Q 7.9

9. Suppose we design a discrete-time filter using the impulse invariance technique with an ideal continuous-time lowpass filter as a prototype. The prototype filter has a cutoff frequency of $\Omega_c = 2\pi(1000)$ rad/s, and the impulse invariance transformation uses $T = 0.2$ ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

Q 7.10

10. We wish to design a discrete-time lowpass filter using the bilinear transformation on a continuous-time ideal lowpass filter. Assume that the continuous-time prototype filter has cutoff frequency $\Omega_c = 2\pi(2000)$ rad/s, and we choose the bilinear transformation parameter $T = 0.4$ ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

Q 7.17

17. Suppose that we wish to design a bandpass filter satisfying the following specification:

$$-0.02 < |H(e^{j\omega})| < 0.02, \qquad 0 \le |\omega| \le 0.2\pi,$$
$$0.95 < |H(e^{j\omega})| < 1.05, \qquad 0.3\pi \le |\omega| \le 0.7\pi,$$
$$-0.001 < |H(e^{j\omega})| < 0.001, \qquad 0.75\pi \le |\omega| \le \pi.$$

The filter will be designed by applying impulse invariance with $T = 5$ ms to a prototype continuous-time filter. State the specifications that should be used to design the prototype continuous-time filter.

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Q 7.2 (a)

2. A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}.$$

The specifications for the discrete-time system are those of Example 2, i.e.,

$$0.89125 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.2\pi,$$
$$|H(e^{j\omega})| \leq 0.17783, \quad 0.3\pi \leq |\omega| \leq \pi.$$

Assume, as in that example, that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the desired discrete-time filter.

(a) Sketch the tolerance bounds on the magnitude of the frequency response, $|H_c(j\Omega)|$, of the continuous-time Butterworth filter such that after application of the impulse invariance method (i.e., $h[n] = T_d h_c(nT_d)$), the resulting discrete-time filter will satisfy the given design specifications. Do not assume that $T_d = 1$ as in Example 2.

Q 7.2 (b-c)

(b) Determine the integer order N and the quantity $T_d\Omega_c$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.

(c) Note that if $T_d = 1$, your answer in part (b) should give the values of N and Ω_c obtained in Example 2. Use this observation to determine the system function $H_c(s)$ for $T_d \neq 1$ and to argue that the system function $H(z)$ which results from impulse invariance design with $T_d \neq 1$ is the same as the result for $T_d = 1$ given by Eq. (17).

Q 7.28

28. Consider a continuous-time lowpass filter $H_c(s)$ with passband and stopband specifications

$$1-\delta_1 \le |H_c(j\Omega)| \le 1+\delta_1, \qquad |\Omega| \le \Omega_p,$$
$$|H_c(j\Omega)| \le \delta_2, \qquad \Omega_s \le |\Omega|.$$

This filter is transformed to a lowpass discrete-time filter $H_1(z)$ by the transformation

$$H_1(z) = H_c(s)\big|_{s=(1-z^{-1})/(1+z^{-1})},$$

and the same continuous-time filter is transformed to a highpass discrete-time filter by the transformation

$$H_2(z) = H_c(s)\big|_{s=(1+z^{-1})/(1-z^{-1})}.$$

(a) Determine a relationship between the passband cutoff frequency Ω_p of the continuous-time lowpass filter and the passband cutoff frequency ω_{p1} of the discrete-time lowpass filter.

(b) Determine a relationship between the passband cutoff frequency Ω_p of the continuous-time lowpass filter and the passband cutoff frequency ω_{p2} of the discrete-time highpass filter.

(c) Determine a relationship between the passband cutoff frequency ω_{p1} of the discrete-time lowpass filter and the passband cutoff frequency ω_{p2} of the discrete-time highpass filter.

Q 7.23

23. Consider a continuous-time system with system function

$$H_c(s) = \frac{1}{s}.$$

This system is called an *integrator*, since the output $y_c(t)$ is related to the input $x_c(t)$ by

$$y_c(t) = \int_{-\infty}^{t} x_c(\tau)d\tau.$$

Suppose a discrete-time system is obtained by applying the bilinear transformation to $H_c(s)$.

(a) What is the system function $H(z)$ of the resulting discrete-time system? What is the impulse response $h[n]$?

(b) If $x[n]$ is the input and $y[n]$ is the output of the resulting discrete-time system, write the difference equation that is satisfied by the input and output. What problems do you anticipate in implementing the discrete-time system using this difference equation?

