Q 4.2

4.2. The sequence

\[ x[n] = \cos \left( \frac{\pi}{4} n \right), \quad -\infty < n < \infty, \]

was obtained by sampling the continuous-time signal

\[ x_c(t) = \cos (\Omega_0 t), \quad -\infty < t < \infty, \]

at a sampling rate of 1000 samples/s. What are two possible positive values of \( \Omega_0 \) that could have resulted in the sequence \( x[n] \)?
4.4. The continuous-time signal
\[ x_c(t) = \sin(20\pi t) + \cos(40\pi t) \]
is sampled with a sampling period \( T \) to obtain the discrete-time signal
\[ x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right). \]

(a) Determine a choice for \( T \) consistent with this information.
(b) Is your choice for \( T \) in part (a) unique? If so, explain why. If not, specify another choice of \( T \) consistent with the information given.
4.5. Consider the system of Figure 4.10, with the discrete-time system an ideal lowpass filter with cutoff frequency $\pi/8$ radians/s.

(a) If $x_c(t)$ is bandlimited to 5 kHz, what is the maximum value of $T$ that will avoid aliasing in the C/D converter?

(b) If $1/T = 10$ kHz, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat part (b) for $1/T = 20$ kHz.

Figure 4.10  Discrete-time processing of continuous-time signals.
Q 4.25

4.25. Two bandlimited signals, \( x_1(t) \) and \( x_2(t) \), are multiplied, producing the product signal \( w(t) = x_1(t)x_2(t) \). This signal is sampled by a periodic impulse train yielding the signal

\[
wp(t) = w(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} w(nT)\delta(t - nT).
\]

Assume that \( x_1(t) \) is bandlimited to \( \Omega_1 \), and \( x_2(t) \) is bandlimited to \( \Omega_2 \); that is,

\[
X_1(j\Omega) = 0, \quad |\Omega| \geq \Omega_1
\]

\[
X_2(j\Omega) = 0, \quad |\Omega| \geq \Omega_2.
\]

Determine the maximum sampling interval \( T \) such that \( w(t) \) is recoverable from \( wp(t) \) through the use of an ideal lowpass filter.