# Q 8.4

**8.4.** Consider the sequence $x[n]$ given by $x[n] = \alpha^n u[n]$. Assume $|\alpha| < 1$. A periodic sequence $\tilde{x}[n]$ is constructed from $x[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n + rN].$$

**(a)** Determine the Fourier transform $X(e^{j\omega})$ of $x[n]$.
**(b)** Determine the DFS coefficients $\tilde{X}[k]$ for the sequence $\tilde{x}[n]$.
**(c)** How is $\tilde{X}[k]$ related to $X(e^{j\omega})$?

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# Q 8.6

**8.6.** Consider the complex sequence

$$x[n] = \begin{cases} e^{j\omega_0 n}, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$$

**(a)** Find the Fourier transform $X(e^{j\omega})$ of $x[n]$.
**(b)** Find the $N$-point DFT $X[k]$ of the finite-length sequence $x[n]$.
**(c)** Find the DFT of $x[n]$ for the case $\omega_0 = 2\pi k_0/N$, where $k_0$ is an integer.

# Q 8.8

**8.8.** Let $X(e^{j\omega})$ denote the Fourier transform of the sequence $x[n] = (0.5)^n u[n]$. Let $y[n]$ denote a finite-duration sequence of length 10; i.e., $y[n] = 0, n < 0$, and $y[n] = 0, n \geq 10$. The 10-point DFT of $y[n]$, denoted by $Y[k]$, corresponds to 10 equally spaced samples of $X(e^{j\omega})$; i.e., $Y[k] = X(e^{j2\pi k/10})$. Determine $y[n]$.

# Q 8.10

**8.10.** The two eight-point sequences $x_1[n]$ and $x_2[n]$ shown in Figure P8.10 have DFTs $X_1[k]$ and $X_2[k]$, respectively. Determine the relationship between $X_1[k]$ and $X_2[k]$.

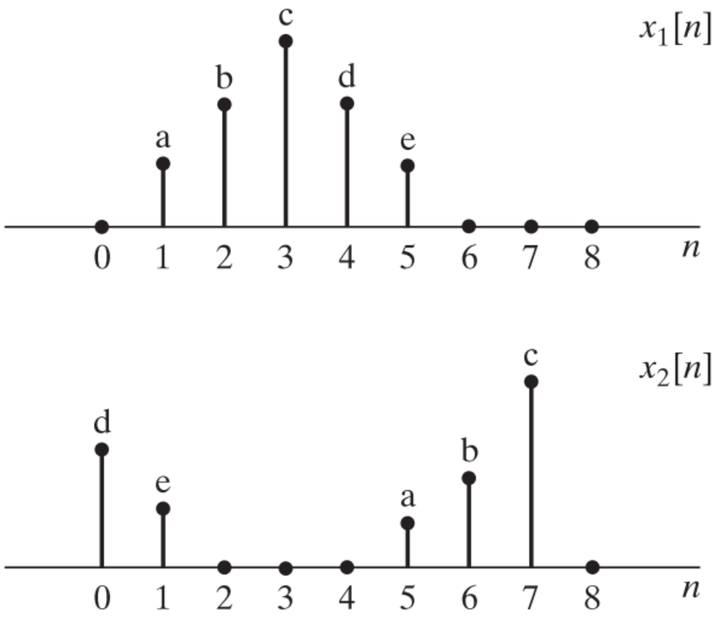


**Figure P8.10**

# Q 8.12

**8.12.** Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \cos\left(\frac{\pi n}{2}\right), \quad n = 0, 1, 2, 3,$$
$$h[n] = 2^n, \quad n = 0, 1, 2, 3.$$

**(a)** Calculate the four-point DFT $X[k]$.
**(b)** Calculate the four-point DFT $H[k]$.
**(c)** Calculate $y[n] = x[n] \textcircled{4} h[n]$ by doing the circular convolution directly.
**(d)** Calculate $y[n]$ of part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

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# Q 8.11

**8.11.** Figure P8.11 shows two finite-length sequences $x_1[n]$ and $x_2[n]$. Sketch their six-point circular convolution.