(c) Obtain an expression for the frequency response $H(e^{j\omega})$ of the system. Sketch the magnitude and phase of the discrete-time system for $0 \leq |\omega| \leq \pi$. Compare them with the magnitude and phase of the frequency response $H_c(j\Omega)$ of the continuous-time integrator. Under what conditions could the discrete-time "integrator" be considered a good approximation to the continuous-time integrator?

Q 7.5

7.5. We wish to use the Kaiser window method to design a discrete-time filter with generalized linear phase that meets specifications of the following form:

$$
\begin{aligned}
|H(e^{j\omega})| &\le 0.01, && 0 \le |\omega| \le 0.25\pi, \\
0.95 \le |H(e^{j\omega})| &\le 1.05, && 0.35\pi \le |\omega| \le 0.6\pi, \\
|H(e^{j\omega})| &\le 0.01, && 0.65\pi \le |\omega| \le \pi.
\end{aligned}
$$

(a) Determine the minimum length $(M+1)$ of the impulse response and the value of the Kaiser window parameter β for a filter that meets the preceding specifications.

(b) What is the delay of the filter?

(c) Determine the ideal impulse response $h_d[n]$ to which the Kaiser window should be applied.

Q 7.15

7.15. We wish to design an FIR lowpass filter satisfying the specifications

$$0.95 < H(e^{j\omega}) < 1.05, \qquad 0 \leq |\omega| \leq 0.25\pi,$$

$$-0.1 < H(e^{j\omega}) < 0.1, \qquad 0.35\pi \leq |\omega| \leq \pi,$$

by applying a window $w[n]$ to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.3\pi$. Which of the windows listed in Section 7.5.1 can be used to meet this specification? For each window that you claim will satisfy this specification, give the minimum length $M + 1$ required for the filter.

TABLE 7.2 COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

Q 7.22

7.22. In the system shown in Figure P7.22, the discrete-time system is a linear-phase FIR lowpass filter designed by the Parks–McClellan algorithm with $\delta_1 = 0.01$, $\delta_2 = 0.001$, $\omega_p = 0.4\pi$, and $\omega_s = 0.6\pi$. The length of the impulse response is 28 samples. The sampling rate for the ideal C/D and D/C converters is $1/T = 10000$ samples/sec.

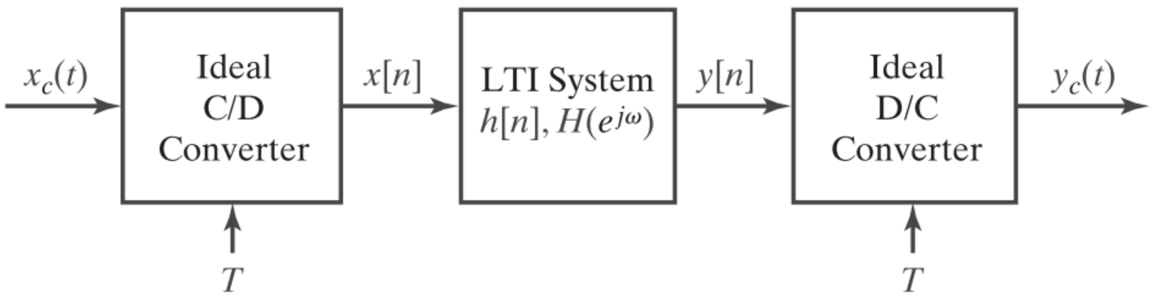

Figure P7.22

(a) What property should the input signal have so that the overall system behaves as an LTI system with $Y_c(j\Omega) = H_{eff}(j\Omega)X_c(j\Omega)$?

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Q 7.22

(b) For the conditions found in (a), determine the approximation error specifications satisfied by $|H_{eff}(j\Omega)|$. Give your answer as either an equation or a plot as a function of Ω.

(c) What is the overall delay from the continuous-time input to the continuous-time output (in seconds) of the system in Figure P7.22?

Q 7.27

7.27. Suppose that we are given an ideal lowpass discrete-time filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/4, \\ 0, & \pi/4 < |\omega| \leq \pi. \end{cases}$$

We wish to derive new filters from this prototype by manipulations of the impulse response $h[n]$.

(a) Plot the frequency response $H_1(e^{j\omega})$ for the system whose impulse response is $h_1[n] = h[2n]$.

(b) Plot the frequency response $H_2(e^{j\omega})$ for the system whose impulse response is

$$h_2[n] = \begin{cases} h[n/2], & n = 0, \pm 2, \pm 4, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Plot the frequency response $H_3(e^{j\omega})$ for the system whose impulse response is $h_3[n] = e^{j\pi n} h[n] = (-1)^n h[n]$.