Tutorial 4

Q 7.9

* For impulse invariance, we have:

$$\omega_c = \Omega_c T_d$$

$T_d \rightarrow$ sampling interval.

$$\omega_c = [2\pi * 1000] * 0.2 \times 10^{-3}$$
$$= 0.4\pi \text{ rad.}$$

Q 7.10

* For the bilinear transformation method:

$$\omega_c = 2\tan^{-1}\left(\Omega_c \frac{T_d}{2}\right)$$
$$= 2\tan^{-1}\left[\frac{2\pi(2000)(0.4\times 10^{-3})}{2}\right]$$

$$\omega_c = 0.76\pi \text{ rad}$$

Q 7.15

max. passband error: (δ_p)

$1-\delta_p = 0.95$ or $1+\delta_p = 1.05$

$\therefore \delta_p = 0.05$

$$\delta_p)_{dB} = 20\log_{10}(0.05) \cong -26 \text{ dB}$$

max. stopband error: $\delta_s' = 0.1$

$$\delta_s)_{dB} = 20\log_{10}(0.1) = -20 \text{ dB}$$

* Therefore, this needs a window with peak approximation error of -26dB

* From Table 2 in the Book:
  Hann, Hamming, Blackman windows meet this requirement.

* Han: $0.1\pi = \frac{8\pi}{M} \Rightarrow M = 80$

filter length: $L = M + 1$

* Hamming: $0.1\pi = \frac{8\pi}{M} \Rightarrow M = 80$

* Blackman: $0.1\pi = \frac{12\pi}{M} \rightarrow M = 120$

Q 7.17

* From impulse invariance:
  $\omega = \Omega T_d$

∴ specs of continuous-time filter are:

$-0.02 < H(j\Omega) < 0.02 \qquad 0 \leq |\Omega| \leq 2\pi(20)$

$0.95 < H(j\Omega) < 1.05 \qquad 2\pi(30) \leq |\Omega| \leq 2\pi(70)$

$-0.001 < H(j\Omega) < 0.001 \qquad 2\pi(75) \leq |\Omega| \leq 2\pi(100)$

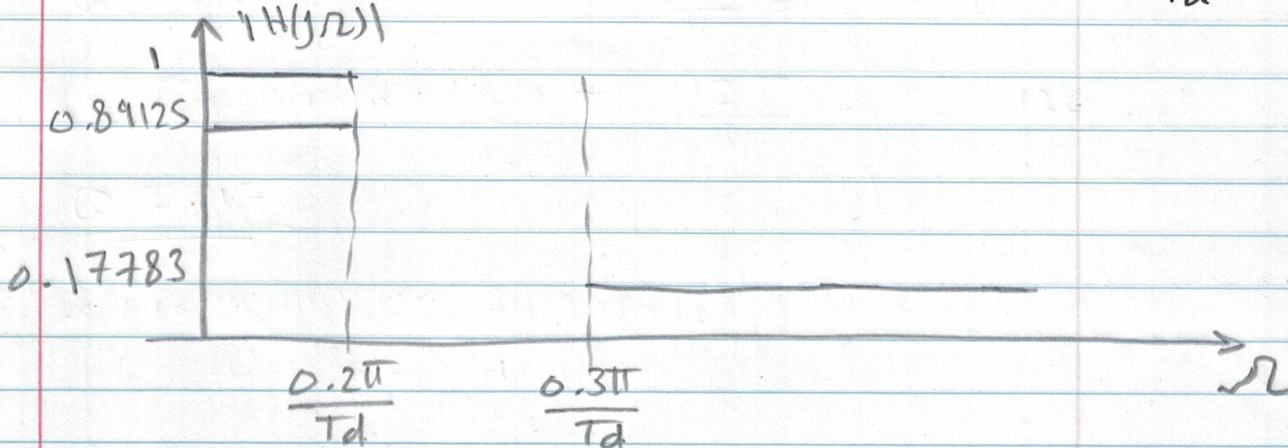
Q 7.2

* $\omega = \Omega T_d$

(a) From the specs of the discrete-time filter, we can get the specs of continuous-time filter $\Rightarrow \omega = \Omega T_d$

