Q 5.2

$$y[n-1] - \frac{10}{3} y[n] + y[n+1] = x[n]$$

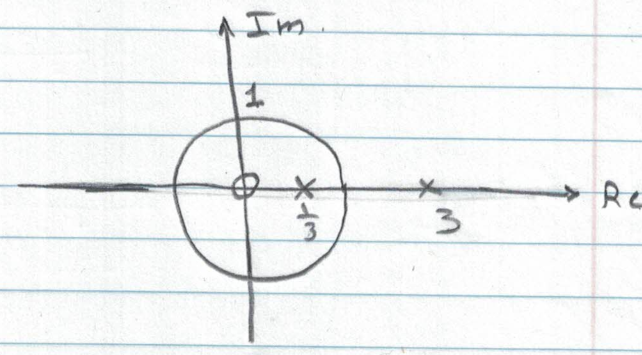
Take The Z-Transform:

$$Y(z) z^{-1} - \frac{10}{3} Y(z) + Y(z) z = X(z)$$

$X(z) = 1$ for $x[n] = \delta[n]$

$$\therefore H(z) = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z}{z^2 - \frac{10}{3} z + 1}$$

$$= \frac{z}{(z-3)(z-1/3)}$$



To have a stable system, Roc must include The unit Circle :

Roc : $\frac{1}{3} < |z| < 3$

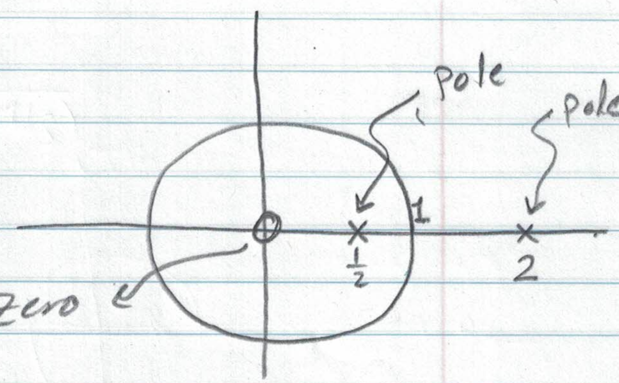
Q 5.9

$$y[n-1] - \tfrac{5}{2}y[n] + y[n+1] = x[n]$$

$$z^{-1}Y(z) - \tfrac{5}{2}Y(z) + Y(z)\,z = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{5}{2} + z} = \frac{z}{z^2 - \frac{5}{2}z + 1}$$

$$= \frac{z}{(z-2)(z-1/2)} = \frac{2/3}{z-2} - \frac{2/3}{z-\frac{1}{2}}$$



* Three ROC:

(a) $|z| < \frac{1}{2}$

$\therefore h[n] = -\frac{2}{3}(2)^n u[-n-1] + \frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1]$

(b) $\frac{1}{2} < |z| < 2 \Rightarrow h[n] = -\frac{2}{3}(2)^n u[-n-1] - \frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$

$|z| = 1$ is in the ROC $\Rightarrow$ The system is stable.

(c) $|z| > 2$

$h[n] = \frac{2}{3}(2)^n u[n] - \frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$

ROC extends from the outermost pole, hence the system is causal.

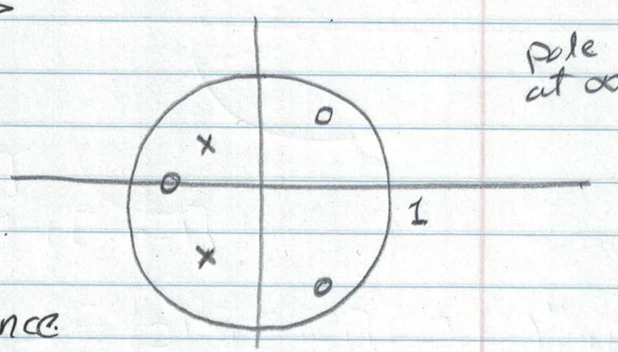
Co 5.10

$H(z)\, H_i(z) = 1$

* The Pole-Zero plot shows Two Zeros & three Poles. since number of poles is equal to number of Zeros, there must be another Zero, which is at $z = \infty$

* The system $H(z)$ is Causal, therefore the Roc extends from the outermost pole

* In the inverse system $H_i(z)$, Poles & Zeros are switched



The system Can be stable if the Roc includes the unit circle. However, it Cannot be Causal, since there is a pole at $z = \infty$.

