}(k)=\sum_{h=1}^{k} P_{s d h}$ as the penalty for rejecting $k=\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]$ lightpath demands of $(s, d)$. Taking Fig. 1 as an example, when $\sum_{0<n<H_{s d}}\left[1-\alpha_{s d n}\right]=1$, we have $P_{s d}(1)=40$. Likewise we obtain $\bar{P}_{s d}(2)=110$, and $P_{s d}(3)=210$. Note that $P_{s d}(0)=0$.

In general, when there are $k$ - 1 lightpath demands already rejected, the penalty to reject one more lightpath demand is $P_{s d k}=\left(P-\left(N_{s d}-k\right) S\right)$, where $S$ is a constant rejection penalty step size. When $S=0, P_{s d}(k)=k \times P$ and no fairness is considered, i.e., the penalty for rejecting any lightpath demand is the same. To accommodate the source-destination pairs with $N_{s d}<X_{s d}^{\prime}$ (for which there are in total $f=X_{s d^{-}}^{\prime} N_{s d}$ 'dummy' demands), we can simply set $P_{s d 1}=P_{s d 2}=\ldots$

$$
\sum_{j \in V} \sum_{0<c \leq n_{i j}} \delta_{i j c}^{s d n}-\sum_{j \in V} \sum_{0<c \leq n_{j i}} \delta_{j i c}^{s d n}=\left\{\begin{array}{cl}
\alpha_{s d n} & \text { if } i=s  \tag{2}\\
-\alpha_{s d n} & \text { if } i=d \\
0 & \text { otherwise }
\end{array} \quad \forall(s, d), 0<n \leq H_{s d}\right.
$$

$=P_{s d f}=0$, so that $f$ demands will be naturally rejected, resulting in the least overall cost. In summary, one would have $P_{s d k}=\left(P-\left(H_{s d}-k\right) \cdot S\right)$ and if $f>0, P_{s d 1}=P_{s d 2}=\ldots=P_{s d f}$ $=0$.

Please note that the definition of the fairness is very network-operation-dependent and different fairness objectives can be achieved by adopting different rejection penalty assignment schemes. For example, if we set the rejection penalty for one demand to the direct cost (which is the potential increase of revenue by accepting this demand), we are then maximizing the overall revenue. On the other hand, the network operator might also want to consider other kinds of indirect costs, such as the complaints from the clients for the unfair assignments or the difficulties of scheduling traffic in the future. By assigning higher rejection penalty, some demand can be more difficult to reject than the others, and we can thus achieve the desired assignment fairness. The basis of our penaltybased optimization is that each objective (including fairness and congestion) can be evaluated in cost/money.

For the convenience of our study, we use the same simple definition as $[13,30]$ to measure how evenly the traffic acceptance is distributed among the source-destination pairs that have lightpath demands. To measure the fairness (according our definition) of the scheduling result, we introduce a performance measure disconnection ratio defined to be $D$ (the number of source-destination pairs that have lightpath demands, but are not assigned any lightpath) to $T$ (the overall number of source-destination pairs that have the lightpath demands). In essence, this measure gives us an idea on the percentage of the source-destination pairs (with lightpath demands) that are totally disconnected.
2) Constraints: The constraints can be classified into General Constraints and Session Relationship Constraints.

General Constraints: General Constraints are used to confines the network operation in an independent session, just like those in the classic static RWA problems. Since the existing lightpaths are the accepted lightpath demands from the previous session, they conform to the General Constraints.
a) Lightpath flow continuity constraints:

Lightpath continuity means that if a demand is admitted, the lightpath assigned to it has to be continuous along the path between the source-destination pair. Since the assigned lightpath terminates at the two end nodes, we have (2), where $\delta_{i j c}^{s d n}$ is a $0-1$ integer variable, which equals to one if WC $w_{i j c}$ is used by $s_{s d n}$, and zero otherwise. Note that $\delta_{i j c}^{s d n}$ equals 0 , if $\alpha_{s d n}=0$. If $s_{s d n}$ is accepted, (i.e. $\alpha_{s d n}=1$ ) at the source node ( $i=s$ ), there is one unit of flow going out of this node, thus equation (2) equals 1 . At the destination node $(i=d)$, there is a flow of 1 coming into this node, thus equation (2) equals -1 . Finally, at the intermediate nodes, equation (2) equals 0 due to the conservation of flows. If the $s_{s d n}$ is rejected (i.e., $\alpha_{s d n}=0$ ), equation (2) equals 0 at any node.
b) Wavelength conversion constraints:

$$
\phi_{j, a b}^{s d n}= \begin{cases}1 & \text { if } \exists m, k \in V \text { and } b \neq a, \delta_{m j a}^{s d n}=\delta_{j k b}^{s d n}=1  \tag{3}\\ 0 & \text { otherwise }\end{cases}
$$

where $\phi_{j, a b}^{s d n}$ is a 0-1 integer variable representing the use of wavelength converters by $s_{s d n}$ at node $j$ to convert a signal from wavelength $a$ to wavelength $b$. The value ' 1 ' means in-use and ' 0 ' not-in-use. These constraints are due to the lightpath flow continuity constraints in (a). They stipulate that a converter with index $a$ on an intermediate node $j$ is used only when different wavelengths are assigned to $s_{s d n}$ for the incoming and outgoing signals at this node.
c) Wavelength channel capacity constraints:

$$
\begin{equation*}
\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \delta_{i j c}^{s d n} \leq 1 \quad \forall(i, j), 0<c \leq n_{i j} \tag{4}
\end{equation*}
$$

These constraints restrict every WC on a fiber to have only one lightpath routed in the same direction.
d) Limited wavelength conversion degree constraints:

$$
\begin{equation*}
\delta_{i j c}^{s d n}=\sum_{y \in V} \sum_{w_{j y x} \in I_{j}(c) \cap W_{j y}} \delta_{j y x}^{s d n} \quad \forall(s, d), 0<n \leq H_{s d}, 0< \tag{5}
\end{equation*}
$$

$\mathrm{c} \leq n_{i j} ; \forall(i, j), j \neq d, i \neq s$
These constraints stipulate that a wavelength can only be converted to a certain set of wavelengths that are allowed by the wavelength converters and physical links.
e) Converter capacity constraints:

$$
\begin{equation*}
\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \phi_{i, c a}^{s d n} \leq F_{i c} \quad \forall i, 0 \leq c<W \tag{6}
\end{equation*}
$$