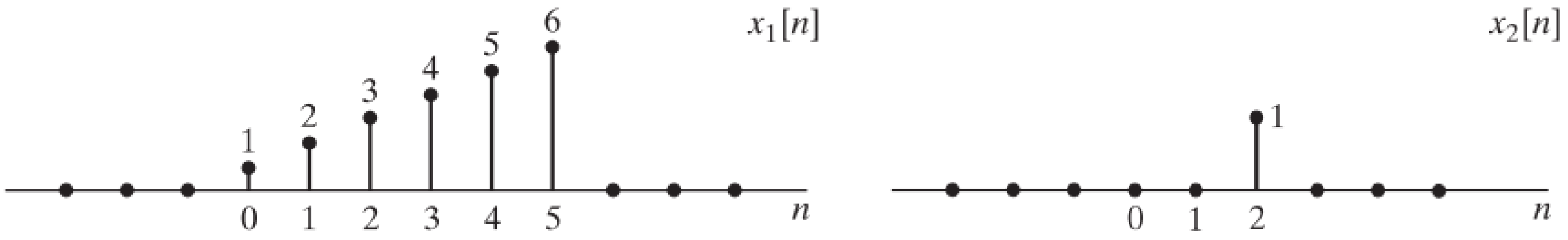


**Figure P8.11**

# Q 8.14

**8.14.** Two finite-length signals, $x_1[n]$ and $x_2[n]$, are sketched in Figure P8.14. Assume that $x_1[n]$ and $x_2[n]$ are zero outside of the region shown in the figure. Let $x_3[n]$ be the eight-point circular convolution of $x_1[n]$ with $x_2[n]$; i.e., $x_3[n] = x_1[n] \circledast_8 x_2[n]$. Determine $x_3[2]$.

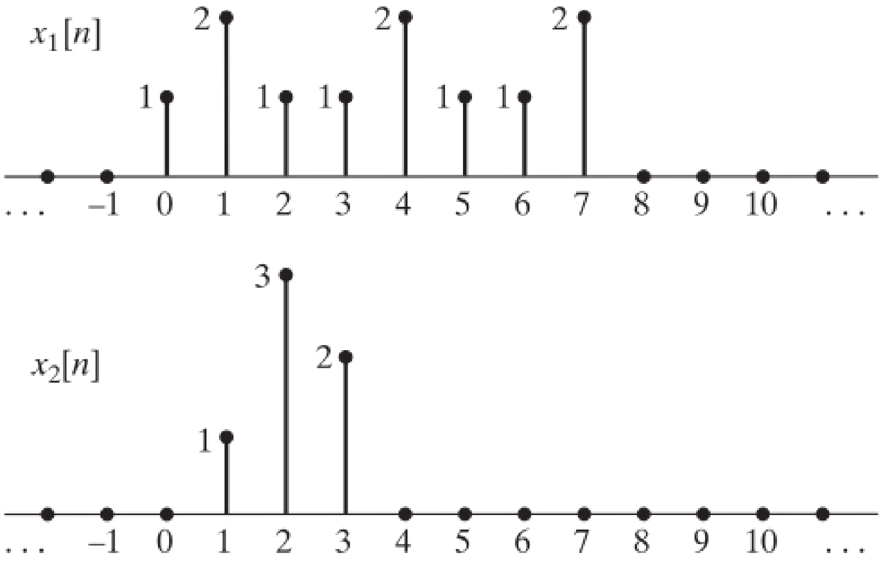


Figure P8.14

# Q 8.18

**8.18.** Figure P8.18-1 shows a sequence $x[n]$ for which the value of $x[3]$ is an unknown constant $c$. The sample with amplitude $c$ is not necessarily drawn to scale. Let

$$X_1[k] = X[k]e^{j2\pi 3k/5},$$

where $X[k]$ is the five-point DFT of $x[n]$. The sequence $x_1[n]$ plotted in Figure P8.18-2 is the inverse DFT of $X_1[k]$. What is the value of $c$?

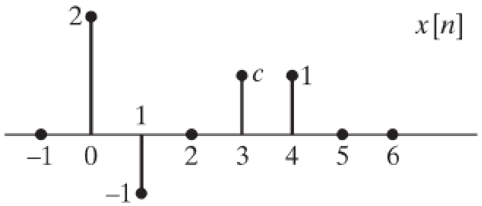


Figure P8.18-1

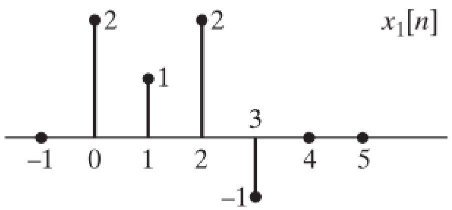


Figure P8.18-2

# Q 8.38

**8.34.** Suppose that $x_1[n]$ and $x_2[n]$ are two finite-length sequences of length $N$, i.e., $x_1[n] = x_2[n] = 0$ outside $0 \le n \le N-1$. Denote the $z$-transform of $x_1[n]$ by $X_1(z)$, and denote the $N$-point DFT of $x_2[n]$ by $X_2[k]$. The two transforms $X_1(z)$ and $X_2[k]$ are related by:

$$X_2[k] = X_1(z)\Big|_{z=\frac{1}{2}e^{-j\frac{2\pi k}{N}}}, \quad k = 0, 1, \ldots, N-1$$

Determine the relationship between $x_1[n]$ and $x_2[n]$.

