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CLASSIFICATION OF THREE-VALUED LOGICAL FUNCTIONS PRESERVING 0

Masahiro MIYAKAWA
Electrotechnical Laboratory, 1-1-4 Umezono, Tsukuba, Ibaraki 305, Japan

Ivo G. ROSENBERG
Mathématiques et Stat., Université de Montréal, C.P.6128, Succ. "A", Montréal, Que., Canada H3C 3J7

Ivan STOJMENOVIĆ
Institute of Mathematics, University of Novi Sad, dr Ilije Djuričića 4, 21000 Novi Sad, Yugoslavia

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The set T_0 of three-valued logical functions preserving 0 is classified into 253 classes using the known classification of P_3 (the whole set of three-valued logical functions). This enables one to determine all 883,720 classes of T_0-bases.

1. Introduction

Let $E=\{0,1,2\}$. The set of three-valued logical functions (i.e., $f:E^n\rightarrow E$ for $n=1,2,\dots$) is denoted by P_3. A subset F of P_3 is said to be *closed* if it contains all superpositions (i.e., compositions or substitutions) of its members (cf. [1, 11-13]). For closed sets F and H such that $F\subset H$ (proper inclusion), F is an *H-maximal* set if $F\subset G\subset H$ for no closed set G. A subset F of H is *complete in H* if H is the least closed set containing F. We assume throughout that H has finitely many H-maximal sets and that each proper closed subset of H extends to a maximal one (or, equivalently, it is finitely generated, i.e., there is a finite F complete in H). Clearly a subset of H is complete in H if and only if it is not contained in any H-maximal set (completeness condition, cf. [1]).

Completeness (also called functional completeness or primality) is directly related to universal algebra and to logical circuit design. A complete set F in H is called an *H-base* if no proper subset of F is complete in H. A subset F of H is called *pivotal* in H, if for each function $f\in F$ there exists an H-maximal set M such that $f\notin M\supseteq F\setminus\{f\}$. From these definitions it follows that an H-base is a complete and pivotal set in H. The *rank* of a set is the number of its elements.

Let $H_1,\dots,H_m$ be all the H-maximal sets. For $f,g\in H$ put $f\approx g$ if either both $f,g\in H_i$ or both $f,g\notin H_i$ for all $i=1,\dots,m$. The relation $\approx$ is an equivalence relation partitioning the set H into equivalence classes. Now we can discuss the completeness in H in terms of these classes instead of individual functions: if a set is complete, then by replacing a function in the set by any function in its equivalence class we get another complete set. The *characteristic vector* of $f \in H$ is the zero-one m-vector $a_1 \cdots a_m$, where $a_i = 0$ if $f \in H_i$ and $a_i = 1$ otherwise $(1 \le i \le m)$. All functions $f \in H$ with the same characteristic vector form an equivalence *class of functions*. The completeness and nonredundancy of $F \subseteq H$ can be checked using characteristic vectors of functions of F. All bases with the same set of characteristic vectors form a *class of bases*. If we have the complete list of characteristic vectors, we can enumerate all classes of bases.

Our description of the H-maximal sets is based on relations. For $h \ge 1$ an h-ary relation on E is a subset of E^h (i.e., a set of h-tuples over E). The relation ϱ is written as an $h \times |\varrho|$ matrix whose columns are the elements of the relation ϱ in any fixed order.

If $\boldsymbol{a}_i = (a_{i1}, \dots, a_{in}) \in E^n$ $(i = 1, \dots, h)$ are such that $(a_{1i}, \dots, a_{hi}) \in \varrho$ for all $i = 1, \dots, n$ we write $(\boldsymbol{a}_1, \dots, \boldsymbol{a}_h)^{\mathrm{T}} \in \varrho^n$. We say that an n-ary function f *preserves* ϱ if $(f(\boldsymbol{a}_1), \dots, f(\boldsymbol{a}_h)) \in \varrho$ whenever $(\boldsymbol{a}_1, \dots, \boldsymbol{a}_h)^{\mathrm{T}} \in \varrho^n$. The set of functions preserving ϱ is denoted by $\operatorname{Pol} \varrho$. In the following theorem $T_0, \dots, T_{12}$ are determined by unary relations (i.e., subsets of E), M_0, M_1, M_2 by linear orders (chains) on E, U_0, U_1, U_2 by the nontrivial equivalence relations on E, B_0, B_1, B_2 by the so-called central relations, T is the Słupecki clone (of all essentially unary or nonsurjective functions), L is the clone of all linear or affine (mod 3) functions and S of all functions selfdual with respect to the cyclic permutation (012). Throughout this paper $x+y$ and xy denote the element of E congruent (mod 3) to $x+y$ and xy, respectively. Intersection of sets $X_1, \dots, X_r$ will be denoted by $X_1 \cdots X_r$. Finally, for $x \in E$ let x^r denote the vector $x \cdots x$ (r times).

Theorem 1.1 [1]. *P_3 has exactly the following 18 maximal sets:*

$$T_0 = \operatorname{Pol}(0), \qquad T_1 = \operatorname{Pol}(1), \qquad T_2 = \operatorname{Pol}(2),$$

$$T_{01} = \operatorname{Pol}(01), \qquad T_{02} = \operatorname{Pol}(02), \qquad T_{12} = \operatorname{Pol}(12),$$

$$M_0 = \operatorname{Pol}\begin{pmatrix} 012220 \\ 012011 \end{pmatrix}, \quad M_1 = \operatorname{Pol}\begin{pmatrix} 012001 \\ 012122 \end{pmatrix}, \quad M_2 = \operatorname{Pol}\begin{pmatrix} 012112 \\ 012200 \end{pmatrix},$$

$$U_0 = \operatorname{Pol}\begin{pmatrix} 01212 \\ 01221 \end{pmatrix}, \quad U_1 = \operatorname{Pol}\begin{pmatrix} 01202 \\ 01220 \end{pmatrix}, \quad U_2 = \operatorname{Pol}\begin{pmatrix} 01201 \\ 01210 \end{pmatrix},$$