These constraints restrict the number of occupied converters with an index $c$ in node $i$ to be no more than the number of the converters available on this node.
f) Link congestion constraints:

$$
\begin{equation*}
\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \sum_{0 \leq c<n_{i j}} \delta_{i j c}^{s d n} \leq \gamma\left|W_{i j}\right| \tag{7}
\end{equation*}
$$

where $W_{i j}$ denotes the wavelength set available in the physical link $e_{i j}, \gamma$ represents the congestion (note that $0 \leq \gamma \leq 1$ ) and $|\cdot|$ denotes the number of elements in the set. Since all links are assumed to have the same number of wavelengths, $\left|W_{i j}\right|=W$. These constraints have the same meaning as $W \gamma=$ $\max _{(i, j)}\left\{\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \sum_{0 \leq c<n_{i j}} \delta_{i j c}^{s d n}\right\}$. We use this formulation to facilitate the mathematical solution.

Session Relationship Constraints: Session Relationship Constraints stipulate the relationship between the two consecutive sessions. In other words, the relationship between the existing lightpaths and the new lightpath demands.
g) Re-routing constraints:

$$
\beta_{s d n}= \begin{cases}1, & \text { if } \Delta_{s d n} \neq \Delta_{s d n}^{\prime} \text { and } 0<n \leq X_{s d}^{\prime}  \tag{8}\\ 0, & \text { otherwise }\end{cases}
$$

These constraints require $\beta_{s d n}$ be set to 1 , if a re-routing of an existing lightpath $t_{s d n}^{\prime}$ take place (i.e., the associated $s_{s d n}$ takes a different RWA from $\left.t_{s d n}^{\prime}\right)$. $\beta_{s d n}$ is set to 0 , if there is no re-routing or if $s_{s d n}$ is not associated with any existing lightpath.
h) Non-disconnection constraints:

$$
\left\{\begin{array}{c}
\sum_{0<n \leq N_{s d}} \alpha_{s d n}=N_{s d}, \text { if } N_{s d}<X_{s d}^{\prime}  \tag{9}\\
\sum_{0<n \leq N_{s d}} \alpha_{s d n} \geq X_{s d}^{\prime}, \text { if } N_{s d} \geq X_{s d}^{\prime}
\end{array} .\right.
$$

These constraints ensure that the bandwidth promised in the previous session is observed in the new session, by confining that the existing lightpaths (say $t_{s d n}^{\prime}$ ) can be disconnected in the new session only when there are less lightpath demands between $(s, d)$ in the new session. If there are more or the same number of lightpath demands between $(s, d)$ in the new session, i.e., $N_{s d} \geq X_{s d}^{\prime}$, the number of accepted new lightpaths cannot be less than the number of existing lightpaths ( $\sum_{0<n \leq N_{s d}} \alpha_{s d n} \geq X_{s d}^{\prime}$ ); if there are less lightpath demands between $(s, d)$ in the new session, i.e, $N_{s d}<X_{s d}^{\prime}$, only ( $X_{s d}^{\prime}$ $\left.-N_{s d}\right)$ lightpaths should be disconnected $\left(\sum_{0<n \leq N_{s d}} \alpha_{s d n}=\right.$ $N_{s d}$ ).

## IV. Solution Methodology

By properly relaxing some constraints, we will derive in this section the DP (Dual Problem), which can be decomposed into independent subproblems.

## A. The LR Solution Procedure

We first use the Lagrange multipliers $\xi_{i j c}, \lambda_{i c}, \pi_{i j}$ to relax respectively the wavelength channel capacity constraints (c), converter capacity constraints (e) and link congestion constraints (f). This leads to the following Lagrangean DP (Dual Problem):

$$
\begin{aligned}
& \max _{\xi, \lambda, \pi \geq 0} q=\min _{A, B, \Delta, \Phi, \gamma}\left\{\sum _ { ( s , d ) } \left[P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right.\right. \\
& \left.+\sum_{0<n \leq H_{s d}} Q \beta_{s d n}+G \gamma\right] \\
& +\sum_{(i, j)} \sum_{0<c \leq n_{i j}} \xi_{i j c}\left(\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \delta_{i j c}^{s d n}-1\right) \\
& +\sum_{i \in V} \sum_{0 \leq c<W} \lambda_{i c}\left(\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \phi_{i, c a}^{s d n}-F_{i c}\right) \\
& +\sum_{(i, j)} \pi_{i j}\left(\sum_{(s, d)} \sum_{0<n \leq H_{s d}} \sum_{0 \leq c<n_{i j}} \delta_{i j c}^{s d n}-\gamma W\right),
\end{aligned}
$$

subject to the constraints (a), (b), (d), (g) and (h), where $\xi, \lambda$, $\pi$ are respectively the vectors of Lagrange multipliers $\left\{\xi_{i j c}\right\}$, $\left\{\lambda_{i c}\right\},\left\{\pi_{i j}\right\}$.

After regrouping the relevant terms, the dual function leads to the following problem:

$$
\min _{A, B, \Delta, \Phi, \gamma}\left\{\sum _ { ( s , d ) } \left[P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right.\right.
$$

$$
\begin{align*}
& +\sum_{0<n \leq H_{s d}} Q \beta_{s d n}+\sum_{(i, j)} \sum_{0<c \leq n_{i j}} \delta_{i j c}^{s d n}\left(\xi_{i j c}+\pi_{i j}\right) \\
& \left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right\}_{i \in V} \sum_{0 \leq c<W} \sum_{i, j c} \sum_{i c} F_{j c}+\gamma\left(G-W \sum_{i, n_{i j}} \pi_{i j}\right\}
\end{align*}
$$

