For impulse invariance, we have:

\[ w_c = \frac{\omega_c}{T_d} \rightarrow \text{sampling interval.} \]

\[ w_c = \left[ 2\pi \times 1000 \right] \times 0.2 \times 10^{-3} = 0.4 \pi \text{ rad.} \]

For the bilinear transformation method:

\[ w_c = 2 \tan^{-1} \left( \frac{\omega_c T_d}{2} \right) \]

\[ = 2 \tan^{-1} \left[ \frac{2\pi (2000) (0.4 \times 10^{-3})}{2} \right] \]

\[ w_c = 0.76 \pi \text{ rad} \]

max. passband error: \( (\delta p) \)

\[ 1 - \delta p = 0.95 \quad \text{or} \quad 1 + \delta p = 1.05 \]

\[ \delta p = 0.05 \]

\[ \delta p) \text{dB} = 20 \log_{10} (0.05) \approx -26 \text{ dB} \]

max. stopband error: \( \delta s' = 0.1 \)

\[ \delta s') \text{dB} = 20 \log_{10} (0.1) = -20 \text{ dB} \]
Therefore, this needs a window with peak approximation error of -26 dB.

* From Table 2 in the book: Hann, Hamming, Blackman windows meet this requirement.

* Hann: \( 0.1 \pi = \frac{8 \pi}{M} \Rightarrow M = 80 \)
  filter length: \( L = M + 1 \)

* Hamming: \( 0.1 \pi = \frac{8 \pi}{M} \Rightarrow M = 80 \)

* Blackman: \( 0.1 \pi = \frac{12 \pi}{M} \Rightarrow M = 120 \)

\[ 7.17 \]

* From impulse invariance:
  \[ \omega = \pi \frac{\tau}{T} \]

Specs of continuous-time filter are:

\[ -0.02 < H(j \omega) < 0.02 \quad 0 \leq |\omega| \leq 2\pi(20) \]

\[ 0.95 < H(j \omega) < 1.05 \quad 2\pi(30) \leq |\omega| \leq 2\pi(70) \]

\[ -0.001 < H(j \omega) < 0.001 \quad 2\pi(75) \leq |\omega| \leq 2\pi(100) \]
(a) From the specs of the discrete-time filter, we can get the specs of continuous-time filter \( \omega = \frac{\omega_c}{T_d} \).

\[
0.89125 \leq |H_c(j \omega)| \leq 1 \quad 0 \leq |\omega| \leq \frac{0.2 \pi}{T_d}
\]

\[
|H_c(j \omega)| \leq 0.17783 \quad \frac{0.3 \pi}{T_d} \leq |\omega| \leq \frac{\pi}{T_d}
\]

(b) The Butterworth filter response is monotonic:

So at \( \omega = \frac{0.2 \pi}{T_d} \) \( |H_c(0.2 \pi/T_d)|^2 = \frac{1}{1 + \left(\frac{0.2 \pi}{\omega_c T_d}\right)^{2N}} \)

and \( |H_c(0.3 \pi/T_d)|^2 = \frac{1}{1 + \left(\frac{0.3 \pi}{\omega_c T_d}\right)^{2N}} \)

Solving these give \( \omega_c T_d = 0.76474 \), \( N = 5.8858 \).
* N has to be integer
  \[ N = 6 \]

We note that these values \((R_c T_d & N)\) don't satisfy the discrete-time specs

So, we solve again with \(N = 6\) to find \(R_c T_d = 0.7032\)

\[ N = 6, \quad R_c T_d = 0.7032 \rightarrow \text{meet the specs.} \]

C) \(R_c T_d = 0.7032\), the poles of \(|H_c(j\omega)|^2\) are evenly distributed around a circle of radius 0.7032. Therefore, \(H_c(\omega)\) is formed from the left-half plane of \(|H_c(j\omega)|^2\), and the result doesn't depend on \(T_d\), hence \(H(z)\) doesn't depend on \(T_d\).
(a) \[ H_1(z) = H_C(s) \bigg| s = \frac{1 - z^{-1}}{1 + z^{-1}} \]
\[ S = \frac{1 - z^{-1}}{1 + z^{-1}} = \frac{1 - e^{-j\omega}}{1 + e^{-j\omega}} = j \Omega \]
\[ \Omega = \frac{e^{j\omega/2} - e^{-j\omega/2}}{e^{j\omega/2} + e^{-j\omega/2}} = \tan(w/2) \]
\[ \Rightarrow \Omega_P = \tan\left(\frac{\omega P_1}{2}\right) \Rightarrow \omega P_1 = 2 \tan^{-1}(\Omega_P) \]

(b) \[ H_2(z) = H_C(s) \bigg| s = \frac{1 + z^{-1}}{1 - z^{-1}} \]
\[ S = \frac{1 + z^{-1}}{1 - z^{-1}} = \frac{1 + e^{j\omega}}{1 - e^{j\omega}} = j \Omega \]
\[ \Omega = \frac{e^{j\omega/2} + e^{-j\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}} = -\cot(w/2) \]
\[ = \tan\left(\frac{\omega - \pi}{2}\right) \]
\[ \Rightarrow \Omega_P = \tan\left(\frac{\omega P_2 - \pi}{2}\right) \Rightarrow \omega P_2 = \pi + 2 \tan^{-1}(\Omega_P) \]

(c) \[ \tan\left(\frac{\omega P_2 - \pi}{2}\right) = \tan\left(\frac{\omega P_1}{2}\right) \]
\[ \omega P_2 = \omega P_1 + \pi \]
Q. 7.5

(a)
Using the min. specifications:

\[ R_{S} = 0.01 \quad \Omega \]
\[ \Delta \omega = 0.05 \pi \]
\[ A = 20 \log_{10} (8) = -20 \cdot (-2) = 40 \]
\[ M = \frac{A - 8}{2.285 \Delta \omega} = 80.2 \Rightarrow 90 \]
\[ R = 0.58 + 0.21(4) + 0.07886(4) \approx 3.395 \]

(b) The filter is a linear phase filter.