$$B_0 = \operatorname{Pol}\begin{pmatrix} 0120102 \\ 0121020 \end{pmatrix}, \quad B_1 = \operatorname{Pol}\begin{pmatrix} 0120112 \\ 0121021 \end{pmatrix}, \quad B_2 = \operatorname{Pol}\begin{pmatrix} 0120212 \\ 0122021 \end{pmatrix},$$

$$T = \operatorname{Pol}(\{(a,b,c)^{\mathrm{T}} \in E^3\colon\ a = b \text{ or } a = c \text{ or } b = c\}),$$

$$L = \operatorname{Pol}(\{(a,b,c)^{\mathrm{T}} \in E^3\colon\ c = 2(a+b)\}),$$

$$S = \operatorname{Pol}\begin{pmatrix} 012 \\ 120 \end{pmatrix}.$$

The classes of functions of P_3 are determined in [3, 14]. Classes of P_3-bases are determined in [5, 14]. For other classifications we refer [9, 10, 18].

The complete list of maximal sets for each of the 18 P_3-maximal sets has been given by Lau [2]. Classes of functions and classes of bases for the set B_1 are determined in [6], for the set M_1 in [15], for the sets T, L and S in [7], for the set T_{01} in [16] and for the set U_2 in [17]. Recall that T_0 is the set of all three-valued logical functions f such that $f(0,\dots,0)=0$. In [8] the classes of functions and bases for T_0 are given. In this paper we give much simpler description of it using the classification of P_3. We recall:

Theorem 1.2 [2]. *T_0 has exactly the following* 12 *maximal sets*:

Group I.

(1) $K_{10}=\mathrm{Pol}\begin{pmatrix}012\\021\end{pmatrix}.$

(2) $K_{11}=\mathrm{Pol}\begin{pmatrix}00102\\01020\end{pmatrix}.$

(3) $K_{12}=\mathrm{Pol}\begin{pmatrix}0010212\\0102021\end{pmatrix}.$

Group II.

(4) $T_0M_1=\mathrm{Pol}(0)\mathrm{Pol}\begin{pmatrix}012001\\012122\end{pmatrix}.$

(5) $T_0M_2=\mathrm{Pol}(0)\mathrm{Pol}\begin{pmatrix}012121\\012200\end{pmatrix}.$

(6) $T_0U_{12}=\mathrm{Pol}(0)\mathrm{Pol}\begin{pmatrix}01212\\01221\end{pmatrix}.$

(7) $T_0B_0=\mathrm{Pol}(0)\mathrm{Pol}\begin{pmatrix}0120012\\0121200\end{pmatrix}.$

Group III.

(8) $T_0T_1=\mathrm{Pol}(0)\mathrm{Pol}(1)$.

(9) $T_0T_2=\mathrm{Pol}(0)\mathrm{Pol}(2)$.

(10) $T_0T_{01}=\mathrm{Pol}(0)\mathrm{Pol}(01)$.

(11) $T_0T_{12}=\mathrm{Pol}(0)\mathrm{Pol}(12)$.

(12) $T_0T_{20}=\mathrm{Pol}(0)\mathrm{Pol}(20)$.

Note that only the three sets K_{10}, K_{11} and K_{12} are not P_3-maximal. In what follows we delete the prefix T_0 to denote the above maximal sets of T_0. In Section 2 we need the following 14 technical lemmas which are of independent interest (as statements about the lattice of closed sets ordered by $\subseteq$). First we list them together (as Lemmas 1.3–1.16) and then proceed with their proofs.

Lemma 1.3. $K_{10}K_{12} \subseteq K_{11}$.

Lemma 1.4. $T_1K_{10} \subseteq T_2$, $T_2K_{10} \subseteq T_1$.

Lemma 1.5. $T_{01}K_{10} \subseteq T_{02}$.

Lemma 1.6. $U_0K_{12} \subseteq K_{10}$.

Lemma 1.7. $T_1K_{12} \subseteq T_{02}$, $T_2K_{12} \subseteq T_{01}$.

Lemma 1.8. $B_0T_{01}T_{02}U_0 \subseteq K_{11}$.

Lemma 1.9. $K_{10}K_{12} \subseteq B_0$.

Lemma 1.10. $U_0K_{12} \subseteq B_0$.

Lemma 1.11. $M_1K_{10} \subseteq M_2$.

Lemma 1.12. $M_1K_{10} \subseteq U_0$.

Lemma 1.13. $B_0K_{12} \subseteq K_{11}$.

Lemma 1.14. $K_{12}T_{12} \subseteq B_0$.

Lemma 1.15. $K_{10}B_0 \subseteq K_{12}$.

Lemma 1.16. $M_1T_{02}K_{12} \subseteq K_{11}$.

Proofs. We must prove inclusions of the form $\operatorname{Pol}\varrho_1 \cdots \operatorname{Pol}\varrho_i \subseteq \operatorname{Pol}\varrho_0$ (where $i=4$ in Lemma 1.8, $i=3$ in Lemma 1.16 and $i=2$ otherwise). The inclusion holds if we can express ϱ_0 by a logical formula based on $\exists$, &, $=$ and membership in ϱ_j $(1 \leq j \leq i)$.

We show what we mean by an example. Let (see Fig. 1)

$$\kappa_{10} := \begin{pmatrix} 012 \\ 021 \end{pmatrix}, \quad \kappa_{12} := \begin{pmatrix} 0010212 \\ 0102021 \end{pmatrix}, \quad \kappa_{11} := \begin{pmatrix} 00102 \\ 01020 \end{pmatrix}.$$

Put

$$\lambda := \{(x,y) \colon (x,y) \in \kappa_{12},\ (x,u) \in \kappa_{10},\ (u,y) \in \kappa_{12} \text{ for some } u\}.$$

This may be written as $\lambda = \kappa_{12} \cap (\kappa_{10} \circ \kappa_{12})$ where $\circ$ denotes the relational (de Morgan) product or composition.