By using the fact that $\delta_{i j c}^{s d n}=\alpha_{s d n} \delta_{i j c}^{s d n}, \phi_{i, c a}^{s d n}=\alpha_{s d n} \phi_{i, c a}^{s d n}$ and $\beta_{\text {sdn }}=\alpha_{\text {sdn }} \beta_{\text {sdn }}$, we can rewrite (10) as:
$\min _{A, B, \Delta, \Phi, \gamma}\left\{\sum_{(s, d)}\left[P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right.\right.$
$+\sum_{0<n \leq H_{s d}} \alpha_{s d n}\left(Q \beta_{s d n}\right.$
$+\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n}$
$\left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right)$
$\left.+\gamma\left(G-W \sum_{(i, j)} \pi_{i j}\right)-\sum_{(i, j)} \sum_{0<c \leq n_{i j}} \xi_{i j c}-\sum_{i \in V} \sum_{0 \leq c<W} \lambda_{i c} F_{j c}\right\}$.
Since the last two terms are independent of the decision variables, the problem can be further simplified as:

$$
\begin{align*}
& \min _{A, B, \Delta, \Phi}\left\{\sum _ { ( s , d ) } \left[P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right.\right. \\
& +\sum_{0<n \leq H_{s d}} \alpha_{s d n}\left(Q \beta_{s d n}+\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n}\right. \\
& \left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right) \\
& +\min _{\gamma}\left\{\gamma\left(G-W \sum_{(i, j)} \pi_{i j}\right)\right\} \\
& =\sum_{(s, d)} \min _{A_{s d}}\left\{P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right. \\
& +\sum_{0<n \leq H_{s d}}\left[\alpha _ { s d n } \cdot \operatorname { m i n } _ { \beta _ { s d n } , \Delta _ { s d n } , \Phi _ { s d n } } \left(Q \beta_{s d n}\right.\right. \\
& +\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n} \\
& \left.\left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right)\right] \\
& +\min _{\gamma}\left\{\gamma\left(G-W \sum_{(i, j)} \pi_{i j}\right)\right\}, \tag{11}
\end{align*}
$$

which we shall refer to as RP (Relaxed Problem).


Fig. 2. Schematic depiction of the overall algorithm.

## B. RWSS and CGSS

We can see that RP is composed of two minimization subproblem sets. The first subproblem set RWSS (RWA Subproblem Set) is

$$
\begin{align*}
& \sum_{(s, d)} \min _{A_{s d}}\left\{P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right. \\
& +\sum_{0<n \leq H_{s d}}\left[\alpha _ { s d n } \cdot \operatorname { m i n } _ { \beta _ { s d n } , \Delta _ { s d n } , \Phi _ { s d n } } \left(Q \beta_{s d n}\right.\right. \\
& +\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n} \\
& \left.\left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right)\right] \tag{12}
\end{align*}
$$

subject to the constraints (a), (b), (d), (g) and (h). RWSS can be decomposed into source-destination-level sub-problems (denoted as $\mathbf{S D S}_{s d}$ ), each corresponding to one $(s, d)$ :

$$
\begin{align*}
\mathrm{SDS}_{s d}= & \min _{A_{s d}}\left\{P_{s d}\left(\sum_{0<n \leq H_{s d}}\left[1-\alpha_{s d n}\right]\right)\right. \\
& +\sum_{0<n \leq H_{s d}} \alpha_{s d n} \cdot I_{s d n}, \tag{13}
\end{align*}
$$

where $I_{s d n}$ (corresponding to every $s_{s d n}$ ) is defined as shown in (14), subject to the constraints (a), (b), (d), and (g). In RWSS, there are altogether $Z$ lightpath-level subproblems ( $I_{s d n}$ 's).

In the subproblem set CGSS (Congestion Subproblem Set), there is only one subproblem:

$$
\begin{equation*}
\min _{0 \leq \gamma \leq 1}\left\{\gamma\left(G-W \sum_{(i, j)} \pi_{i j}\right)\right\} \tag{15}
\end{equation*}
$$

## V. The LRSM Framework

The overall solution architecture is based on the LRSM framework and can be described in Fig. 2. The LRSM [13] basically solves the relaxed dual problem iteratively by adjusting the Lagrange multipliers. The feasible optimization result is obtained by applying the heuristic algorithm to the dual solution in every iteration. The algorithm stops when the stopping criterion is satisfied. We have modified and extended the LRSM framework by solving two independent subproblem sets in the DP, and developed a heuristic algorithm to obtain a feasible solution based on the dual solution. We will provide these details while summarizing the essentials.

## A. Solving RWSS and CGSS

The overall dual problem is optimized when the two subproblem sets (i.e., RWSS and CGSS) are optimized separately. Each subproblem in RWSS corresponds to one lightpath demand. CGSS has only a single subproblem of solving the congestion problem.

1) Solving RWSS: We first solve the $I_{s d n}$ in (14) without considering Constraint (h) or $\alpha_{s d n}$. After we obtain the results of all $I_{s d n}$ 's for a particular $(s, d)$, we launch a sorting process for the corresponding $\mathrm{SDS}_{s d}$ to comply with Constraint (h) and optimize on $\alpha_{s d n}$. We repeat that for all $(s, d)$.

In order to solve the subproblem introduced in Section IVB, we can rewrite $I_{s d n}$ in (14) as:

$$
\begin{equation*}
I_{s d n} \equiv \min _{\beta_{s d n}}\left\{Q \beta_{s d n}+\min _{\Delta_{s d n}, \Phi_{s d n}}\left\{D_{s d n}\right\}\right\}, \tag{16}
\end{equation*}
$$

subject to the constraints (a), (b), (d), and (g), where

$$
\begin{aligned}
D_{s d n} & =\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n} \\
& \left.\left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right)\right\}
\end{aligned}
$$

This can be solved by adapting the Modified Minimum Cost Lightpath (MMCSLP) in [13]. The overall solution to RWSS are depicted in Step A and Step B below.

