

Construction and application of sparse connected wireless ad hoc and sensor networks

Tutorials on: www.site.uottawa.ca/~ivan/adhoc.html

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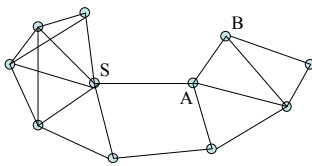
Outline

- MST, LMST
- Finding minimum common transmission radii and constructing MST
- RNG, GG, Yao graphs
- Applications of sparse graphs:
 - Minimal energy broadcasting,
 - Searching and broadcasting in peer to peer networks, over Internet, and in one-to-one wireless networks
 - Routing,
 - Bluetooth scatternet formation.

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Ad hoc wireless networks



Unit graphs
Transmission radius

Sensor networks
Mobile ad hoc networks
Rooftop networks

Transmission radius R normally same for all nodes

Devices can adjust transmission radii

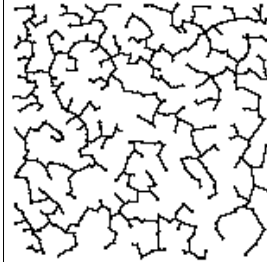
Two nodes communicate iff their distance is $\leq R$

Problems: How each node learns R so that the network is connected? What is minimal such R ?

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Minimal Spanning Tree MST



Each edge has weight

MST = connected tree with the minimal total sum of weights

Weight = distance in ad hoc networks

Problem: the construction of MST is centralized

To know which edges belong to MST, each node needs either global knowledge or needs to be informed from a central node

MST

Communication overhead to collect global network info is prohibitive in dynamic networks⁴

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Connectivity issues

- n nodes deployed in a region
- Minimum r (transmitting range) for connectivity?
- node placement known
 - longest edge in MST Penrose 1997.
- random node placement, localized knowledge
 - $r = O((\log n/n)^{1/2})$ Gupta, Kumar 1998.
- average #of neighbors $d \approx (n-1)r^2\pi/A$ (A = area)
 - $d = O(\log n)$ is average density for critical connectivity
 - each node needs to have, on average, $O(\log n)$ neighbors for the network to be connected.

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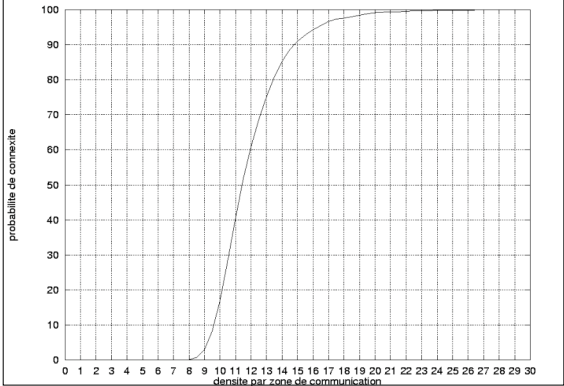
Connectivity continued

- the longest nearest neighbor and the longest MST edge have asymptotically (when $n \rightarrow \infty$) the same value (Penrose)
- Generally, when the minimum node degree becomes k , the graph becomes k -connected
- a graph is k -connected if it remains connected after removing any $k-1$ of its nodes.
- Santi and Blough 2003:
- Halving the critical transmission range, largest connected component contains $\approx 90\%$ of nodes.

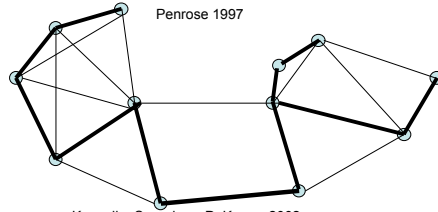
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Connectivity vs density: sharp transition



Minimal R = longest edge in MST



Narayanaswamy, Kawadia, Sreenivas, P. Kumar 2002:

Run multiple routing daemons (RDs), one at each power level.

R = smallest power level at which the number of reachable nodes is the same as that of the max power level.