$\therefore \quad 0.89125 \leq |H_c(j\Omega)| \leq 1 \qquad 0 \leq |\Omega| \leq \dfrac{0.2\pi}{T_d}$

$|H_c(j\Omega)| \leq 0.17783 \qquad \dfrac{0.3\pi}{T_d} \leq |\Omega| \leq \dfrac{\pi}{T_d}$



(b) The Butterworth filter response is monotonic :

$\therefore$ at $\Omega = \dfrac{0.2\pi}{T_d} \Rightarrow \left|H_c\left(j\,\dfrac{0.2\pi}{T_d}\right)\right|^2 = \dfrac{1}{1 + \left(\dfrac{0.2\pi}{\Omega_c T_d}\right)^{2N}}$

and $|H_c(0.3\pi/T_d)|^2 = \dfrac{1}{1 + \left(\dfrac{0.3\pi}{\Omega_c T_d}\right)^{2N}}$

Solving these give $\Rightarrow \Omega_c T_d = 0.70474$, $N = 5.8858$

* N has to be integer
  $\therefore N = 6$

we note that these values ($\Omega_c T_d$ & N) don't satisfy the discrete-time specs

So, we solve again with $N = 6$ to find $\Omega_c T_d = 0.7032$

$\therefore N = 6$, $\Omega_c T_d = 0.7032 \Rightarrow$ meet the specs.

(c) from the previous result $\Rightarrow \Omega_c T_d = 0.7032$,

The poles of $|H_c(j\Omega)|^2$ are evenly distributed around a circle of radius (0.7032). Therefore, $H_c(s)$ is formed from the left-half plane of $|H_c(j\Omega)|^2$, and the result doesn't depend on T_d, hence $H(z)$ doesn't depend on T_d.

Ce 7.28

(a) $H_1(z) = H_c(s)\Big|_{s = \frac{1-z^{-1}}{1+z^{-1}}}$

$$S = \frac{1-z^{-1}}{1+z^{-1}} = \frac{1-e^{-j\omega}}{1+e^{-j\omega}} = j\Omega$$

$$\Omega = \frac{e^{j\omega/2} - e^{-j\omega/2}}{e^{j\omega/2} + e^{j\omega/2}} = \tan(\omega/2)$$

$$\therefore \Omega_p = \tan\left(\frac{\omega_{p_1}}{2}\right) \Rightarrow \omega_{p_1} = 2\tan^{-1}(\Omega_p)$$

(b) $H_2(z) = H_c(s)\Big|_{s = \frac{1+z^{-1}}{1-z^{-1}}}$

$$S = \frac{1+z^{-1}}{1-z^{-1}} = \frac{1+e^{-j\omega}}{1 \cancel{+} - e^{-j\omega}} = j\Omega$$

$$\Omega = \frac{e^{j\omega/2} + e^{-j\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}} = -\cot(\omega/2)$$

$$= \tan\left(\frac{\omega - \pi}{2}\right)$$

$$\therefore \Omega_p = \tan\left(\frac{\omega_{p_2} - \pi}{2}\right) \Rightarrow \omega_{p_2} = \pi + 2\tan^{-1}(\Omega_p)$$

(c) $$\tan\left(\frac{\omega_{p_2} - \pi}{2}\right) = \tan\left(\frac{\omega_{p_1}}{2}\right)$$

$$\omega_{p_2} = \omega_{p_1} + \pi$$

Tutorial 9

Q 7.5

(a) Using the min. specifications:

$\delta = 0.01$

$\Delta\omega = 0.05\pi$

$A = -20\log_{10}(\delta) = -20\cdot(-2) = 40$

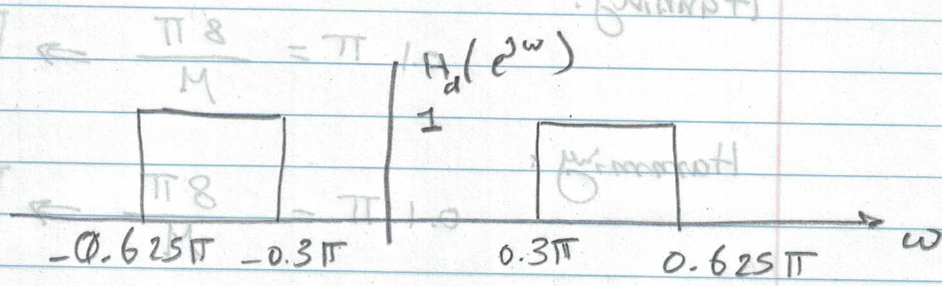
$$M = \frac{A-8}{2.285\,\Delta\omega} = 89.2 \rightarrow 90$$

$$\beta = 0.5842(A-21)^{0.4} + 0.07886(A-21)$$
$$= 3.395$$

(b) The filter is a linear phase filter

$\therefore$ The filter delay is $90/2 = 45$ samples.

(c)



$$h_d[n] = \frac{\sin(0.625\pi(n-45)) - \sin(0.3\pi(n-45))}{\pi(n-45)}$$

Q 7.15

max. Passband error $\delta_p = 0.05$
max stop band error $\delta_s = 0.1$

$$\delta_p = -26 \text{ dB} \qquad \delta_s = -20 \text{ dB}$$

∴ The required window of peak approximation error less than -26 dB:

- Hanning
- Hamming
- Blackman

} Table 7.1

* From the Table, main lobe width can be obtained.

Hanning:

$$0.1\pi = \frac{8\pi}{M} \Rightarrow \boxed{M = 80}$$

Hamming:

$$0.1\pi = \frac{8\pi}{M} \Rightarrow \boxed{M = 80}$$

Blackman:

$$0.1\pi = \frac{12\pi}{M} \Rightarrow \boxed{M = 120}$$

Q 7.22

$h[n] \rightarrow$ linear phase, $\delta_1 = 0.01$, $\delta_2 = 0.001$,
$w_p = 0.4\pi$, $w_s = 0.6\pi$, $L = 28$

$f_s = 10{,}000$ samples/sec.

(a) Since $f_s = 10$ ksamples/sec $\Rightarrow x_c(t)$ must be bandlimited to 5 KHz to ensure no aliasing will happen.

Practically: $x_c(t)$ can be limited to 7 KHz

The freq. components from 5 to 7 KHz will aliase to the range

$\Omega = 2\pi(3000)$ to $2\pi(5000)$

$\omega = \frac{2\pi}{f_s} = 0.6\pi$ to π

which fall in the stopband of the low pass filter.

(b) for continuous time specs:

$\Omega_p = w_p/T = 0.4\pi * 10{,}000 = 2\pi(2000)$ rad/sec.

$\Omega_s = w_s/T = 0.6\pi \cdot 10{,}000 = 2\pi(3000)$ rad/sec.

$\therefore \quad (1-\delta_1) \leq |H_{eff}(j\Omega)| \leq (1+\delta_1) \qquad |\Omega| \leq \Omega_p$

$|H_{eff}(j\Omega)| \leq \delta_2 \qquad \Omega_s \leq \Omega \leq 2\pi(5000)$

$0.99 \leq |H_{eff}(j\Omega)| \leq 1.02 \qquad |\Omega| \leq 2\pi(2000)$

$|H_{eff}(j\Omega)| \leq 0.001 \qquad 2\pi(3000) \leq \Omega \leq 2\pi(5000)$

(c) The filter given is a linear phase with length $= 28 \Rightarrow$ group delay $= 27/2 = 13.5$ samples.

