Tutorial 8

Q 4.2

$f_s = 1000$ samples/sec. $\Rightarrow T = \frac{1}{1000}$ sec.

$x[n] = \cos(\pi n/4)$

sampling $x_c(t) = \cos(\Omega_0 t)$

$x_c(nT) = \cos(\Omega_0 nT)$

$\therefore \omega = \Omega_0 T \Rightarrow \frac{\pi}{4} = \Omega_0 \cdot \frac{1}{1000}$

$\Omega_0 = \frac{1000}{4}\pi = 250\pi$

Another possible positive frequency is:

$\Omega_0 = (2\pi + \pi/4) \cdot 1000 = 2250\pi$

Q 4.4

$x_c(t) = \sin(20\pi t) + \cos(40\pi t)$

(a) $x_c(nT) = \sin(20\pi nT) + \cos(40\pi nT)$

if T is chosen to be $\frac{1}{100}$ sec.

$\therefore x_c(nT) = \sin(20\pi n \cdot \frac{1}{100}) + \cos(40\pi n \cdot \frac{1}{100})$

$= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$

$= x[n] \Rightarrow \therefore T = \frac{1}{100}$ is one solution

(b) The choice of T is not unique.

For example, let $T = \frac{11}{100}$

$$x_c(nT) = \sin\left(20\pi n \cdot \frac{11}{100}\right) + \cos\left(40\pi n \frac{11}{100}\right)$$

$$= \sin\left(\frac{\pi n}{5} \cdot 11\right) + \cos\left(\frac{22\pi n}{5}\right)$$

$$= \sin\left(\frac{\pi n}{5} + \frac{10\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5} + \frac{20\pi n}{5}\right)$$

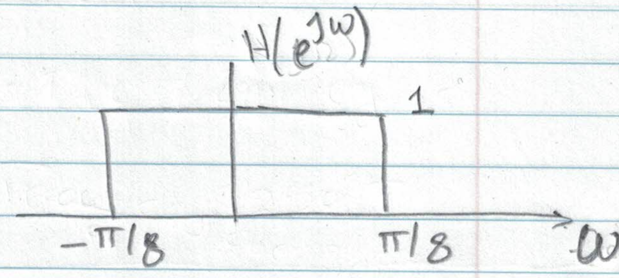
$$= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) = x[n]$$

Q 4.5

$\cancel{\Omega}\omega_c = \pi/8 \Rightarrow$

$\downarrow$

cutt-off frequency.



(a) $x_c(t)$ is bandlimited to 5KHz

$$\Omega = 2\pi \cdot 5000$$

Nyquist rate is $\Omega_s = 2\Omega = 4\pi \cdot 5000 = \frac{2\pi}{T_s}$

$T_s = T = \frac{1}{10{,}000}$ sec. (avoids aliasing)

(b) $\frac{1}{T} = 10\text{ kHz} \Rightarrow T = \frac{1}{10{,}000}$

$$\omega = \Omega T$$
$$\pi/8 = \frac{1}{10{,}000}\,\Omega_c$$

$\therefore\ \Omega_c = 2\pi \cdot 625$ rad/sec.

$\therefore\ f_c = 625$ Hz.

(c) $\frac{1}{T} = 20\text{ kHz} \Rightarrow T = \frac{1}{20{,}000}$ sec.

$$\omega = \Omega T$$
$$\pi/8 = \Omega_c \cdot \frac{1}{20{,}000}$$

$\Omega_c = 2\pi \cdot 1250$ rad./sec.

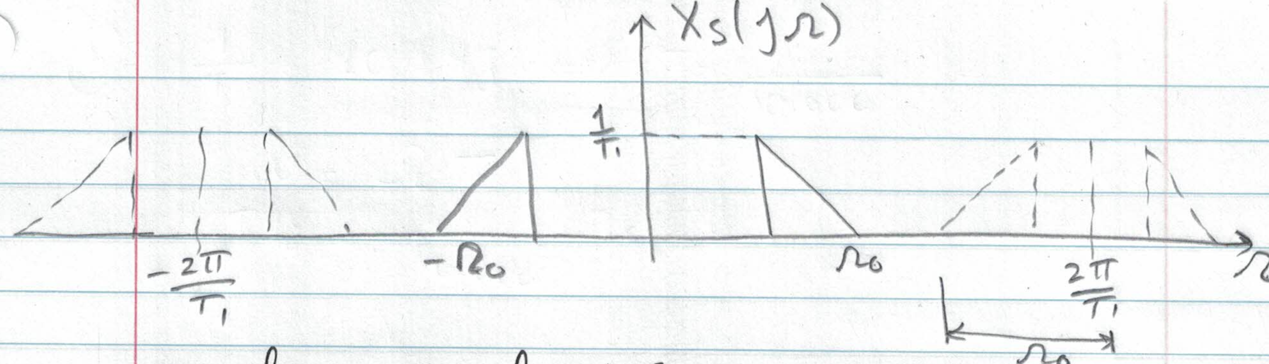
$f_c = 1250$ Hz.