Q 3.8

$$H(z) = \frac{1 - z^{-1}}{1 + (3/4) z^{-1}}$$

a) $$H(z) = \frac{1}{1+(3/4)z^{-1}} - \frac{z^{-1}}{1+(3/4)z^{-1}}$$

The system is causal $\Rightarrow$ $\therefore$ $H(z)$ is Right-sided

$$\frac{1}{1+(3/4)z^{-1}} \longrightarrow (-3/4)^n u[n]$$

$$\frac{z^{-1}}{1+(3/4)z^{-1}} \Rightarrow (z^{-1}) \text{ is a shift in time-domain}$$

$$\therefore (-3/4)^{n-1} u[n-1]$$

$$\therefore h[n] = \left(\frac{-3}{4}\right)^n u[n] + (-3/4)^{(n-1)} u[n-1]$$

b) $$Y(z) = X(z)\,H(z) = X(z) \cdot \frac{1-z^{-1}}{1+(3/4)z^{-1}}$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-z^{-1}} = \frac{1-z^{-1} - 1 + \frac{1}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-z^{-1})}$$

$$= \frac{(-2/3)z^{-1}}{(1-\frac{1}{3}z^{-1})(1-z^{-1})}$$

$$Y(z) = \frac{(-2/3)z^{-1}}{(1-\frac{1}{3}z^{-1})\cancel{(1-z^{-1})}} \cdot \frac{\cancel{(1-z^{-1})}}{(1+3/4\,z^{-1})}$$

$$= \frac{(-2/3)z^{-1}}{(1-\frac{1}{3}z^{-1})(1+3/4\,z^{-1})} \qquad \text{ROC: } |z| > 3/4$$

$$y[n] = \frac{-8}{13}\left(\frac{1}{3}\right)^n u[n] + \frac{8}{13}\left(\frac{-3}{4}\right)^n u[n]$$

c) ROC of $h[n]$ is $|z| > 3/4$ $\Rightarrow$ includes the unit circle. Therefore, $h[n]$ is absolutely summable.

Tutorial 5

Q 5.52

$$X(z) = \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{5}z^{+1})}{(1-\frac{1}{6}z)}$$

$$= \frac{6}{5} \cdot \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-5z^{-1})}{(1-6z^{-1})}$$

$$\alpha^n x[n] \xrightarrow{Z-T} X(\alpha^{-1}z)$$

$$X(\alpha^{-1}z) = \frac{6}{5} \cdot \frac{(1-\frac{1}{2}\alpha z^{-1})(1-\frac{1}{4}\alpha z^{-1})(1-5\alpha z^{-1})}{(1-6\alpha z^{-1})}$$

* A min. phase sequence has all poles and zeros inside the unit circle.:

$|\alpha/2| < 1 \longrightarrow |\alpha| < 2$

$|\alpha/4| < 1 \longrightarrow |\alpha| < 4$

$|5\alpha| < 1 \longrightarrow |\alpha| < 1/5$

$|6\alpha| < 1 \longrightarrow |\alpha| < 1/6$

$\therefore$ To have a min phase sequence ($\alpha^n x[n]$)

$|\alpha| < 1/6$

Q 5.28

(a) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{(1 - e^{j\frac{\pi}{3}} z^{-1})(1 - e^{-j\frac{\pi}{3}} z^{-1})(1 + 1.17 z^{-1})}{(1 - 0.9 e^{j\pi/3} z^{-1})(1 - 0.9 e^{-j\pi/3} z^{-1})(1 + 0.85 z^{-1})}$

$$= \frac{1 + 0.1765 z^{-1} - 0.1765 z^{-2} + 1.1765 z^{-3}}{1 - 0.05 z^{-1} + 0.045 z^{-2} + 0.6885 z^{-3}}$$

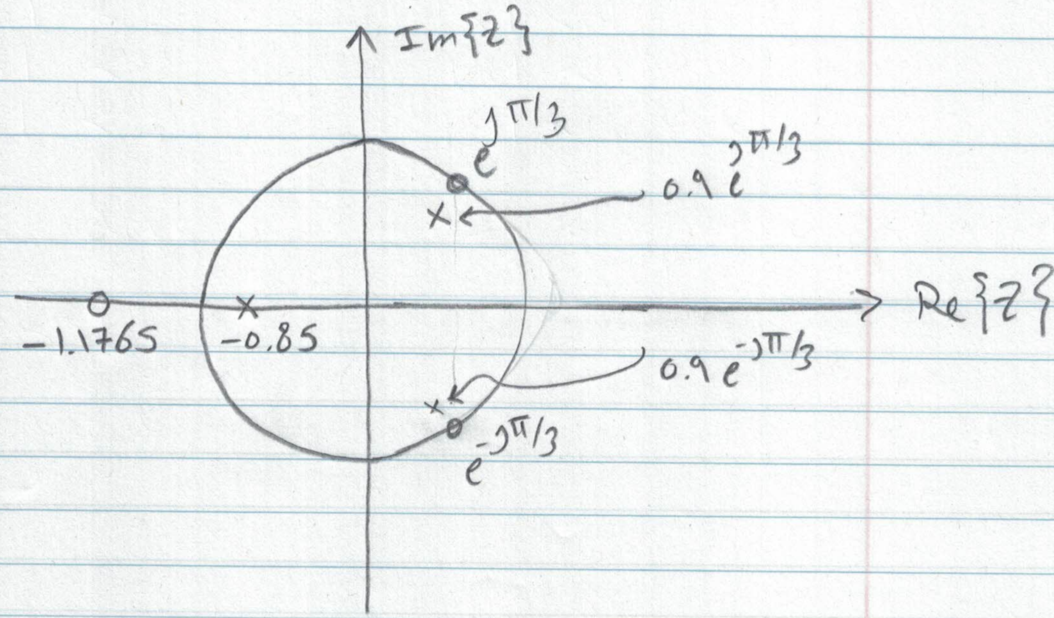
$y[n] - 0.05\, y[n-1] + 0.045\, y[n-2] + 0.68\, y[n-3]$