We prove $\kappa_{11} = \lambda$ by a direct check. First clearly $\lambda \subseteq \kappa_{12}$. We have $(0,0), (0,1), (0,2) \in \kappa_{10} \circ \kappa_{12}$ (choose $u=0$ in all 3 cases), $(2,0) \in \kappa_{10}$ (choose $u=1$) and $(1,0) \in \kappa_{10} \circ \kappa_{12}$ (choose $u=2$) and so $\kappa_{11} \subseteq \lambda \subseteq \kappa_{12}$. Next $(1,2) \notin \kappa_{10} \circ \kappa_{12}$ (if it were we would need

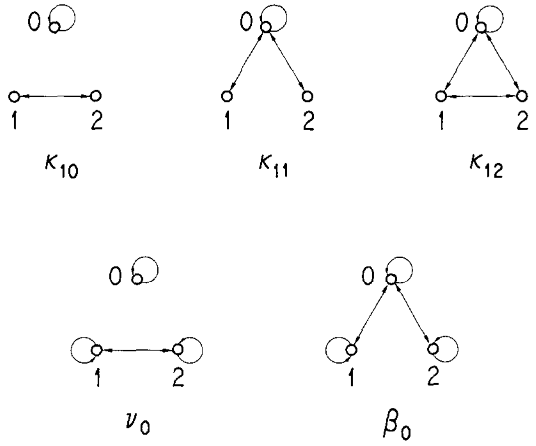

Fig. 1.

$u=2$ but $(2,2)\notin\kappa_{12}$) and similarly $(2,1)\notin\kappa_{10}\circ\kappa_{12}$ (we need $u=1$ but $(1,1)\notin\kappa_{12}$). It follows that $\kappa_{11}=\lambda$.

The above fact $\operatorname{Pol}\varrho_1\cdots\operatorname{Pol}\varrho_i\subseteq\operatorname{Pol}\varrho_0$ is well known ([12, §4], for more information cf. [11, §1.1, Ch. 2]), and may be proved directly (it has also an interesting and basic converse called Galois polytheory, cf. ibid).

In the sequel κ_{ij} denotes the relation in $K_{ij}=\operatorname{Pol}\kappa_{ij}$ (see Theorem 1.2, group I), similarly $U_i=\operatorname{Pol}\nu_i$, $M_i=\operatorname{Pol}\mu_i$, and $B_0=\operatorname{Pol}\beta_0$.

Lemma 1.3. $\kappa_{11}=\{(x,y)\colon (x,y)\in\kappa_{12},\ (x,u)\in\kappa_{10}$ and $(y,u)\in\kappa_{12}$ for some $u\}$ (see above).

Lemma 1.4. $\{2\}=\{x\colon (x,u)\in\kappa_{10}$ for some $u\in\{1\}\}$ (as $T_i=\operatorname{Pol}\{i\}$ where $\{i\}$ is a unary relation; of course $u\in\{1\}$ means $u=1$). Similarly $\{1\}=\{x\colon (x,2)\in\kappa_{10}\}$.

Lemma 1.5. $\{0,2\}=\{x\colon (x,u)\in\kappa_{10}$ for some $u\in\{0,1\}\}$.

Lemma 1.6. $\kappa_{10}=\nu_0\cap\kappa_{12}$.

Lemma 1.7. $\{0,2\}=\{x\colon (x,1)\in\kappa_{12}\}$, $\{0,1\}=\{x\colon (x,2)\in\kappa_{12}\}$.

Lemma 1.8. $\kappa_{11}=\{(x,y)\colon (x,y)\in\beta_0,\ (x,u)\in\mu_0,\ (u,v)\in\beta_0,\ (v,y)\in\nu_0$ for some $u\in\{0,1\}$ and $v\in\{0,2\}\}$. To see $\subseteq$ consider the following (x,u,v,y): $(0,0,2,1)$, $(1,1,0,0)$, $(0,0,2,2)$, $(2,1,0,0)$ and $(0,0,0,0)$. The inclusion $\supseteq$ is obtained as follows. If $(1,u)\in\mu_0$ and $(v,1)\in\nu_0$ for some $u\in\{1,0\}$ and $v\in\{0,2\}$, then $u=1$ and $v=2$ and hence $(u,v)\notin\beta_0$ proving $(1,1)$ does not belong to the right side. The proof for $(2,2)$ is similar. As the right side is a subrelation of β_0 this completes the proof.

Lemma 1.9. $\beta_0=\{(x,y)\colon (x,u),(v,y)\in\kappa_{10},\ (x,v),(u,y)\in\kappa_{12}$ for some u and $v\}$.

Lemma 1.10. Combine Lemmas 1.6 and 1.9.

Lemma 1.11. $\mu_2=\{(x,y)\colon (x,u),(v,y)\in\kappa_{10}$ for some $u\geq v\}$.

Lemma 1.12. $\nu_0=\{(x,y)\colon (u,v),(w,t)\in\kappa_{10},\ u\leq x\leq t,\ w\leq y\leq v\}$.

Lemma 1.13. $\kappa_{11}=\beta_0\cap\kappa_{12}$.

Lemma 1.14. $\beta_0=\{(x,y)\colon (x,u),(u,y)\in\kappa_{12}$ for some $u\in\{1,2\}\}$.

Lemma 1.15. $\kappa_{12}=\{(x,y)\colon (x,u),(v,y)\in\kappa_{10},\ (x,v),(u,y)\in\beta_0$ for some u and $v\}$. To prove $\subseteq$ we take the following quadruples (x,u,v,y): $(0,0,0,0)$, $(0,0,2,1)$, $(0,0,1,2)$ and $(1,2,1,2)$ (the right side is obviously symmetric). For $\supseteq$ note that neither $(1,1)$ nor $(2,2)$ belong to the right side (if $(1,1)$ would, then $u=2$ in contradiction to $(2,1)\notin\beta_0$ and similarly for $(2,2)$).