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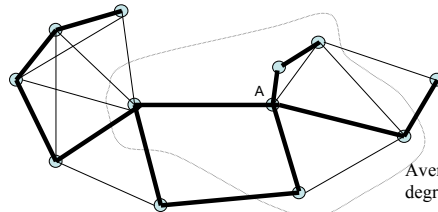
Computing minimum R and Wave propagation

- *Dai and Wu 2003:*
- 3 global algorithms to compute minimal R:
- *Area-based binary search,*
- *Prim's MST,*
- *extension with Fibonacci heap implementation.*
- *Dulman, Havinga, Jurink, 2002:*
- *Wave Propagation Leader Election Protocol*

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Localized structure: LMST



LMST = Localized Minimal Spanning Tree: Li, Hou, Sha 2003

Each node constructs MST of subgraph of its neighbors; keep edge if both endpoints have it in their MSTs

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LMST: required knowledge

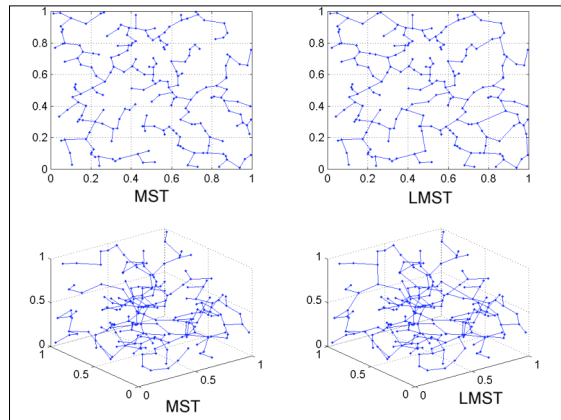
- Each node needs to know position of itself and all its direct neighbors (collected by a round of 'hello' messages)
- After computing MST of the subgraph of its 1-hop neighbors, each node needs to *send one message* to inform neighbors about selected edges

Unique edge weights:

$$w(AB) = (|AB|, \min(ID(A), ID(B)), \max(ID(A), ID(B)))$$

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MST \subseteq LMST

Kruskal's algorithm for *MST*: sort edges, consider edges for *MST* in increasing length order

Add edge to *MST* iff it does not create a cycle

$LMST(A) = MST$ of $n(A) = \{A \text{ and its 1-hop neighbors}\}$.

If an edge from *MST* has endpoints in $n(A)$ then it belongs to *LMST(A)*.

Suppose that this is not correct, and let e be the shortest such edge.

LMST(A) constructed by Kruskal's algorithm.

Ovalle-Martinez, Solano-Gonzalez, Garcia-Nocetti, Stojmenovic, IJDC 2005

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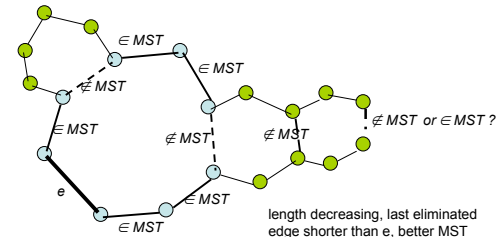


MST \subseteq LMST

$e =$ shortest edge in *MST* not in *LMST(A)*

○ Nodes from *LMST(A)*

● Nodes from *MST*, may not be in *LMST(A)*



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MST \subseteq LMST

When e is considered, since it is not included in *LMST(A)*, it creates a cycle C in *LMST(A)*, e the longest edge in that cycle.

Some of the edges from C are not in *MST*. Consider now expanded cycle C' constructed from C as follows.

Let f be an edge from C which is not in *MST*. Addition of f into *MST* creates a cycle B , with f being the longest edge in the cycle. The cycle consists of f and a path consisting of edges from *MST*. Replace f in C with all the edges from that path. Each such replacement enlarges the cycle C , but does not add any edge longer than f , and consequently longer than e . At the end, after replacing all non-*MST* edges with the corresponding paths of *MST* edges, e remains the longest edge of C' , but all the other edges of C' are now also in *MST*. This is a contradiction, since *MST* has no cycles. Therefore each edge AB from *MST* belongs to both *LMST(A)* and *LMST(B)*, and therefore to *LMST*.