Step A: Solve all $I_{s d n}$ 's first.
A.1): Construct the wavelength graph of the network
(see Part B of [13]).
A.2): Find a shortest path between the source and the destination columns using the MCSLP algorithm
(see Part C of [13]) to get $\min _{\Delta_{s d n}, \Phi_{s d n}}\left\{D_{s d n}\right\}$.
A.3): Check the dual cost,
$D_{s d n}^{\prime}=\sum_{(i, j)} \sum_{0<c \leq X_{i j}^{\prime}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{\prime} s d n$
$\left.\left.+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<X_{i j}^{\prime}} \lambda_{i c} \phi_{i, c a}^{\prime}{ }^{s d n}\right)\right\}$ for the $t_{s d n}^{\prime}$.
A.4): If $D_{s d n}^{\prime}<Q+\min _{\Delta_{s d n}, \Phi_{s d n}}\left\{D_{s d n}\right\}$,
A.4.1): set $\Delta_{s d n} \stackrel{\Delta_{s d n}, \Phi_{s d n}}{=} \Delta_{s d n}^{\prime}$ and $\beta_{s d n}=0$ (i.e., use the RWA of the existing lightpath).
Set $I_{s d n}=\min _{\Delta_{s d n}, \Phi_{s d n}}\left\{D_{s d n}\right\}$.
A.4.2): Otherwise, $\operatorname{set} \beta_{s d n}=1$ and
$I_{s d n}=Q+\min _{\Delta_{s d n}, \Phi_{s d n}}\left\{D_{s d n}\right\}$. The tie is broken arbitrarily.
After the $I_{s d n}$ value for every $s_{s d n}$ is obtained, we sort for every $(s, d)$ to solve $\mathrm{SDS}_{s d}$, in order to comply with Constraint (h) and to optimize on $\alpha_{s d n}$. The following is the

$$
\begin{equation*}
I_{s d n}=\min _{\beta_{s d n}, \Delta_{s d n}, \Phi_{s d n}}\left\{Q \beta_{s d n}+\sum_{(i, j)} \sum_{0<c \leq n_{i j}}\left(\xi_{i j c}+\pi_{i j}\right) \delta_{i j c}^{s d n}+\sum_{i \in V} \sum_{0 \leq c<W} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \lambda_{i c} \phi_{i, c a}^{s d n}\right\} \tag{14}
\end{equation*}
$$

sorting process, where $k$ is a temporary variable, representing the number of demands rejected for $(s, d)$.

Step B: (Sorting all $(s, d)$ ).
B.1): Sort all $s_{s d n}$ 's in a particular $(s, d)$ with respect to the $I_{s d n}$ value obtained from Step A. Initialize all $\alpha_{s d n}=0$, $k=0$.
B.2): If $N_{s d} \geq X_{s d}^{\prime}$, continue with Step B.3).

Otherwise go to Step B.5).
B.3): Set the $X_{s d}^{\prime} s_{s d n}$ 's with the lowest $I_{s d n}$ values accepted ( $\operatorname{set} \alpha_{s d n}=1$ ).
B.4): If all $s_{s d n}$ 's have been calculated, go to Step B.6).
B.4.1): Otherwise, for the rest $s_{s d n}$ 's, find the one with the highest $I_{s d n}$ value (A tie is broken arbitrarily).
B.4.2): If $I_{s d n}>P_{s d k}$, set $k=k+1$ and $\alpha_{s d n}=0$. Go to Step B.4).
B.4.3): Otherwise set $\alpha_{s d n}=1$. Go to Step B.4).
B.5): If $N_{s d}<X_{s d}^{\prime}$,
B.5.1): Set the $N_{s d} s_{s d n}$ 's with the lowest $I_{s d n}$ values accepted (set $\alpha_{s d n}=1$ ).
B.5.2): Set other $s_{s d n}$ 's rejected (set $\alpha_{s d n}=0$ ).
B.6): If not all source-destination pairs are calculated, go to B.1) for the next $(s, d)$; otherwise finish the DP minimization.

Through the above procedure, we can solve all $\mathrm{SDS}_{s d}$ 's optimally, and RWSS can thus be optimized.
2) Solving CGSS: To obtain the optimum of the congestion problem CGSS, we set $\gamma$ to 0 when $G-W \Sigma_{(i, j)} \pi_{i j}<0$, and to 1 when $G-W \Sigma_{(i, j)} \pi_{i j}>0$. If $G-W \Sigma_{(i, j)} \pi_{i j}=0, \gamma$ is set to 0 or 1 arbitrarily.

## B. Updating Lagrange Multipliers

Since there are integer variables involved in the formulation, we now employ the subgradient method [28] to solve the DP maximization.

The multiplier vector $z=(\xi, \lambda, \pi)$ are first updated by the following formula:

$$
z^{(h+1)}=z^{(h)}+\alpha^{(h)} g\left(z^{(h)}\right),
$$

where $z^{(h)}$ denotes the value of vectors $z$ obtained at the $h$ th iteration, and $\alpha^{(h)}$ denote the step size at the $h$ th iteration. The vector $g(z)$ is the subgradient of the dual function $q$ with respect to $z$, i.e. $g(z)=\{g(\xi), g(\lambda), g(\pi)\}$. The vectors $g(\xi)$, $g(\lambda)$, and $g(\pi)$ are composed of $g_{i j c}(\xi), g_{i c}(\lambda)$, and $g_{i j}(\pi)$ respectively, where

$$
\begin{aligned}
& g_{i j c}(\xi)=\sum_{(s, d)} \sum_{0<n \leq N_{s d}} \delta_{j i c}^{s d n}-1, \\
& g_{i c}(\lambda)=\sum_{(s, d)} \sum_{0<n \leq N_{s d}} \sum_{j \in V} \sum_{0 \leq a<n_{i j}} \phi_{i, c a}^{s d n}-F_{i c},
\end{aligned}
$$



Fig. 3. The flowchart of the heuristic algorithm to obtain a feasible solution.

$$
g_{i j}(\pi)=\sum_{(s, d)} \sum_{0<n \leq N_{s d}} \sum_{0 \leq c<n_{i j}} \delta_{i j c}^{s d n}-W \gamma
$$

The step size is determined by

$$
\begin{equation*}
\alpha^{(h)}=\mu \times \frac{q^{U}-q^{(h)}}{g^{T}\left(z^{(h)}\right) g\left(z^{(h)}\right)}, \tag{17}
\end{equation*}
$$

where $q^{U}$ is an estimate of the optimal solution (which generally takes the best value of the objective function $J$ obtained), and $q^{(h)}$ is the value of $q$ at the $h$ th iteration. The parameters $\mu$ and $q^{U}$ are changed adaptively as the algorithm converges. Convergence can also be sped up by using the method in [13].

## C. Constructing a Feasible RWA Scheme

Since some of the constraints are relaxed by the Lagrange multipliers, the solution to DP might have an infeasible routing scheme. In other words, some wavelength channel capacity constraints in (c), converter capacity constraints in (e), or link congestion constraints in (f) might have been violated at some links and nodes. On the other hand, other constraints will not be violated because this is guaranteed by the solution to RWSS in (12) and the solution to CGSS in (15).