Lemma 1.16. $\kappa_{11}=\{(x,y)\in\kappa_{12}\colon x\leq u,\ v\geq y,\ (x,v),(u,y)\in\kappa_{12}$ for some $u,v\in\{0,2\}\}$. To see $\subseteq$ note that the right side is symmetric and take the quadruples (x,u,v,y): $(0,0,0,0)$, $(0,2,2,1)$ and $(0,0,2,2)$. For $\supseteq$ note the following. First the right side is symmetric. If $(1,2)$ belongs to the right side, then $u\geq 1$, $u\in\{0,2\}$ means $u=2$ in contradiction to $(2,2)\notin\kappa_{12}$. □

Lemma 1.17. $U_0B_0\subseteq T_{01}\cup T_{02}\cup K_{11}$.

Proof. Suppose there exists an n-ary $f\in U_0B_0\bar{T}_{01}\bar{T}_{02}\bar{K}_{11}$. Then there are $\binom{a}{b}\in\kappa_{11}^n$ such that $\binom{f(a)}{f(b)}\notin\kappa_{11}$, i.e., $\in\binom{1212}{1221}$. When $\binom{f(a)}{f(b)}\in\binom{12}{21}$, in view of $\kappa_{11}\subseteq\beta_0$ we would have $f\notin B_0$. Next suppose $f(\boldsymbol{a})=f(\boldsymbol{b})=1$. Define a vector $\boldsymbol{c}$ so that

$$\begin{pmatrix}\boldsymbol{a}\\ \boldsymbol{b}\\ \boldsymbol{c}\end{pmatrix}\in\begin{pmatrix}01020\\ 00102\\ 01010\end{pmatrix}.$$

Now $\binom{\boldsymbol{a}}{\boldsymbol{c}}\in\nu_0^n$ and $f\in U_0$ imply $f(\boldsymbol{c})\neq 0$. Next $\binom{\boldsymbol{b}}{\boldsymbol{c}}\in\beta_0^n$ and $f\in B_0$ imply $\binom{1}{f(\boldsymbol{c})}\in\beta_0$ and therefore together we have $f(\boldsymbol{c})\neq 2$ and $f(\boldsymbol{c})\neq 1$. Since $f\notin T_{01}$, there is a vector $\boldsymbol{d}\in\{0,1\}^n$ such that $f(\boldsymbol{d})=2$. From $f(\boldsymbol{c})=1$, $f(\boldsymbol{d})=2$ and $\binom{\boldsymbol{c}}{\boldsymbol{d}}\in\binom{0110}{0011}$ we conclude $f\notin B_0$, a contradiction. Finally if $f(\boldsymbol{a})=f(\boldsymbol{b})=2$ the proof is quite similar. □

Lemma 1.18. *The set* $M_1\bar{T}_2T_{02}$ *consists of constant functions with value* 0 *only and so* $M_1\bar{T}_2T_{02}\subseteq K_{01}K_{11}K_{12}$.

Proof. From $f\in\bar{T}_2T_{02}$ follows $f(\mathbf{2})\in\{0,2\}$ and $f(\mathbf{2})\neq 2$, i.e., $f(\mathbf{2})=0$. From $f\in M_1$ and $y\leq 2$ for all $y\in E$ we get $f(x)\leq 0$ for all $x\in E^n$, i.e., f is a constant function with value 0 which is an element of $K_{10}K_{11}K_{12}$. □

2. Classification of T_0

The sets T_1, T_2, T_{01}, T_{02}, T_{12}, U_0, B_0, M_1 and M_2 are P_3-maximal sets. Among the 406 classes of P_3 exactly 248 classes are subsets of T_0. However, only 93 classes are obtained from the above nine P_3-maximal sets (as intersections of the sets or

Table 1. T_0-classes among P_3 classes.

No.	Is	My	Sim.	M_1M_2	U_0	B_0	T_1T_2	$T_{01}T_{12}T_{20}$	#classes	Lemmas
1	7	7	1	11	1	1	11	111	6	9
2	20	20	1	11	1	1	11	101	4	14
3	21	21	2	11	1	1	11	011	4	5
4	23	23	2	11	1	1	01	111	2	4, 7
5	26	26	1	11	1	0	11	111	4	13, 15
6	34	34	1	11	0	1	11	111	4	10
7	48	48	1	11	1	1	11	010	6	9
8	52	52	2	11	1	1	01	110	4	4
9	53	53	2	11	1	1	01	101	2	4, 7
10	54	54	2	11	1	1	01	011	2	4, 7
11	55	55	1	11	1	1	00	111	4	7
12	63	63	1	11	1	0	11	101	4	13, 15
13	64	64	2	11	1	0	11	011	3	5, 13
14	74	74	1	11	0	1	11	101	4	10
15	75	75	2	11	0	1	11	011	2	5, 10
16	76	76	1	11	0	0	11	111	2	6, 15, 17
17	88	88	2	11	1	1	01	010	4	4
18	89	89	2	11	1	1	01	001	2	4, 7
19	91	91	1	11	1	1	00	101	4	7
20	92	92	2	11	1	1	00	011	2	5, 7
21	99	99	1	11	1	0	11	010	4	13, 15
22	101	101	2	11	1	0	01	011	2	4, 7
23	114	114	1	11	0	1	11	010	4	10
24	116	116	2	11	0	1	01	101	2	4, 7
25	118	118	1	11	0	0	11	101	2	6, 15, 17
26	119	119	2	11	0	0	11	011	2	5, 6
27	133	133	2	01	1	1	10	101	2	4, 7
28	134	134	2	01	1	1	10	011	4	4
29	137	137	1	11	1	1	00	010	6	9
30	138	138	2	11	1	1	00	001	2	5, 7
31	149	149	2	11	1	0	01	010	3	4, 13
32	150	150	2	11	1	0	01	001	2	4, 7
33	162	162	2	11	0	1	01	001	2	4, 7
34	163	163	1	11	0	1	00	101	4	7
35	166	166	1	11	0	0	11	010	2	6, 8, 15
36	183	183	2	01	1	1	10	100	2	4, 7
37	184	184	2	01	1	1	10	010	3	4, 16
38	185	185	2	01	0	1	10	101	2	4, 7
39	191	194	1	11	1	1	00	000	4	14
40	194	197	1	11	1	0	00	010	4	13, 15
41	204	297	2	11	0	1	00	001	2	5, 7
42	210	213	2	11	0	0	01	001	2	4, 7
43	232	235	2	01	1	1	00	001	2	5, 7
44	234	237	2	01	1	0	10	010	3	4, 13
45	235	238	2	01	0	1	10	100	2	4, 7
46	254	263	1	11	1	0	00	000	4	13, 15
47	258	267	1	11	0	1	00	000	4	10
48	282	291	2	01	1	1	00	000	2	12, 14

Table 1 (continued).