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MST/LMST Longest edges ratio

- Longest *LMST* edge $\approx 1/3$ times longer than longest *MST* edge
- Longest *LMST* edge spends \approx twice energy vs. longest *MST* edge
- Average degrees 1.99 vs. 2.04
- $\leq 3\%$ more edges, but they are relatively long

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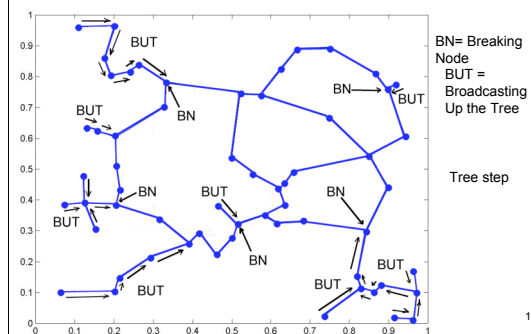
Distributed scheme for finding longest *LMST* edge

- **Wave propagation quazi-localized scheme**
- Each node maintains a record of the longest edge it has seen so far
- In each round, each node receiving longer edge in the previous round will broadcasts its new longest edge.
- At end, all nodes will receive the same longest edge.

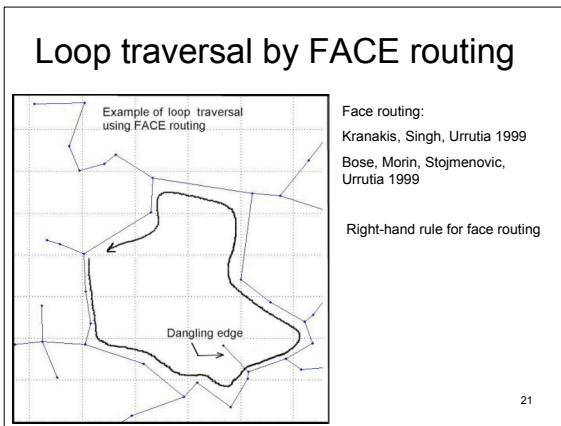
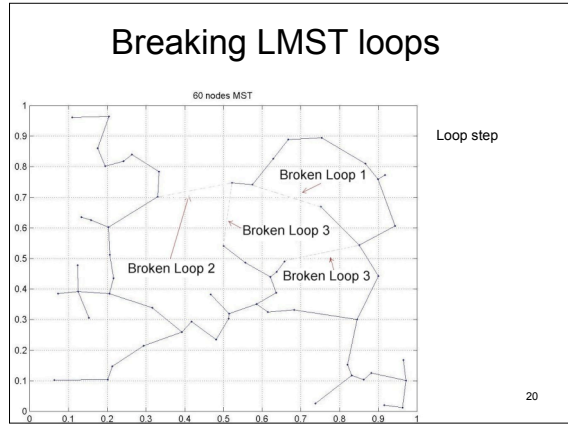
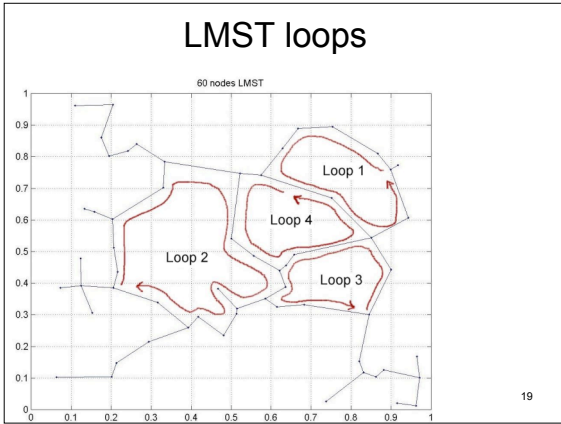
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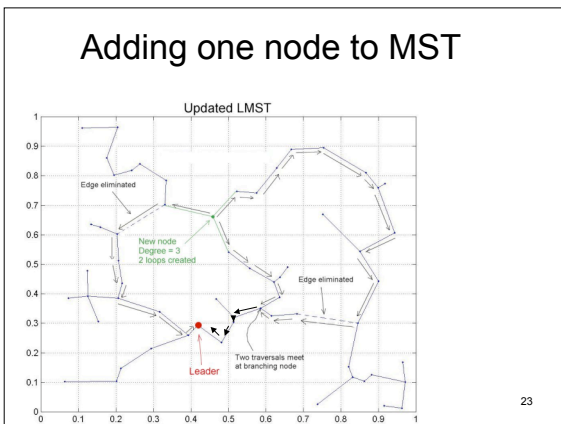
Constructing *MST* from *LMST*