$= x[n] + 0.176\, x[n-1] - 0.1765\, x[n-2] +$

$1.1765\, x[n-3]$

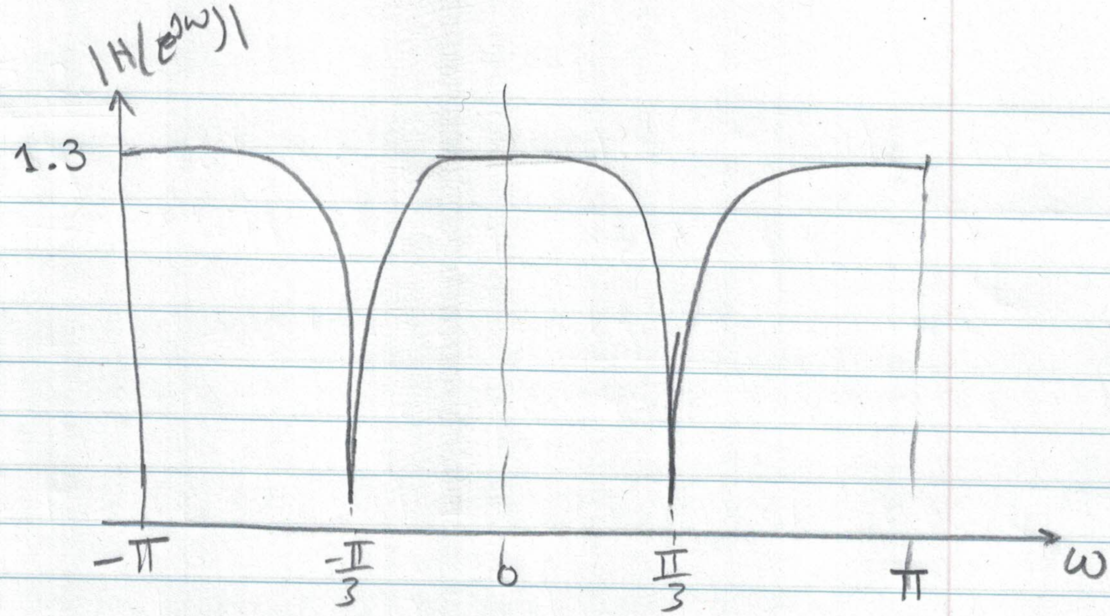
(b) Poles:
$0.9 e^{j\pi/3}$, $0.9 e^{-j\pi/3}$, -0.85

Zeros: $e^{j\pi/3}$, $e^{-j\pi/3}$, -1.1765



The system is causal, hence ROC extends from the outermost pole $\Rightarrow$ $|z| > 0.9$

(c)



Q5.8

(a) $y[n] = 3/2\, y[n-1] + y[n-2] + x[n-1]$

$Y(z) = 3/2\, z^{-1} Y(z) + z^{-2} Y(z) + z^{-1} X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - \frac{3}{2}z^{-1} - z^{-2}}$$

$$= \frac{z^{-1}}{(1-2z^{-1})(1+\frac{1}{2}z^{-1})}$$

ROC: $|z| > 2$

Note that it is a causal LTI system.

(b) $$H(z) = \frac{2/5}{(1-2z^{-1})} + \frac{-2/5}{(1+\frac{1}{2}z^{-1})}$$

$$h[n] = \frac{2}{5}(2)^n u[n] - \frac{2}{5}\left(\frac{-1}{2}\right)^n u[n]$$

(c) Going back to $H(z) = \frac{2/5}{(1-2z^{-1})} - \frac{2/5}{(1+\frac{1}{2}z^{-1})}$

To have a stable system, the ROC must include the unit circle. Therefore,

ROC: $1/2 < |z| < 2$

$\therefore\ h[n] = -\frac{2}{5}(2)^n u[-n-1] - \frac{2}{5}\left(-\frac{1}{2}\right)^n u[n]$

Q5.24

* Min. phase system has all poles and zeros inside the unit circle.

* Therefore, we first find Hmin(z) by moving Poles and zeros of H(z) inside the unit circle (i.e., move them to their conjugate reciprocal).

$$H_{min}(z) = K \cdot \frac{(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

* Note: $H(z) = H_{Min}(z)\ \underset{\text{all-pass.}}{\underbrace{H_{ap}(z)}}$

* Now, we choose Hap(z) to bring poles and zeros back to their original locations.

$$H_{ap}(z) = z^{-1}\left(\frac{z^{-1}-3}{1-3z^{-1}}\right)\left(\frac{z^{-1}-1/4}{1-\frac{1}{4}z^{-1}}\right)$$

* Note also that this decomposition is unique up to the scale factor (K).