.\ The filter delay is 90/2 = 45 samples.

(c)
\[ h[n] = \frac{\sin(0.625 \pi (n-45)) - \sin(0.3 \pi (n-45))}{\pi (n-45)} \]
max. passband error $SP = 0.05$
max. stopband error $SS = 0.1$

$SP = -26$ dB $SS = -20$ dB

The required window of peaks approximation error less than $-26$ dB:
- Hanning
- Hamming
- Blackman

* From the table, main lobe width can be obtained.

Hanning:

$$0.1 \pi = \frac{8 \pi}{M} \Rightarrow M = 80$$

Hamming:

$$0.1 \pi = \frac{8 \pi}{M} \Rightarrow M = 80$$

Blackman:

$$0.1 \pi = \frac{12 \pi}{M} \Rightarrow M = 120$$
Problem 7.22

\[ h[n] \rightarrow \text{linear phase, } \delta_1 = 0.01, \delta_2 = 0.001, \]
\[ \omega_p = 0.4\pi, \omega_s = 0.6\pi, \quad L = 28 \]
\[ f_s = 10,000 \text{ samples/sec}. \]

(a) Since \( f_s = 10 \text{ ksamples/sec} \Rightarrow \chi_c(t) \) must be bandlimited to 5 kHz to ensure no aliasing will happen.

Practically: \( \chi_c(t) \) can be limited to 7 kHz.

The freq. components from 5 to 7 kHz will alias to the range

\[ \Omega = 2\pi(3000) \text{ to } 2\pi(5000) \]

\[ \omega = \frac{2\pi}{f_s} = 0.6\pi \text{ to } \pi \]

which fall in the stop band of the low pass filter.

(b) For continuous time specs:

\[ \Omega_p = \omega_p / T = 0.4\pi \times 10,000 = 2\pi(2000) \text{ rad/sec.} \]

\[ \Omega_s = \omega_s / T = 0.6\pi \times 10,000 = 2\pi(3000) \text{ rad/sec.} \]

\[ \Rightarrow (1 - \delta_1) \leq |H_{\ell 0}(j\Omega)| \leq (1 + \delta_1) \quad |\Omega| \leq \Omega_p \]

\[ |H_{\ell 0}(j\Omega)| \leq \delta_2 \quad \Omega_s \leq \Omega \leq 2\pi(5000) \]

\[ 0.99 \leq |H_{\ell 0}(j\Omega)| \leq 1.02 \quad |\Omega| \leq 2\pi(2000) \]

\[ |H_{\ell 0}(j\Omega)| \leq 0.001 \quad 2\pi(3000) \leq \Omega \leq 2\pi(5000) \]
The filter given is a linear phase with:

\[ \text{length} = 28 \Rightarrow \text{group delay} = \frac{27}{2} = 13.5 \text{ samples.} \]

\[ \therefore \text{Delay} = T \cdot (13.5) = 10.4 \cdot 13.5 \]

\[ = 1.35 \text{ msec} \]

(a) \[ h_1[n] = h[2n] \]

\[ H_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_1[n] e^{j\omega n} = \sum_{n=-\infty}^{\infty} h[2n] e^{j\omega n} \]

\[ = \sum_{n \text{ even}} h[n] e^{j\omega n/2} \]

\[ = \sum \frac{1}{2} [h[n] + (-1)^n h[n]] e^{j\omega n/2} \]

\[ H_1(e^{j\omega}) = \frac{1}{2} H(e^{j\omega/2}) + \frac{1}{2} H(e^{j(\omega/2 + \pi)}) \]

\[ H_1(e^{j\omega}) \]

\[ H(e^{j\omega/2}) \]

\[ H(e^{j(\omega/2 + \pi)}) \]

\[ \omega \geq \pi \]

\[ (\omega + 1) = |(\omega)| \geq (\omega - 1) \]

\[ |h(n)| \geq \frac{1}{2} \]

\[ -\pi/2 \leq \omega \leq \pi/2 \]

\[ H(e^{j\omega}) \]

\[ H(e^{j\omega/2}) \]

\[ H(e^{j(\omega/2 + \pi)}) \]

\[ -\pi/4 \leq \omega \leq \pi/4 \]

\[ H(e^{j\omega}) \]

\[ H(e^{j\omega/2}) \]

\[ H(e^{j(\omega/2 + \pi)}) \]
(b) \[ h_2[n] = \begin{cases} h[n/2] & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases} \]

\[ H_2(e^{j\omega}) = \sum_{n \text{ even}} h[n/2] e^{-j\omega n} = \sum_{n = -\infty}^{\infty} h[n] e^{-j2\omega n} \]

\[ H_2(e^{j\omega}) = H(e^{j2\omega}) \]

(C) \[ h_3[n] = e^{j\pi n} h[n] = (-1)^n h[n] \]

\[ H_3(e^{j\omega}) = H(e^{j(\omega + \pi)}) \]