To construct a feasible routing scheme, a heuristic algorithm has to be employed to decide which lightpath demands should be rejected or re-routed and which paths the re-routed lightpath demands should take. The heuristic we propose is shown in the flowchart in Fig. 3, which is similar but more complicated than the structure in [13].

The most prioritized lightpath will be naturally deployed first to use the critical resources. Given two colliding lightpath
demands $c 1$ and $c 2$, corresponding to source destination pairs $(s 1, d 1)$ and $(s 2, d 2)$, respectively. For convenience, let $c 1<$ $c 2$ mean that $c 1$ has a higher priority than $c 2$. Then the following priority rules are used to determine which lightpath demands should occupy the 'critical resources'.

1) If $(s 1, d 1)$ is different from $(s 2, d 2)$, then use Rule 2. Otherwise, if $c 1$ has lower dual cost than $c 2$, then $c 1<$ $c 2$; and vice versa. The tie is broken arbitrarily.
2) If $(s 1, d 1)$ has more 'must-accept' demands (i.e., $\left.\min \left(N_{s d}, X_{s d}^{\prime}\right)-\sum_{0<n \leq N_{s d}} \alpha_{s d n}\right)$ to satisfy Constraints (h) than ( $s 2, d 2$ ), then $c \overline{1}<c 2$; and vice versa. If $c 1=c 2$, then use Rule 3.
3) If $(s 1, d 1)$ has fewer lightpaths accepted than the demands (i.e., $\sum_{0<n \leq N_{s d}} \alpha_{s d n}<N_{s d}$ ) while ( $s 2, d 2$ ) does not, then $c 1<c \overline{2}$; and vice versa. If $c 1=c 2$, then use Rule 4.
4) If $c 1$ is associated with an existing lightpath while $c 2$ is not, then $c 1<c 2$; and vice versa. If $c 1=c 2$, then use Rule 5.
5) If $c 1$ is accepted in the dual solution while $c 2$ is not, then $c 1<c 2$; and vice versa. The tie is broken arbitrarily.
After determining the priority, we find a feasible route based on the dual solution for each $s_{s d n}$ by launching the Feasible Route Searching Algorithm (FRSA) below. The algorithm is implemented on the Wavelength Graph (WG) of the network, which is formed by separating every node into $W$ vertices representing the wavelengths available on the node, and connecting these vertices with arcs representing the WCs and the wavelength converters. The details to form the WG can be found in Appendix B of [13], and the weight $\left(W E_{i j c}\right)$ corresponding to a WC $w_{i j c}$ is estimated by $W E_{i j c}=\sigma \cdot\left(G \cdot M_{i j}\right) / W+\rho \cdot\left(P \cdot m_{i j c}\right)$, where $M_{i j}$ and $m_{i j c}$ are the total number of lightpaths routed through $e_{i j}$ and $w_{i j c}$ in DP solution respectively; $\sigma$ and $\rho$ are the estimate coefficients for the congestion and the rejection respectively. We use $\sigma=0.01$ and $\rho=0.005$ in the current implementation. This is a procedure similar to the procedure in [13].

The Feasible Route Searching Algorithm (FRSA) [13]: Step 1 (Check the Solution of the DP):

Check if the Wavelength Assignment (WA) derived from the dual solution is viable, i.e., no WC in the WA is occupied by some other lightpath demands while at the same time there are still wavelength converters in the WA.
1.1) If the WA from the dual solution is viable, the WA is assigned to this lightpath. Then go on to the next lightpath demand in the priority list, and repeat Step 1.
1.2) If the WA from the dual solution is not viable, go to Step 2.

## Step 2 (Find the WA in the Same Route):

Find a viable WA for the lightpath demand in the same route derived from the dual solution, but with a different WA. The procedure of this step is the same as MMCSLP [13]. However, the WG is only made of the fibers and nodes traversed by the lightpath demand in the dual solution.
2.1) If a viable WA can be found, this lightpath demand shall monopolize this WA. Then go on to the next
lightpath demand demand, and repeat Step 1.
2.2) If no viable WA can be found, go to Step 3.

Step 3 (Find the WA for One Lightpath Demand with the MEWA):

Apply the Modified Esau-Williams algorithm (MEWA) ${ }^{2}$ [13] to find a viable RWA for the lightpath demand.
3.1) If a viable WA can be found, this lightpath demand shall monopolize all the WCs and converters assigned by this WA. Then go on to the next lightpath demand, and repeat Step 1.
3.2) If no viable WA can be found, reject the lightpath demand. Then go on to the next lightpath demand, and repeat Step 1.
After we obtain the RWA for $s_{s d n}$, if $s_{s d n}$ is to be associated with an existing lightpath $t_{s d n}^{\prime}$ (see Section III-A), we have to decide whether $s_{s d n}$ should be re-routed (or use the RWA of the existing lightpath), by using the following estimation procedure:

1) If there is a feasible route found in FRSA, we compute the estimated cost of $s_{s d n}$ as $C_{s d n}=$ $\sum_{(i, j)} \sum_{0<c \leq X_{i j}^{\prime}} W E_{i j c} \delta_{i j c}^{s d n}$. Otherwise, $C_{s d n}=\infty$.
2) We compute the estimate cost of the existing route as $C_{s d n}^{\prime}=\sum_{(i, j)} \sum_{0<c \leq X_{i j}^{\prime}} \delta_{i j c}^{\prime}{ }^{s d n} W E_{i j c}$. The 'gain' for rerouting $s_{s d n}$ is then computed as $G_{s d n}=C_{s d n}^{\prime}-C_{s d n}$.
3) If $\varsigma \cdot G_{s d n}>Q$, then use the newly found route $\left(\beta_{s d n}=\right.$ $1)$. Otherwise use the existing route ( $\beta_{s d n}=0$ ), where $\varsigma$ is the estimated coefficient, and we use $Q / 6$ in the current implementation.
In the last stage of the heuristic algorithm, we use the following Insolvency Disconnection Procedure to disconnect the 'insolvent' $s_{s d n}$ 's. Let $M C$ be the number of the most congested links (i.e., the number of links having the same congestion $\gamma$ ). The 'congestion cost' of going through one most-congested link is defined as $C C=G /(W \cdot M C)$. The 'congestion cost' of $s_{s d n}$ is defined as $C C_{s d n}=N M_{s d n} \cdot C C$, where $N M_{s d n}$ is the number of most congested links $s_{s d n}$ goes through. The 'revenue' of $s_{s d n}$ (denoted as $R_{s d n}$ ) equals its rejection penalty $P_{s d k}$ (See the definition of $P_{s d k}$ in Section III-B). Then an $s_{s d n}$ is 'insolvent', if $R_{s d n}<C C_{s d n}$, and will be disconnected according to the following procedure.
4) If $\gamma=0$, stop the heuristic algorithm. Otherwise calculate $M C$ and $C C_{s d n}$.
5) For each of the $M C$ most congested links $e_{i j}$, find one 'insolvent' $s_{s d n}$ that goes through $e_{i j}$, and store it into a list $L$.
6) If each of the $M C$ most congested links has at least one 'insolvent' $s_{s d n}$, then disconnect all the $s_{s d n}$ 's stored in list L. Go to Step.1;
Otherwise terminate the heuristic algorithm.