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# Q 8.24

**8.24.** Figure P8.24 shows a finite-length sequence $x[n]$. Sketch the sequences

$$x_1[n] = x[((n-2))_4], \qquad 0 \le n \le 3,$$

and

$$x_2[n] = x[((-n))_4], \qquad 0 \le n \le 3.$$

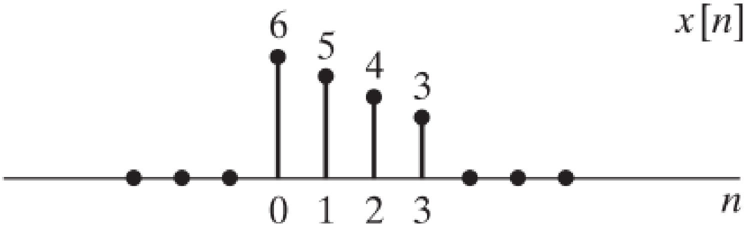


**Figure P8.24**

# Q 8.25

**8.25.** Consider the signal $x[n] = \delta[n-4] + 2\delta[n-5] + \delta[n-6]$.

**(a)** Find $X(e^{j\omega})$ the discrete-time Fourier transform of $x[n]$. Write expressions for the magnitude and phase of $X(e^{j\omega})$, and sketch these functions.

**(b)** Find all values of $N$ for which the $N$-point DFT is a set of real numbers.

**(c)** Can you find a three-point causal signal $x_1[n]$ (i.e., $x_1[n] = 0$ for $n < 0$ and $n > 2$) for which the three-point DFT of $x_1[n]$ is:

$$X_1[k] = |X[k]| \qquad k = 0, 1, 2$$

where $X[k]$ is the three-point DFT of $x[n]$?

# Q 8.33

**8.33.** An FIR filter has a 10-point impulse response, i.e.,

$$h[n] = 0 \qquad \text{for } n < 0 \text{ and for } n > 9.$$

Given that the 10-point DFT of $h[n]$ is given by

$$H[k] = \frac{1}{5}\delta[k-1] + \frac{1}{3}\delta[k-7],$$

find $H(e^{j\omega})$, the DTFT of $h[n]$.

# Q 8.46

**8.46.** Each part of this problem may be solved independently. All parts use the signal $x[n]$ given by

$$x[n] = 3\delta[n] - \delta[n-1] + 2\delta[n-3] + \delta[n-4] - \delta[n-6].$$

**(a)** Let $X\left(e^{j\omega}\right)$ be the DTFT of $x[n]$. Define

$$R[k] = X\left(e^{j\omega}\right)\Big|_{\omega=\frac{2\pi k}{4}}, \qquad 0 \le k \le 3$$

Plot the signal $r[n]$ which is the four-point inverse DFT of $R[k]$.

**(b)** Let $X[k]$ be the eight-point DFT of $x[n]$, and let $H[k]$ be the eight-point DFT of the impulse response $h[n]$ given by

$$h[n] = \delta[n] - \delta[n-4].$$

Define $Y[k] = X[k]H[k]$ for $0 \le k \le 7$. Plot $y[n]$, the eight-point DFT of $Y[k]$.

# Q 8.50

**8.50.** The DFT of a finite-duration sequence corresponds to samples of its $z$-transform on the unit circle. For example, the DFT of a 10-point sequence $x[n]$ corresponds to samples of $X(z)$ at the 10 equally spaced points indicated in Figure P8.50-1. We wish to find the equally spaced samples of $X(z)$ on the contour shown in Figure P8.50-2; i.e., we wish to obtain

$$X(z)\big|_{z=0.5e^{j[(2\pi k/10)+(\pi/10)]}}.$$

Show how to modify $x[n]$ to obtain a sequence $x_1[n]$ such that the DFT of $x_1[n]$ corresponds to the desired samples of $X(z)$.

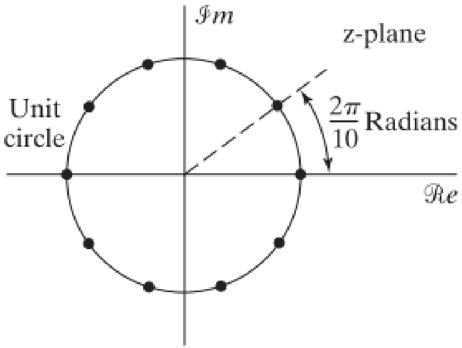


**Figure P8.50-1**

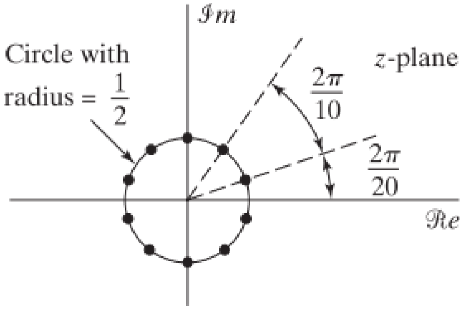


**Figure P8.50-2**