No.	Is	My	Sim.	M_1M_2	U_0	B_0	T_1T_2	$T_{01}T_{12}T_{20}$	#classes	Lemmas
49	284	293	2	01	0	1	00	001	2	5, 7
50	309	321	2	01	1	0	11	011	3	5, 13
51	315	327	1	11	0	0	00	000	2	6, 8, 15
52	335	347	2	01	1	0	00	000	3	12, 13
53	336	348	2	01	0	1	00	000	2	10, 11
54	378	390	2	01	1	0	10	100	2	4, 7
55	381	393	2	01	1	0	01	100	2	4, 7
56	390	402	1	00	0	0	00	000	2	6, 8, 15
57	396	408	2	00	0	0	01	001	2	4, 7
58	405	417	1	00	0	0	11	010	1	18

their complements). The interchange 1 and 2 in the definition of each maximal set T_1, T_2, T_{01}, T_{12}, T_{02}, U_0, B_0, M_1, M_2, K_{10}, K_{11} and K_{12} yields T_2, T_1, T_{02}, T_{12}, T_{01}, U_0, B_0, M_2, M_1, K_{10}, K_{11} and K_{12}, respectively. The class T_0 is mapped onto itself. Two classes are *similar* if the characteristic vectors are obtained by one from the other by applying the above mapping to all coordinates of the vector, i.e., $a_i' = a_{i'}$, where $'$ denotes the above mapping of maximal sets. Among the 93 classes (the sum of the fourth column in Table 1), 58 are pairwise nonsimilar.

The complete classification of T_0 is obtained by checking all 8 possible cases with respect to the sets K_{10}, K_{11} and K_{12} for each of the above 93 classses. From Lemmas 1.3–1.18 we can show that many classes are empty. In Table 1 for each of the 58 nonsimilar classes with respect to the first 9 maximal sets we give the ordinal number of one of the corresponding classes of P_3 from [18, 3] (the second and the third column of the table). In the next to the last column we give the number of corresponding classes of the set T_0 obtained by concatenating the characteristic vectors corresponding to K_{10}, K_{11} and K_{12}. In the last column we indicate the lemmas, on the basis of which some of the 8 cases do not occur.

For each of the remaining 169 (the sum of the numbers of the next to the last column) classes, a representative function is shown in Fig. 1 (163 nonunary representatives, the other 6 representatives are unary, which are shown in the table directly by using the notation $s_{f(0)f(1)f(2)}$ or c_0 (0-constant function); a three-variable function $g(x, y, z)$ is represented in a matrix form, where the ith row corresponds to $x = i - 1$, $i = 1, 2, 3$ and the column is in the order of $yz = 00, 01, 10, 11, 12, 21, 22, 20, 02$). Counting the similarity (summing sim-column multiplied by #classes-column for all rows), we have:

Theorem 2.1 [8]. *The number of the classes of T_0 is* 253.

The classes are listed in Table 2. The representatives of the classes are listed in Table 3.

Table 2. Classes of T_0. The coordinates are: $K_{10}K_{11}K_{12}$, M_1M_2, U_0B_0, T_1T_2 and $T_{01}T_{12}T_{20}$.

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
12	1	111	11	11	11	111	
11	2	111	11	11	11	110	$g'4$
11	3	111	11	11	11	101	
11	4	111	11	11	11	011	
11	5	111	11	11	10	111	$g'6$
11	6	111	11	11	01	111	
11	7	111	11	10	11	111	
11	8	111	11	01	11	111	
11	9	110	11	11	11	111	
11	10	101	11	11	11	111	
11	11	011	11	11	11	111	
10	12	111	11	11	11	010	
10	13	111	11	11	10	110	
10	14	111	11	11	10	101	$g'17$
10	15	111	11	11	10	011	$g'16$
10	16	111	11	11	01	110	
10	17	111	11	11	01	101	
10	18	111	11	11	01	011	$g'13$
10	19	111	11	11	00	111	
10	20	111	11	10	11	110	
10	21	111	11	10	11	101	
10	22	111	11	10	11	011	$g'20$
10	23	111	11	01	11	110	
10	24	111	11	01	11	101	
10	25	111	11	01	11	011	$g'23$
10	26	110	11	11	11	110	
10	27	110	11	11	11	011	$g'26$
10	28	101	11	11	11	110	
10	29	101	11	11	11	101	
10	30	101	11	11	11	011	$g'28$
10	31	101	11	11	10	111	$g'32$
10	32	101	11	11	01	111	
10	33	101	11	10	11	111	
10	34	101	11	01	11	111	
10	35	100	11	11	11	111	
10	36	011	11	11	11	101	
10	37	011	11	01	11	111	
10	38	001	11	11	11	111	
9	39	111	11	11	10	100	$g'42$
9	40	111	11	11	10	010	$g'41$
9	41	111	11	11	01	010	
9	42	111	11	11	01	001	
9	43	111	11	11	00	110	
9	44	111	11	11	00	101	
9	45	111	11	11	00	011	$g'43$
9	46	111	11	10	11	010	
9	47	111	11	10	10	110	
9	48	111	11	10	01	011	$g'47$

Table 2 (continued).