## D. Computation Complexity

The dual solution is minimized when both RWSS and CGSS are minimized (See Section V-A). The complexity of solving

[^2]

Fig. 4. NSFNET with 14 nodes and 21 links.

| 0 | 1 | 0 | 3 | 1 | 2 | 1 | 0 | 2 | 0 | 0 | 2 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 2 | 0 | 3 | 1 | 2 | 1 | 3 | 0 | 0 | 0 | 3 |
| 1 | 2 | 0 | 3 | 0 | 1 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 0 |
| 3 | 2 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 2 | 3 |
| 1 | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 2 | 0 | 1 | 2 | 3 |
| 1 | 2 | 1 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 1 | 1 | 2 |
| 2 | 2 | 3 | 2 | 0 | 3 | 0 | 0 | 3 | 1 | 2 | 0 | 0 | 0 |
| 3 | 1 | 2 | 1 | 1 | 0 | 1 | 0 | 0 | 3 | 2 | 0 | 3 | 0 |
| 3 | 0 | 1 | 2 | 3 | 0 | 1 | 0 | 0 | 2 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 3 | 0 | 2 | 0 | 1 | 0 | 2 | 0 | 1 | 1 |
| 1 | 0 | 0 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 | 3 | 2 | 2 |
| 1 | 0 | 3 | 3 | 0 | 1 | 0 | 3 | 2 | 0 | 1 | 0 | 1 | 3 |
| 3 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 1 | 0 | 0 | 3 | 0 | 1 |
| 2 | 0 | 3 | 1 | 2 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 3 | 0 |

Fig. 5. Lightpath demand matrix for the coming rearrangement session.
each $I_{s d n}$ is the same as the complexity of MMCSLP [13], i.e., $O((N+W) N W)$. By grouping the $I_{s d n}$ 's with the same source node, we can solve all $I_{\text {sdn }}$ 's with $O\left((N+W) N^{2} W\right)$. The worst-case time complexity for the sorting operation is $\Sigma_{(s, d)} O\left(H_{s d} \log \left(H_{s d}\right)\right)$, and thus the overall complexity to solve DP is $O\left((N+W) N^{2} W\right)+\Sigma_{(s, d)} O\left(H_{s d} \log \left(H_{s d}\right)\right)$.

The worst case complexity for the Insolvency Disconnection Procedure is $O\left(E W \log (E)+Z W^{2}\right)$, where $E$ is the number of links, and $Z$ is the number of $s_{s d n}$ 's. The complexity of the rest part of the heuristic algorithm is $O\left(Z\left(Z \log Z+(N W)^{2}\right)\right)$, which has been analyzed in [13]. Therefore the overall complexity of the heuristic algorithm is $O\left(Z^{2} \log Z+Z\right.$ $\left.(N W)^{2}+E W \log (E)\right)$.

## E. Evaluation of a Feasible Routing Scheme

We use the duality gap, which is defined as $\left(J^{*}-q^{*}\right)$ (generally $>0$ ) [28], to evaluate the performance of our routing scheme. The value of the objective function $J$ of any feasible routing scheme obtained is an upper bound on the optimal objective $J^{*}$. The optimum value of the dual function $q^{*}$, on the other hand, is a lower bound of $J^{*}$. The value $(J-q)$ provides an upper bound to the duality gap, so that even without obtaining the exact optimum, we can still know that the distance of the sub-optimal solution is within a certain range from the optimum.

## VI. Verification and Performance Evaluation

We compute the duality gap on the 14-node NSFNET topology as shown in Fig. 4 to test the capability of our proposed algorithm. Each link has only one fiber and each fiber has $W$ WCs. The lightpath demands are shown in Fig. 5, where the horizontal and vertical indexes are the source and destination nodes, respectively. Specifically, the number

| 0 | 1 | 3 | 1 | 1 | 1 | 3 | 0 | 2 | 0 | 1 | 2 | 0 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 1 | 0 | 1 | 3 |
| 3 | 2 | 0 | 3 | 0 | 1 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 0 |
| 3 | 1 | 0 | 0 | 1 | 1 | 2 | 3 | 2 | 2 | 1 | 2 | 1 | 3 |
| 1 | 3 | 0 | 2 | 0 | 1 | 0 | 2 | 0 | 3 | 0 | 1 | 1 | 3 |
| 1 | 2 | 1 | 3 | 2 | 0 | 1 | 3 | 3 | 1 | 0 | 1 | 1 | 2 |
| 2 | 2 | 3 | 1 | 3 | 3 | 0 | 0 | 3 | 1 | 2 | 0 | 3 | 3 |
| 3 | 1 | 2 | 3 | 1 | 0 | 1 | 0 | 0 | 3 | 2 | 0 | 3 | 0 |
| 3 | 0 | 1 | 3 | 3 | 3 | 1 | 0 | 0 | 2 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 1 | 0 | 1 | 0 | 0 | 3 |
| 1 | 0 | 0 | 2 | 0 | 3 | 0 | 1 | 0 | 3 | 0 | 3 | 1 | 3 |
| 2 | 3 | 1 | 1 | 3 | 2 | 3 | 2 | 2 | 2 | 2 | 0 | 1 | 3 |
| 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 3 | 0 | 2 | 1 | 0 | 3 |
| 1 | 1 | 0 | 2 | 1 | 0 | 1 | 3 | 0 | 1 | 2 | 1 | 3 | 0 |