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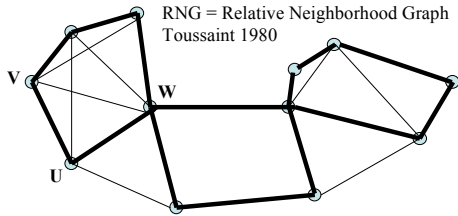


- ### Constructing MST from LMST
- Repeat
 - Tree step
 - Loop step
 - Until a single node remains (leader)
 - *Correctness:*
 - $MST \subseteq LMST$
 - All eliminated edges are longest in a loop
 - Proof by contradiction
 - Performance: $\approx \log_2 n - 2$ messages per node
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- ### Summary for MST construction
- MST leads to 1/3 shorter transmission radius vs. LMST
 - MST construction from LMST is worthwhile when network is mostly stable: rooftop networks for wireless internet access, sensor networks with long lasting nodes
 - *Open:* 3D construction of MST from LMST
 - *Problem:* face routing works only in 2D
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Localized structures: RNG



RNG = Relative Neighborhood Graph
Toussaint 1980

$UV \in \text{RNG} \rightarrow UV$ not the longest edge in any triangle UVW

Average degree ≈ 2.5

Weighted RNG: arbitrary weights on edges instead of distances
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(Escalante, Perez, Solano, Stojmenovic 2005)

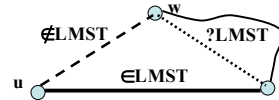
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LMST \subseteq RNG

Suppose $uv \in \text{LMST}$ but $uv \notin \text{RNG}$.

$\exists w, uw < uv, vw < uv$ (unique edge lengths)

uv can be replaced by uw in $\text{MST}(n(u))$, giving a spanning tree with lower overall weight (total sum of all edge lengths) than the minimal one ($\text{MST}(n(u))$), which is a contradiction.

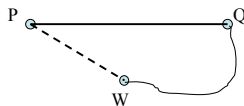


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RNG contains Minimal Spanning Tree

$\text{MST} \subseteq \text{LMST} \subseteq \text{RNG} \subseteq \text{GG}$



Proof 2: By contradiction: Assume

$PQ \in \text{MST}, PQ \notin \text{RNG}$;

$\rightarrow \exists W, PW < PQ$ and $QW < PQ, PW \notin \text{MST}$

Replace PQ by PW in MST

\rightarrow new MST has smaller sum of edge lengths. contradiction

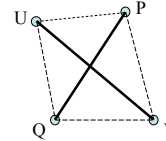
\rightarrow **RNG is connected**

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RNG is planar graph

Planar graph = no two edges intersect



Proof by contradiction: Assume

$UV, PQ \in \text{RNG}(S), UV \cap PQ$

$\rightarrow \angle PUQ < \pi/2, \angle PVQ < \pi/2,$

$\nexists \angle UPV < \pi/2, \angle UQV < \pi/2,$

\rightarrow Sum of angles in $\text{UPVQ} < 2\pi$

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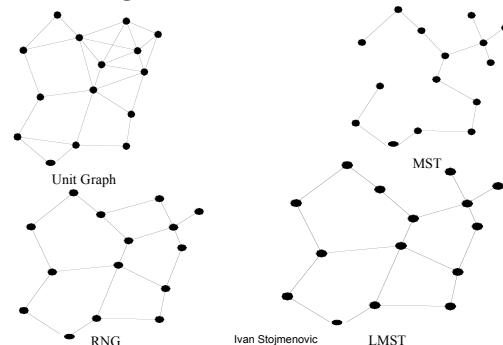
Benefits of RNG