Q 4.21

(a) $x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - nT_1)$

low pass filter cutoff freq $\Omega_c = \pi/T_1$

$$X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\Big[j\big(\Omega - k\underbrace{\tfrac{2\pi}{T_1}}_{\Omega_s}\big)\Big]$$



To have no aliasing, we need

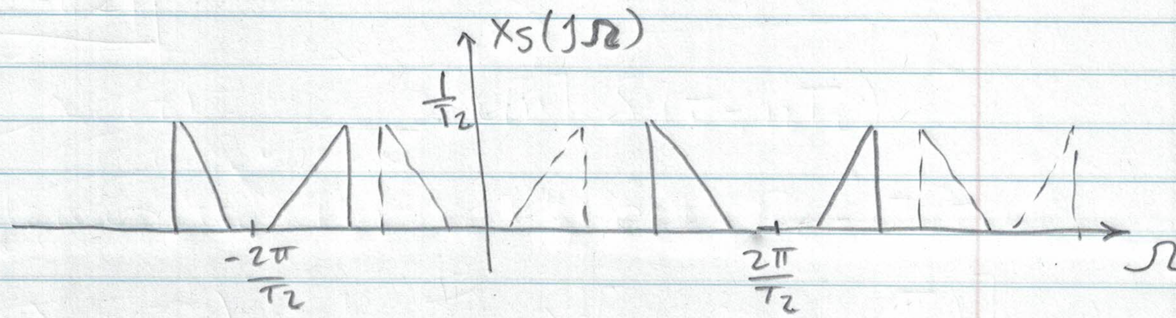
$$\frac{2\pi}{T_1} - \Omega_0 \geq \Omega_0$$

$$\therefore \boxed{T_1 \leq \pi/\Omega_0}$$

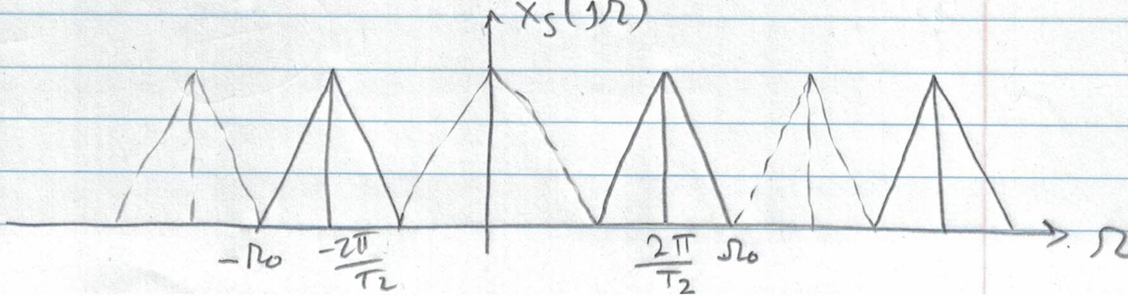
(b) $x_0(t) = x_c(t)$ happens in any of the following:

1) $T_2 \leq \frac{\pi}{\Omega_0}$ $\Rightarrow$ as in part (a)

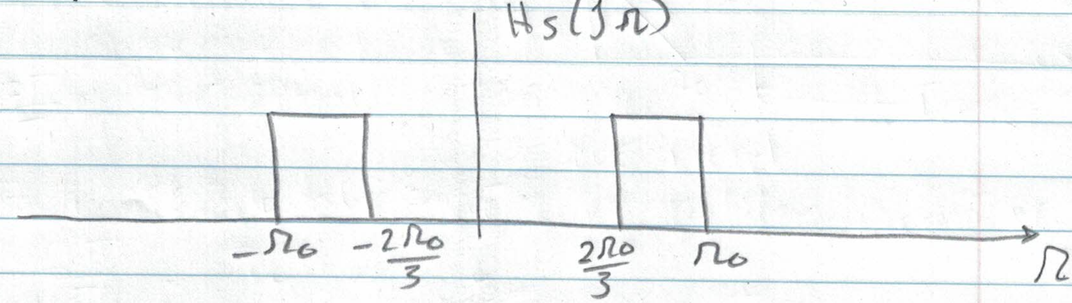
2) $\frac{1.5\pi}{\Omega_0} \leq T_2 \leq \frac{2\pi}{\Omega_0}$



3) $T_2 = \frac{3\pi}{\Omega_0}$



and the frequency response of the filter is:



Q 4.25

$$w(t) = x_1(t)\, x_2(t)$$

Using CTFT ⇒ Continuous-Time Fourier Transform:

$$W(j\Omega) = \frac{1}{2\pi} X_1(j\Omega) \underbrace{*}_{\text{Convolution}} X_2(j\Omega)$$

and the length of $W(j\Omega)$ is $\Omega_1 + \Omega_2$

$\therefore$ $w(t)$ is bandlimited to $(\Omega_1 + \Omega_2)$

for no aliasing:

$$\Omega_s \geq 2(\Omega_1 + \Omega_2)$$

$$\frac{1}{T} \geq 2(\Omega_1 + \Omega_2)$$

$$\therefore \quad T \leq \frac{1}{2(\Omega_1 + \Omega_2)}$$