Fig. 6. Simulated lightpath demand matrix for the previous rearrangement session.
on the $i$ th row and the $j$ th column represents the number of lightpath demands from node $i$ to $j\left(N_{i j}\right)$, which takes random values between 0 and 3 . We first use the same optimization procedure as [13] on the traffic matrix shown in Fig. 6 for 100 iterations in order to generate the existing lightpaths from the previous session. The detailed RWA map of every existing lightpath have been documented in [29]. Most of the results from our algorithm in the NSFNET example can be obtained within 5 minutes running on a personal computer with Windows XP®, Centrino® 1.6 GHz CPU and 512 M RAM. The computation time complexity is similar to the algorithm in [13] because the same LRSM framework used. The readers are referred to [13] for the computation time testing, the large network application and the computation time reduction by reusing Lagrange multiplier values. Note that our optimization algorithm can optimize any input matrix and does not depend on the statistical properties of the incoming demands. Please also note that our algorithm can also be used to deal with the one-by-one style incremental/decremental traffic change, if the new lightpath demand matrix (comparing with the existing lightpath matrix) has one more lightpath demand or one less lightpath demand.

## A. Effect of Different Factors in The Semi-Dynamic Rearrangement

To simplify our study, we set the rejection penalty of the last lightpath demand for every $(s, d)$ to the same value (although not required), i.e., $P_{s d k}=P$, if $k=N_{s d}$ (see Section III-B). We first use number of wavelengths $W=11$, rejection penalty $P=100$, rejection penalty stepsize $S=2$, and congestion penalty $G=100$ to study the network behaviors under heavy traffic.

The new lightpath demands and the existing lightpaths are correlated through Constraints (g) and (h), as well as the rerouting penalty $Q$. Note that when $Q=0$, the situation is the same as the static RWA problems studied in [13] and [14], which means the existing lightpath can be freely re-routed without any penalty. Fig. 7 shows the trade-off between the number of rejected lightpath demands and the number of rerouted lightpaths, as $Q$ changes without wavelength converter $\left(F_{i c}=0\right)$ (the parameters are shown on the top of the graph). We can see that as $Q$ increases (before $Q$ reaches 400), more lightpath demands are rejected, and fewer lightpaths are rerouted. After $Q$ reaches 400, both values stopped changing,


Fig. 7. The number of rejected lightpath demands and the number of rerouted lightpaths.


Fig. 8. The comparison of different number of wavelength converters.
due to the hard constraints that have to be satisfied. This is similar to the dynamic RWA problems, where no existing lightpaths are allowed to be routed.

Fig. 8 shows the influence of $Q$ for different $F_{i c}$ (the number of converters) values. We can see in Fig. 8 that the contribution of the wavelength conversion around $Q=0$ is marginal (i.e., the curves of $F_{i c}=0$ and $F_{i c}=1$ are not far apart), which conforms to the results of [13] and [14]. However, as $Q$ increases, the contribution of the wavelength conversion is becoming more significant (increased by about $300 \%$ ). All curves increase with $Q$, until the limitation from other constraints sets in (around $Q=400$ ). Increasing the conversion capacity (from $F_{i c}=1$ to $F_{i c}=2$ ) or increasing the conversion degree $v$ (see Fig. 9) does not have any further impact. (Note that $v=1$ means no wavelength conversion capability.) We can thus come to the conclusion that the wavelength conversion results in fewer network re-routings, and it is worthwhile to install wavelength converters in a network that does not allow free rearrangements. For the wavelength conversion architecture in this paper, it is demonstrated that more wavelength converters


Fig. 9. Influence of different conversion degrees v.


Fig. 10. The congestion $\gamma$ and the number of rejected lightpath demands.
$\left(F_{i c}>1\right)$ or higher conversion degree $(v>2)$ does not generate better results than the lowest arrangement ( $F_{i c}=1, v=2$ ).

For the lightly-loaded network, we want to distribute the traffic in the network and thus need to minimize $G \gamma$. We can simply use the same traffic demand matrix and increase the number of wavelengths $(W)$ on the links to simulate the lightly-loaded network. Fig. 10 and Fig. 11 show the results obtained for $W=20$. We can see from Fig. 11 that our algorithm generates very good and consistent near-optimum results. The results are mostly within $3 \%$ of the optima. Fig. 10 shows the trade-off between the congestion $(\gamma)$ and the number of rejected lightpath demands. We can see that the congestion goes down and the number of rejected lightpath demands goes up as congestion penalty $G$ increases. We can also see the values vary in stages, because our optimization algorithm tends to distribute the traffic, and there are generally several the most congested links (of congestion ratio $\gamma$ ). So every time when $\gamma$ goes down, more lightpaths on several links are likely to be disconnected at the same time. We can see in Fig. 11 that $J$ and $q$ increase linearly with respect to


Fig. 11. The congestion penalty $G$ vs. the final results of $q$ and $J$.


Fig. 12. The rejection penalty step size $S$ vs. the number of rejected lightpath demands.
$G$. Before $G=18,000, J$ and $q$ increase in response to the increase of the rejections and the increase of $G$. After this point, no more lightpath can be rejected (see Fig. 10) under the Non-disconnection Constraints (h), and $G$ becomes the only contributing factor to the increase of $J$ and $q$ values. Therefore after $G=18,000$, the slope of the curves becomes less steep.

Note that when $P_{s d k}$ and $Q$ are set to 0 , we are optimizing the network congestion similar to [17, 25, 31]. As another interesting experiment, one can disable the congestion objective by simply setting $G$ to 0 . Choosing values for $G$ is quite experience-dependent, and one needs to try several values to find out the approximate congestion $(\gamma)$ in the optimization result and then to decide the best value (as we have done for the experiment in Fig. 10).

## B. Fairness Study

Fig. 12 to 14 demonstrate the results of the fairness study for $W=20, P=100, G=0$, and $Q=100$. We assign the penalty


Fig. 13. The rejection penalty step size $S$ vs. the disconnection ratio $D / T$.