- Planar, very sparse, preserves connectivity
- To construct, each node needs to know the distances (or other type of weight) to neighbors, and between neighbors
- Position of itself and direct neighbors after 'hello' msg \Rightarrow **RNG edges can be learned without additional messages between neighbors!**
- Send a 'hello' containing weight toward each neighbor
- Determining weights may require additional overhead
- Weight = distance, delay, bandwidth, power..

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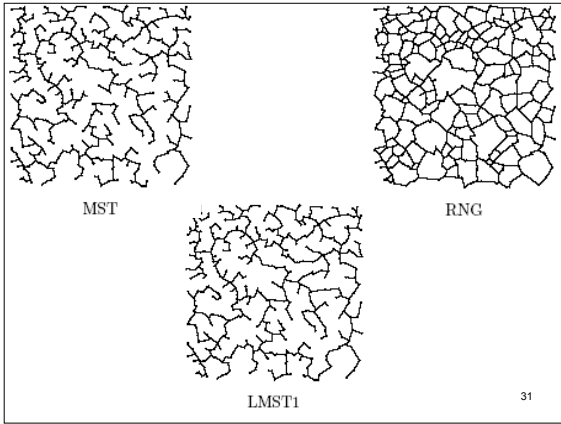
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Unit graph, MST, LMST, RNG

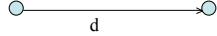


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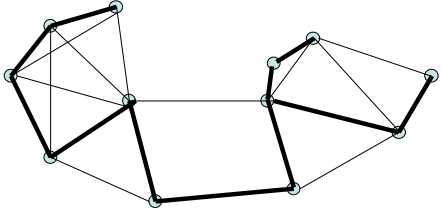


Application: Minimal energy broadcasting

- power = $u(d) = ad^\alpha + c$, $2 \leq \alpha$ 
- Choose transmission radius at each node so that network is connected and sum of transmission powers is minimized (starting from a source)
- Globalized solutions in two dozens of articles
BIP: Wieselthier, Nguyen and Ephremides 2000
- First localized solution:
Cartigny, Simplot, Stojmenovic INFOCOM 2003

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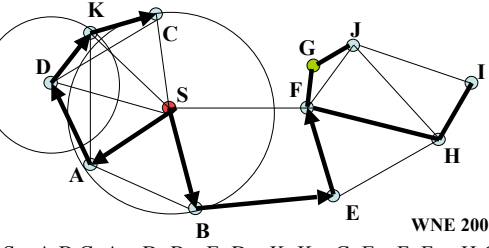
Cover neighbors in a connected structure



Choose transmission radius to **cover remaining neighbors + neighbor elimination** (some nodes may not transmit at all)

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Cover neighbors in connected structure



WNE 2000

S → A, B, C; A → D; B → E; D → K; K → C; E → F; F → H, G, J; H → I; G does not transmit (neighbor elimination F → J)

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Broadcast oriented MST based; **best for reasonable density**

Performance

- RNG: Cartigny, Simplot, Stojmenovic 2003
- LMST: Ingelrest, Cartigny, Simplot, Stojmenovic 2005
- LMST ≈ 50% more energy than MST
- Localized RNG and LMST based protocols remain competitive to globalized BIP protocol (approx 60%-100% more energy)
- Ingelrest, Simplot-Ryl 2005: Run BIP with 2-hop knowledge
- Very competitive to BIP, overall winner!