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
9	49	111	11	01	11	010	
9	50	111	11	01	10	101	$g'51$
9	51	111	11	01	01	101	
9	52	111	11	00	11	110	
9	53	111	11	00	11	011	$g'52$
9	54	111	10	11	01	110	$g'58$
9	55	111	10	11	01	101	$g'57$
9	56	111	10	10	11	110	$g'59$
9	57	111	01	11	10	101	
9	58	111	01	11	10	011	
9	59	111	01	10	11	011	
9	60	110	11	11	11	010	
9	61	110	11	11	10	011	$g'62$
9	62	110	11	11	01	110	
9	63	101	11	11	11	010	
9	64	101	11	11	10	110	
9	65	101	11	11	10	101	$g'68$
9	66	101	11	11	10	011	$g'67$
9	67	101	11	11	01	110	
9	68	101	11	11	01	101	
9	69	101	11	11	01	011	$g'64$
9	70	101	11	11	00	111	
9	71	101	11	10	11	110	
9	72	101	11	10	11	101	
9	73	101	11	10	11	011	$g'71$
9	74	101	11	01	11	110	
9	75	101	11	01	11	101	
9	76	101	11	01	11	011	$g'74$
9	77	101	11	00	11	111	
9	78	100	11	11	11	110	
9	79	100	11	11	11	011	$g'78$
9	80	100	11	10	11	111	
9	81	011	11	11	11	010	
9	82	011	11	11	00	111	
9	83	011	11	01	11	101	
9	84	001	11	11	11	101	
9	85	001	11	01	11	111	
8	86	111	11	11	00	100	$g'88$
8	87	111	11	11	00	010	
8	88	111	11	11	00	001	
8	89	111	11	10	10	100	$g'92$
8	90	111	11	10	10	010	$g'91$
8	91	111	11	10	01	010	
8	92	111	11	10	01	001	
8	93	111	11	01	10	100	$g'94$
8	94	111	11	01	01	001	
8	95	111	11	01	00	101	
8	96	111	10	11	01	010	$g'100$

Table 2 (continued).

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
8	97	111	10	11	01	001	$g'99$
8	98	111	10	01	01	101	$g'101$
8	99	111	01	11	10	100	
8	100	111	01	11	10	010	
8	101	111	01	01	10	101	
8	102	110	11	11	10	010	$g'103$
8	103	110	11	11	01	010	
8	104	110	10	11	01	110	
8	105	110	01	11	10	011	$g'104$
8	106	101	11	11	10	100	$g'109$
8	107	101	11	11	10	010	$g'108$
8	108	101	11	11	01	010	
8	109	101	11	11	01	001	
8	110	101	11	11	00	110	
8	111	101	11	11	00	101	
8	112	101	11	11	00	011	$g'110$
8	113	101	11	10	11	010	
8	114	101	11	10	10	110	
8	115	101	11	10	01	011	$g'114$
8	116	101	11	01	11	010	
8	117	101	11	01	10	101	$g'118$
8	118	101	11	01	01	101	
8	119	101	11	00	11	110	
8	120	101	11	00	11	101	
8	121	101	11	00	11	011	$g'119$
8	122	101	10	11	01	110	$g'126$
8	123	101	10	11	01	101	$g'125$
8	124	101	10	10	11	110	$g'127$
8	125	101	01	11	10	101	
8	126	101	01	11	10	011	
8	127	101	01	10	11	011	
8	128	100	11	11	11	010	
8	129	100	11	11	10	011	$g'130$
8	130	100	11	11	01	110	
8	131	100	11	10	11	110	
8	132	100	11	10	11	101	
8	133	100	11	10	11	011	$g'131$
8	134	011	11	11	00	101	
8	135	011	11	01	11	010	
8	136	001	11	11	11	010	
8	137	001	11	11	00	111	
8	138	001	11	01	11	101	
8	139	000	11	10	11	111	
7	140	111	11	11	00	000	
7	141	111	11	10	00	010	
7	142	111	11	01	00	100	$g'142$
7	143	111	11	01	00	001	
7	144	111	11	00	10	100	$g'145$

Table 2 (continued).

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
7	145	111	11	00	01	001	
7	146	111	10	11	00	100	$g'151$
7	147	111	10	10	10	100	$g'154$
7	148	111	10	10	01	010	$g'153$
7	149	111	10	10	01	001	$g'152$
7	150	111	10	01	01	001	$g'155$
7	151	111	01	11	00	001	
7	152	111	01	10	10	100	
7	153	111	01	10	10	010	
7	154	111	01	10	01	001	
7	155	111	01	01	10	100	
7	156	110	11	11	00	010	
7	157	101	11	11	00	100	$g'159$
7	158	101	11	11	00	010	
7	159	101	11	11	00	001	
7	160	101	11	10	10	100	$g'163$
7	161	101	11	10	10	010	$g'162$
7	162	101	11	10	01	010	
7	163	101	11	10	01	001	
7	164	101	11	01	10	100	$g'165$
7	165	101	11	01	01	001	
7	166	101	11	01	00	101	
7	167	101	11	00	11	010	
7	168	101	10	11	01	010	$g'172$
7	169	101	10	11	01	001	$g'171$
7	170	101	10	01	01	101	$g'173$
7	171	101	01	11	10	100	
7	172	101	01	11	10	010	
7	173	101	01	01	10	101	
7	174	100	11	11	10	010	$g'175$
7	175	100	11	11	01	010	
7	176	100	11	10	11	010	
7	177	100	10	11	01	110	
7	178	100	10	10	11	110	s_{020}
7	179	100	01	11	10	011	$g'177$
7	180	100	01	10	11	011	s'_{020}
7	181	011	11	11	00	010	
7	182	011	11	01	00	101	
7	183	001	11	11	00	101	
7	184	001	11	01	11	010	
7	185	000	11	10	11	101	
7	186	000	11	00	11	111	
7	187	111	11	10	00	000	
7	188	111	11	01	00	000	
7	189	111	10	11	00	000	$g'191$
7	190	111	10	01	00	100	$g'192$
7	191	111	01	11	00	000	
7	192	111	01	01	00	001	

Table 2 (continued).

