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A NOTE ON APPROXIMATE CONVEX HULLS

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1. Introduction

Bentley, Faust and Preparata [1] presented a linear time algorithm for computing an *approximate* convex hull in two-dimensional space. They compute a polygon with vertices in the given set of points in the plane such that the polygon is arbitrarily close to the convex hull of the point set. Soisalon-Soininen [2] modified the approximation scheme of [1] and, among other things, gave a criterion which depends on the required accuracy and which selects edges of the computed approximate hull such that no points can lie 'behind' these edges. However, the criterion given in [2] is not quite correct and an additional condition is required for the edges to be selected. In this note we present a correct form of this criterion.

2. The approximation scheme

We shall shortly describe the algorithm of [2] for computing an approximate convex hull of a planar point set. First, the points with the minimum and maximum x- and y-values are determined. Then the area between the points with extreme x-values is divided into k_1 equally spaced strips, and the area between the points with extreme y-values is divided into k_2 equally spaced strips (see Fig. 1(a)). These strips define $k_1 \cdot k_2$ rectangular areas. Further, the set S_1 is defined to contain the points with maximum and minimum y-values in each strip parallel to the y-axis (if the number of points with a minimum or maximum y-value in some strip is more than one, then the two points among these with the maximum and minimum x-values are taken into account). In an exactly analogous way, the set S_2 is computed to contain the extremes in the strips parallel to the x-axis. In Fig. 1(b) the points of the sets S_1 and S_2 are denoted by squares and circles, respectively. We observe that some points may be marked with both a circle and a square. Finally, the convex hull of $S_1 \cup S_2$ is computed and used as an approximation of the convex hull of the whole point set (Fig. 1(c)).

We note that a vertex of the approximate convex hull is not always a vertex of the true convex hull also (see, for example, point C in Fig. 1(c)).

3. Location of points outside the approximate hull

It is shown in [2] that if p is a point outside the approximate hull (as computed in [2]), then it must lie in a triangle of the form $((x_1, y_1), (x_2, y_2), (x', y'))$, where $((x_1, y_1), (x_2, y_2))$ is an edge of the approximate hull and for (x', y') the following

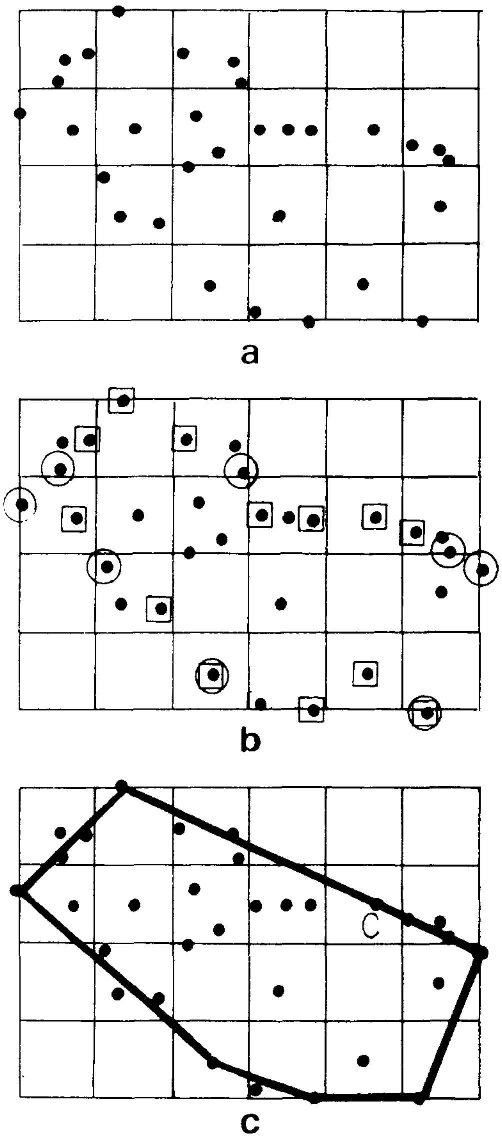

Fig. 1.

condition holds:

$x' = x_1$ and $y' = y_2$

if the triangle $((x_1, y_1), (x_2, y_2), (x_1, y_2))$ lies outside the hull, and

$x' = x_2$ and $y' = y_1$

if the triangle $((x_1, y_1), (x_2, y_2), (x_2, y_1))$ lies outside the hull. Triangle $((x_1, y_1), (x_2, y_2), (x', y'))$ is called the *error triangle* of $((x_1, y_1), (x_2, y_2))$.

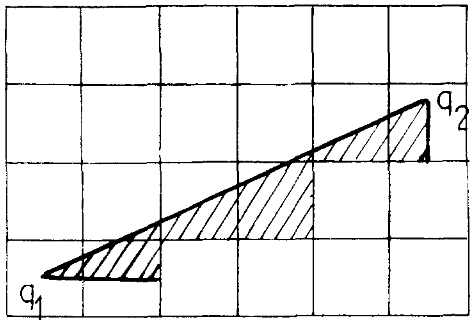

Fig. 2. The interior area of (q_1, q_2) is shaded.

The following incorrect statement was made in [2, Theorem 3]. Let (q_1, q_2) be an edge of the approximate convex hull. If q_1 and q_2 are not in the same strip in either direction, then no point outside the approximate hull can lie in the error triangle of (q_1, q_2). Fig. 1(c) proves that the condition of the statement is not strong enough.

We shall now develop a correct statement. Let $(q_1 = (x_1, y_1), q_2 = (x_2, y_2))$ be an edge of the approximate hull and $(q_1, q_2, (x', y'))$ its error triangle. We say that $(q_1, q_2, (x_1 + x_2 - x', y_1 + y_2 - y'))$ is the *interior triangle* of (q_1, q_2). Further, we define the *interior area* of (q_1, q_2) as the intersection of the interior triangle of (q_1, q_2) and those $k_1 \cdot k_2$ rectangular areas that intersect (q_1, q_2) (see Fig. 2).

The following theorem is now easy to prove.

Theorem. *Let* (q_1, q_2) *be an edge of the approximate convex hull. If* q_1 *and* q_2 *are not in the same strip in either direction and there are no extremes in the interior area of* (q_1, q_2), *then no point outside the approximate hull can lie in the error triangle of* (q_1, q_2).

References

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[2] E. Soisalon-Soininen, On computing approximate convex hulls, Inform. Process. Lett. 16 (1983) 121–126.