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Broadcasting and searching in peer to peer networks and over Internet

- Escalante, Perez, Solano, Stojmenovic 2005
- Weighted RNG and weighted LMST
- Weight = delay or other measurable metric
- Advertise files only on links belonging to RNG or LMST
- Search for files using only LMST or RNG links

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Broadcasting in one-to-one wireless networks

Heinzelman, Kulik, Balakrishnan MOBICOM 1999:

Each node sends *short* message to **all** its neighbors
Long messages are sent to neighbors that request it

Subramanian, Katz 2000:

Construct and maintain a spanning tree
 No short messages but maintenance overhead

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Reducing broadcast search

Seddigh, Solano, Stojmenovic 2001



Localized protocols

Reduce number of short messages by using

backbone nodes and **planar subgraphs**

B-broadcast: edges with at least one endpoint at backbone

P-broadcast: edges of planar subgraph (e.g. **RNG**)

BP-broadcast: backbone nodes, planar subgraph on it

PB-broadcast: planar subgraph, backbone nodes on it

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Performance

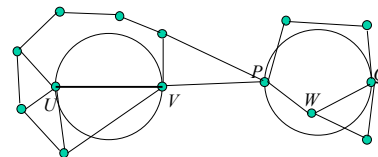
Percentage of edges for re-transmitting short messages for $n=100$ nodes $P=RNG$ $B=Wu/Li$ gateway

Method/degree	4	5	7	9	15	40
P-broadcast	53	45	33	26	16	6
PB-broadcast	53	45	33	26	16	6
BP-broadcast	52	44	32	25	15	5
B-broadcast	61	52	41	35	21	6

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Gabriel graph



Gabriel graph $GG(S)$ contains an edge (U, V)
 iff the disk with diameter (U, V)
 contains no other point from S

Computing GG from unit graph requires no message exchange

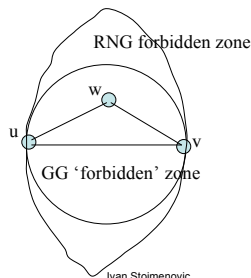
Some nodes may have high degrees; average degree 3.8

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$RNG \subseteq GG$

- $uv \in RNG \rightarrow$ no $w, uw < uv, vw < uv \rightarrow$ disk with diameter uv has no node $\rightarrow uv \in GG$

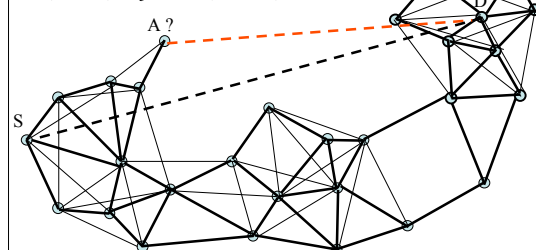


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1. Constructing planar graph: faces

Bose, Morin, Stojmenovic, Urrutia, 1999



Some planar graphs (Gabriel graph) can be constructed
 without message exchange!

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2. Traverse proper face until recovery

Bose, Morin, Stojmenovic, Urrutia, 1999

- Select face containing SD

- Follow that face by **left** hand or **right** hand rule until recovery (= closer node reached)

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Getting closer on the face is guaranteed for GG

$\angle S < \pi/2, \angle D < \pi/2$ since EF is in GG $\rightarrow \angle E > \pi/2$ or $\angle F > \pi/2$

$\angle F > \pi/2 \rightarrow |SD| > |FD| \rightarrow F$ is closer to D than S

Frey, Stojmenovic MOBICOM 2006

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GFG (greedy-face-greedy)

Bose, Morin, Stojmenovic, Urrutia, 1999

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Yao graph

$k=7$

Divide into k equal cones around u

Find closest point in each cone, if any

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Yao directional construct preserves connectivity

Construct MST by considering edges in increasing order, include if no cycle created (Kruskal's scheme)

If AB not in Yao then $\exists C, AC$ in Yao,
 $AB < AC,$
 $BC < AB$ (angle $< \pi/6$)
 $\Rightarrow BC, AC$ already considered for MST and connected,
 $\Rightarrow AB$ not needed in MST
 $\Rightarrow MST$ is included in Yao

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Yao bidirectional is connected

- Contains RNG as subgraph (Wang, Li 2002)
- Unique edges enforced on the graph by $w(AB)$