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
7	193	101	11	11	00	000	
7	194	101	11	10	00	010	
7	195	101	11	01	00	100	$g'196$
7	196	101	11	01	00	001	
7	197	101	11	00	10	100	$g'198$
7	198	101	11	00	01	001	
7	199	101	10	11	00	100	$g'204$
6	200	101	10	10	10	100	$g'207$
6	201	101	10	10	01	010	$g'206$
6	202	101	10	10	01	001	$g'205$
6	203	101	10	01	01	001	$g'208$
6	204	101	01	11	00	001	
6	205	101	01	10	10	100	
6	206	101	01	10	10	010	
6	207	101	01	10	01	001	
6	208	101	01	01	10	100	
6	209	100	11	11	00	010	
6	210	100	11	10	10	010	$g'211$
6	211	100	11	10	01	010	
6	212	100	10	11	01	010	
6	213	100	01	11	10	010	$g'212$
6	214	011	11	11	00	000	
6	215	001	11	11	00	010	
6	216	001	11	01	00	101	
6	217	000	11	10	11	010	
6	218	000	11	00	11	101	s_{021}
5	219	111	10	10	00	000	$g'221$
5	220	111	10	01	00	000	$g'222$
5	221	111	01	10	00	000	
5	222	111	01	01	00	000	
5	223	111	00	00	10	100	$g'224$
5	224	111	00	00	01	001	
5	225	101	11	10	00	000	
5	226	101	11	01	00	000	
5	227	101	10	11	00	000	$g'229$
5	228	101	10	01	00	100	$g'230$
5	229	101	01	11	00	000	
5	230	101	01	01	00	001	
5	231	100	11	10	00	010	
5	232	100	10	10	01	010	s_{010}
5	233	100	01	10	10	010	s'_{010}
5	234	011	11	01	00	000	
5	235	001	11	11	00	000	
5	236	000	11	00	11	010	
4	237	101	11	00	00	000	
4	238	101	10	10	00	000	$g'240$
4	239	101	10	01	00	000	$g'241$
4	240	101	01	10	00	000	

Table 2 (continued).

Wt	No	$K_{10}K_{11}K_{12}$	M_1M_2	U_0B_0	T_1T_2	$T_{01}T_{12}T_{20}$	Similar
4	241	101	01	01	00	000	
4	242	101	00	00	10	100	s'_{011}
4	243	101	00	00	01	001	s_{011}
4	244	100	11	10	00	000	
4	245	001	11	01	00	000	
4	246	000	11	10	00	010	
3	247	100	10	10	00	000	$g'248$
3	248	100	01	10	00	000	
3	249	000	11	10	00	000	
3	250	000	00	00	11	010	c_0
2	251	101	00	00	00	000	
2	252	000	11	00	00	000	
0	253	000	00	00	00	000	s_{012}

Table 3. Representatives of classes of T_0 (163 functions).

	xy								
f	00	01	02	10	11	12	20	21	22
$g1$	0	1	2	1	2	0	2	1	1
$g3$	0	2	0	2	2	1	0	1	1
$g4$	0	1	2	1	0	0	0	2	1
$g6$	0	2	1	2	1	0	1	0	1
$g7$	0	2	0	2	0	2	0	2	1
$g8$	0	2	1	1	0	0	1	0	0
$g10$	0	1	2	0	2	0	0	1	1
$g11$	0	2	1	2	0	2	1	1	0
$g12$	0	1	2	1	0	1	0	0	0
$g13$	0	1	0	1	2	0	0	0	2
$g16$	0	2	0	2	1	2	0	0	0
$g17$	0	2	1	2	1	1	0	1	1
$g19$	0	2	1	2	1	0	1	1	2
$g20$	0	2	2	0	0	0	2	0	0
$g21$	0	0	1	0	2	1	1	1	1
$g23$	0	1	2	2	0	0	2	0	0
$g24$	0	1	2	1	2	1	2	1	1
$g26$	0	2	0	1	2	0	0	2	0
$g28$	0	1	0	0	2	1	0	0	0
$g29$	0	2	0	0	2	1	0	1	1
$g32$	0	2	1	0	1	0	0	0	1
$g33$	0	2	0	0	0	2	0	0	1
$g35$	0	0	1	0	2	2	0	0	0
$g37$	0	2	1	2	0	0	1	0	0
$g38$	0	1	2	0	2	0	0	0	1

Table 3. (continued).

	00	01	02	10	11	12	20	21	22
$g41$	0	1	0	1	1	2	0	2	0
$g42$	0	1	2	0	1	1	2	2	1
$g43$	0	2	0	2	1	2	0	0	2
$g44$	0	0	1	2	1	1	1	1	2
$g46$	0	1	0	1	0	1	0	1	0
$g47$	0	0	2	0	2	0	2	0	2
$g51$	0	2	1	2	1	1	1	1	1
$g52$	0	2	2	2	0	0	2	0	0
$g57$	0	2	2	0	2	2	1	2	2
$g58$	0	0	1	0	0	1	1	1	2
$g59$	0	0	1	0	0	1	1	1	1
$g62$	0	2	0	1	1	0	0	0	0
$g63$	0	1	2	0	0	1	0	0	0
$g64$	0	1	0	0	2	0	0	0	2
$g67$	0	2	0	0	1	2	0	0	0
$g68$	0	2	0	0	1	2	0	1	1
$g70$	0	2	1	0	1	0	0	1	2
$g71$	0	0	0	0	2	1	0	1	0
$g72$	0	0	1	0	2	1	0	1	1
$g75$	0	1	2	0	2	1	0	1	1
$g78$	0	1	0	0	2	0	0	2	0
$g80$	0	0	1	0	2	0	0	0	1
$g81$	0	1	2	1	0	1	2	2	0
$g82$	0	2	1	1	1	0	2	0	2
$g83$	0	2	1	1	2	2	2	1	1
$g87$	0	1	2	0	1	0	2	0	2
$g88$	0	1	1	1	1	1	0	1	2
$g91$	0	0	2	0	1	2	2	2	0
$g92$	0	1	0	0	1	1	1	1	1
$g94$	0	1	1	1	1	2	2	1	1
$g95$	0	2	1	2	1	1	1	1	2
$g99$	0	1	2	0	2	2	2	2	2
$g101$	0	1	1	1	2	2	1	2	2
$g104$	0	2	0	1	1	1	0	2	0
$g108$	0	1	0	0	1	2	0	2	0
$g109$	0	1	0	0	1	1	0	2	1
$g110$	0	2	0	0	1	2	0	0	2
$g113$	0	0	0	0	0	1	0	1	0
$g114$	0	0	0	0	2	0	0	0	2
$g118$	0	2	1	0	1	1	0	1	1
$g119$	0	2	2	0	0	0	0	0	0
$g120$	0	0	0	0	2	1	0	1	1
$g125$	0	0	0	0	2	2	1	2	2
$g126$	0	0	1	0	0	1	0	1	2
$g127$	0	0	1	0	0	1	0	1	1
$g128$	0	0	0	0	0	0	2	1	0
$g130$	0	2	0	0	1	0	0	0	0
$g131$	0	2	0	0	2	0	0	0	0

Table 3. (continued).