$$\therefore \text{Delay} = T \cdot (13.5) = 10^{-4} \cdot (13.5)$$
$$= 1.35 \text{ msec.}$$

Q 7.27

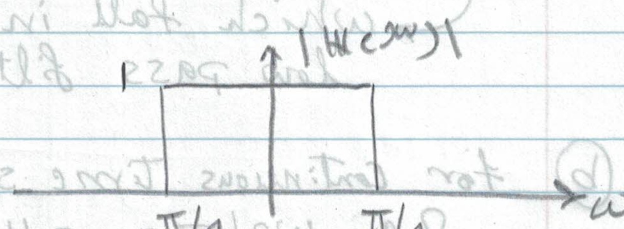
(a) $h_1[n] = h[2n]$

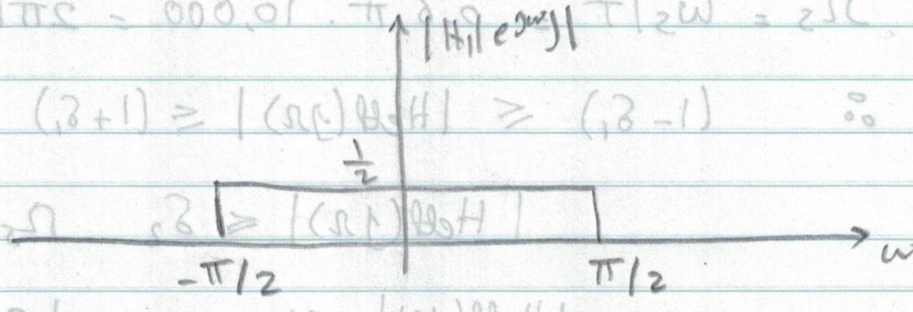
$$H_1(e^{jw}) = \sum_{n=-\infty}^{\infty} h_1[n] e^{jwn} = \sum_{n=-\infty}^{\infty} h[2n] e^{jwn}$$

$$= \sum_{n \text{ even}} h[n] e^{jwn/2}$$

$$= \sum \tfrac{1}{2}\left[h[n] + (-1)^n h[n]\right] e^{jwn/2}$$

$$H_1(e^{jw}) = \tfrac{1}{2} H(e^{jw/2}) + \tfrac{1}{2} H\left(e^{j(\frac{w}{2}+\pi)}\right)$$



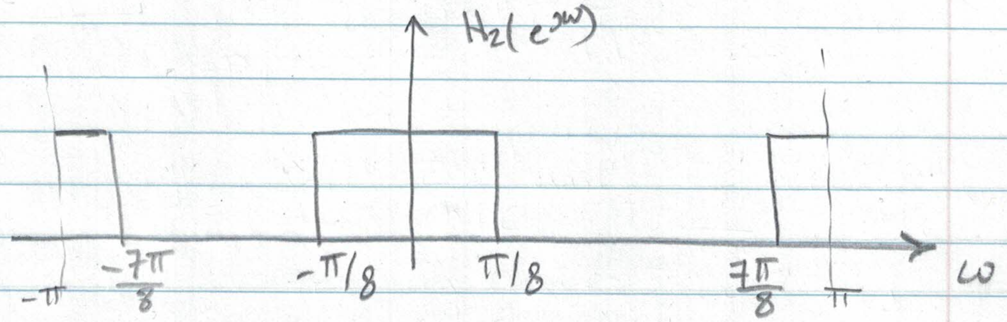


(b) $h_2[n] = \begin{cases} h[n/2] & n \text{ is even} \\ 0 & \text{o.w} \end{cases}$

$$H_2(e^{j\omega}) = \sum_{n \text{ even}} h[n/2]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega 2n}$$

$$H_2(e^{j\omega}) = H(e^{j2\omega})$$



(c) $h_3[n] = e^{j\pi n}\, h[n] = (-1)^n\, h[n]$

$$H_3(e^{j\omega}) = H(e^{j(\omega+\pi)})$$