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Partial Delaunay Triangulation

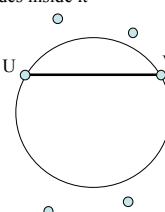
Li, Stojmenovic 2001 Problem: not significantly denser than GG

Delaunay Triangulation = dual Voronoi diagram

UV in DT iff there exist a circle with chord UV without other nodes inside it

Test disk with diameter UV:

- If empty then UV in PDT
- If nodes inside disk on both sides then not in PDT
- Find smallest angle on both sides of UV
- If together $\geq \pi$ then not in PDT
- If together $< \pi$ then in PDT iff both are neighbors, using 1-hop or 2-hop info

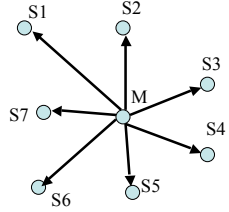


PDT = portion of DT which can be decided locally

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Application: Bluetooth scatternet formation

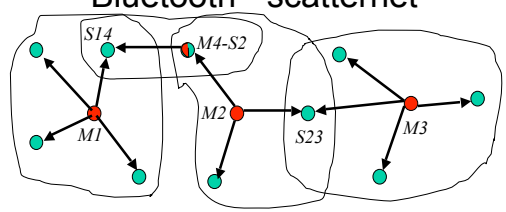
- Short-range
- Master-slave
- Frequency hopping
- PICONET = master + K slaves, $K \leq 7$
- Additional slaves must be parked



Bluetooth - piconet

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Bluetooth - scatternet



Connect piconets into scatternet

Avoid master-slave bridges

Bridges participate in piconets on time division basis

Minimize number of slave roles and # of masters

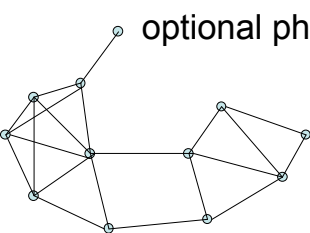
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Degree limited connected scatternet formation

- Li, Stojmenovic, Wang 2001 – clustering based
- Stojmenovic 2002 – dominating set based
- *Phase I* = create unit graph and (optional) construct a planar connected structure in localized manner
- *Phase II* = eliminate some edges in the structure to **limit the degree** of each node to 7
- *Phase III* = **decide master-slave roles** between two nodes of each edge in the structure
- Phases II and III **iterative** (node by node) or concurrently by all nodes **simultaneous** approach
- Phase III **during** or **after** Phase II

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Scatternet formation – optional phase I



Unit graphs radius

Create unit graph

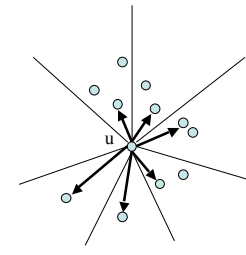
Assumption: Each node is aware of its position and learns position of all neighbors within transmission radius

Construct **planar connected** structure in localized manner:
 $MST \subseteq LMST \subseteq RNG \subseteq GG \subseteq PDT \subseteq DT$

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Yao graph – Phase II

$k=7$



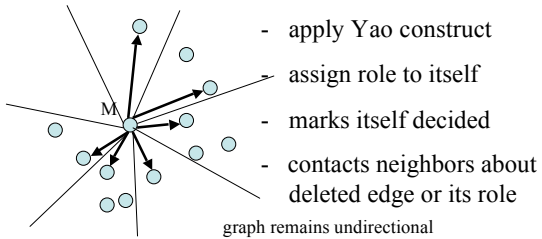
Divide into k equal cones around u

Find closest point in each cone, if any

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Limiting degrees – cluster based

Phase 2: Applied on **active nodes** = nodes with highest keys among undecided neighbors



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Avoiding position information ?

- Single-hop networks (any two nodes communicate = complete graph):
virtual positions
(Wang, Li, Stojmenovic 2004)
- MST can be used instead of LMST, RNG, GG
- Delaunay triangulation can also be used (with Yao structure) for denser planar graph

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