	00	01	02	10	11	12	20	21	22
*g*132	0	0	0	2	2	2	0	1	1
*g*135	0	1	2	1	0	0	2	0	0
*g*136	0	1	2	0	0	1	0	2	0
*g*137	0	2	1	0	1	0	0	0	2
*g*138	0	2	1	0	2	2	0	1	1
*g*139	0	0	0	0	2	0	0	0	1
*g*140	0	1	0	1	1	1	0	2	2
*g*141	0	0	2	0	1	0	2	0	2
*g*151	0	0	1	0	1	1	1	1	2
*g*152	0	0	2	0	2	2	2	2	2
*g*153	0	0	2	0	0	2	2	2	2
*g*154	0	0	1	0	1	1	1	1	1
*g*155	0	1	2	1	2	2	2	2	2
*g*162	0	0	2	0	1	0	0	2	0
*g*163	0	0	0	0	1	2	1	1	1
*g*166	0	0	0	2	1	1	1	1	2
*g*173	0	1	1	0	2	2	0	2	2
*g*175	0	0	2	0	1	1	0	0	0
*g*176	0	0	0	0	0	0	0	1	0
*g*177	0	2	0	0	1	0	0	2	0
*g*182	0	2	1	1	1	2	2	1	2
*g*186	0	2	1	0	0	0	0	0	0
*g*191	0	0	2	1	1	2	2	2	2
*g*192	0	1	1	1	1	1	1	1	2
*g*193	0	1	0	0	1	1	0	2	2
*g*204	0	0	1	0	1	1	0	1	2
*g*205	0	0	2	0	2	2	0	2	2
*g*206	0	0	2	0	0	2	0	2	2
*g*207	0	0	1	0	1	1	0	1	1
*g*208	0	1	2	0	2	2	0	2	2
*g*209	0	0	2	0	1	1	0	0	2
*g*211	0	1	0	0	1	0	0	0	0
*g*216	0	2	1	0	1	1	0	2	2
*g*217	0	0	0	0	0	2	0	1	0
*g*221	0	0	2	0	1	2	2	2	2
*g*222	0	1	2	1	1	2	2	2	2
*g*224	0	1	1	1	1	1	1	1	1
*g*230	0	1	1	0	1	1	0	1	2
*g*231	0	0	2	0	1	0	0	0	2
*g*234	0	1	2	1	1	2	2	1	2
*g*236	0	1	2	0	0	0	0	0	0
*g*240	0	0	2	0	1	2	0	2	2
*g*241	0	1	2	0	1	2	0	2	2
*g*245	0	1	2	0	1	1	0	2	2
*g*246	0	0	0	0	1	0	0	0	2
*g*248	0	0	2	0	1	2	0	1	2
*g*251	0	0	0	0	1	1	0	1	2

Table 3 (continued).

*g*9	*g*34	*g*36	*g*49	*g*60	*g*74
000200000	001211110	000200100	011111222	000100000	002000020
000000000	000000000	100212100	000000000	000000000	001000020
100000000	000000000	200212100	000000000	200000000	001000020

*g*77	*g*84	*g*85	*g*100	*g*103	*g*111
000211100	000012000	002121210	000011200	000100000	000200000
000000000	000211100	000000000	000022211	000100000	000121100
000000000	000222100	000000000	222222222	200000000	000111210

*g*116	*g*134	*g*143	*g*145	*g*156	*g*158
010000002	000200100	010000001	010000001	000100000	000020000
010000001	100112100	001111110	001111110	000100000	000101000
010000002	200212200	001111210	001111110	200000200	000010200

*g*159	*g*165	*g*167	*g*171	*g*172	*g*181
000100000	001112110	000000000	000010200	000000000	011000022
000121100	000111100	010000001	000222200	000022200	000100000
000111210	000111100	010000002	000222200	001222220	000000200

*g*183	*g*184	*g*185	*g*187	*g*188	*g*194
002012010	010112202	000012000	000020000	000111200	000000000
000122100	000000000	000212100	000121100	100111100	000101000
000211200	000000000	000212100	200222200	200111200	000010200

*g*196	*g*198	*g*212	*g*214	*g*215	*g*225
001112110	001000010	000120000	001021020	000112200	000100000
000111100	000111100	001111000	000111101	000101200	000121100
000111200	000111100	000120000	020222200	000120200	000111200

*g*226	*g*229	*g*235	*g*237	*g*244	*g*249
001000020	000020200	000021000	000000000	000010000	001100220
000111100	001121210	000112111	000112200	000112100	000112200
000111200	001122220	022212200	000111200	000212200	000112200

*g*252
001000020
000112200
000112200

Table 4

Rank	1	2	3	4	5	6	Total
Bases	1	4,492	234,031	552,927	91,377	892	883,720
Pivotal incomplete sets	251	21,363	202,689	149,804	6,598	8	380,710

3. Enumeration of bases of T_0

Using the list of the 253 characteristic vectors the T_0-bases and T_0-pivotal incomplete sets are computed [8]: their numbers are 883,720 and 380,710, respectively. The maximal rank of a base of T_0 is 6. Data for each rank are shown in Table 4.

Two algorithms for enumeration of classes of bases are given in [5] and [14] (also cf. [19]).

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