Fig. 14. The rejection penalty step size $S$ vs. the final results of $q$ and $J$.
coefficient $G$ to zero here in order to minimize the influence from the congestion penalty. Fig. 13 shows the change of Disconnection Ratio ( $D / T$ ) defined in Section III-B-1. Similar to [13], when $S$ (rejection penalty stepsize) increases, the algorithm tends to make the rejection fairer (e.g., less totally disconnected pairs $D$ ), at the cost of more rejections. Fig. 14 shows $J$ and $q$ decrease linearly as $S$ increases. This is mostly because when $S$ increases, the overall rejection penalty decreases linearly at the same time. Fig. 14 again demonstrates that our algorithm is highly stable in obtaining the near-optimal results.

Fig. 15 shows another example of a similated lightpath demand matrix for the previous session and we use the same lightpath demand matrix (as Fig. 6) for the coming session. We use wavelength conversion ( $F_{i c}=1$ and $v=2$ ) and more wavelengths $(W=16)$ than the case we studied in Fig. 7. The result we obtained for the changing of $Q$ is shown in Fig. 16. We notice similar trade-off patterns to Fig. 7 as $Q$ increases. However, because there are more wavelengths in the network, the number of rejected demands is significantly lower. Because

| 0 | 1 | 0 | 3 | 1 | 2 | 1 | 0 | 2 | 0 | 0 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 2 | 0 | 3 | 1 | 2 | 1 | 3 | 0 | 0 | 0 |
| 1 | 2 | 0 | 3 | 0 | 1 | 2 | 3 | 1 | 3 | 1 | 2 | 2 |
| 3 | 2 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 2 | 2 |
| 1 | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 2 | 0 | 1 | 2 |
| 1 | 3 |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 2 | 1 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 1 | 1 |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 | 2 | 3 | 2 | 0 | 3 | 0 | 0 | 3 | 1 | 2 | 0 | 0 |
| 3 | 1 | 2 | 1 | 1 | 0 | 1 | 0 | 0 | 3 | 2 | 0 | 3 |
| 3 | 0 | 1 | 2 | 3 | 0 | 1 | 0 | 0 | 2 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 3 | 0 | 2 | 0 | 1 | 0 | 2 | 0 | 1 |
| 1 | 0 | 0 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 | 3 | 2 |
|  | 2 |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 0 | 3 | 3 | 0 | 1 | 0 | 3 | 2 | 0 | 1 | 0 | 1 |
| 3 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 1 | 0 | 0 | 3 | 0 |
| 2 | 0 | 3 | 1 | 2 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 3 |

Fig. 15. Simulated lightpath demand matrix for the previous rearrangement session.


Fig. 16. The number of rejected lightpath demands and the number of re-routed lightpaths $(\mathrm{W}=16)$.
we use wavelength conversion ( $F_{i c}=1$ and $v=2$ ) in this case, the number of re-routed lightpaths are also lower.

We have also used various network matrices and parameters to test our algorithm and study the network behaviors. We have observed stable algorithmic performances and behaviors similar to those shown and discussed above. Generally, to set the penalty coefficients $P, Q$ and $G$, the network operator needs to firstly evalute various costs from the network operations and to estimate the impacts from unfairness/congestion in terms of cost. Then some sample runs of the algorithm with different values should be conducted (similar to Fig. 7 and Fig. 10) to decide the trade-offs and to determine whether the given coefficients generate desired results. Some adjustments could be made based on the sample runs. To set the appropriate coefficients to obtain expected routing scheme, good computation experience is also helpful.

## VII. Conclusion

In this paper, we have successfully formulated and solved the semi-dynamic network rearrangement problem, which has not been systematically analyzed and mathematically solved previously due to formulation difficulties and the lack of mathematical tools. To relate the current session and the previous session in the optimization, we have exerted extensive
effort to formulate the problem properly so that it can be solved mathematically. We proposed a penalty-based objective function, i.e., penalizing the re-routing of a lightpath, the rejection of a lightpath demand and the network congestion. In doing so, the work in [13] was shown to become a special case of our formulation. Two rules were formulated to ensure the stability of the network between any node pair, 1) the capacity allocated has to be more (or equal to) the existing capacity, if there is more (or the same) capacity demand in the new session. 2) the capacity allocated has to be equal to the new demand, if there is less capacity demand in the new session. We employed the LRSM to solve the complicated formulation with polynomial computation complexity. We have discovered many network behaviors not previously observed in the NSFNET computation example, including the real contribution of the wavelength conversion, the trade-offs between the rejection and the re-routing/congestion, and the influence of various parameters on the final optimization results. These behaviors have important bearings for the successful operation of a semi-dynamic network. At the same time, the correctness of our formulation and the high efficiency of our solution methodology have been demonstrated.

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He has managed two "User-Controlled LightPath (UCLP)" projects funded under CANARIE's directed research program involving teams from the Univ. of Ottawa, the i2CAT Foundation Inocybe Technologies Inc. and CRC to develop software that enable users: to dynamically provision dedicated End-to-End connections over shared network resources, and to provide advanced UCLP services with a graphical resource management tool for creating and managing Articulated Private Networks (APNs). The former is based on Web and Grid Services, and Jini and JavaSpaces technologies; while the latter is based on a Service Oriented Architecture (SOA) associated with resource lists comprising virtualized networking, computing, software and instrument resources as Web Services and custom workflows using BPEL representing End-to-End services targeting specific user communities. He was involved with EUCALYPTUS: A Service-oriented Participatory Design Studio project led by Carleton University funded under the CANARIE Intelligent Infrastructure Program (CIIP) which combines the SOA and UCLP to provide a community of architects with an on-demand fully collaborative multi-site design capability. He is also involved with PHOSPHORUS: A Lambda User Controlled Infrastructure for European Research integrated project funded by the European Commission under the IST 6th Framework which addresses End-to-End user empowered service delivery across heterogeneous worldwide network infrastructures including UCLP systems. Mr. Savoie holds a B.Sc. and M.Sc. in Electrical Engineering from the Univ. of New Brunswick.


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[^1]:    ${ }^{1}$ We assign in [13] different rejection penalties to each $s_{s d n}$ for the same $(s, d)$ without differentiation for the fairness consideration.

[^2]:    ${ }^{2}$ In essence, the MEWA algorithm tries to find a viable lightpath that might result in a low overall resource usage. Readers are referred to [13] for details.

