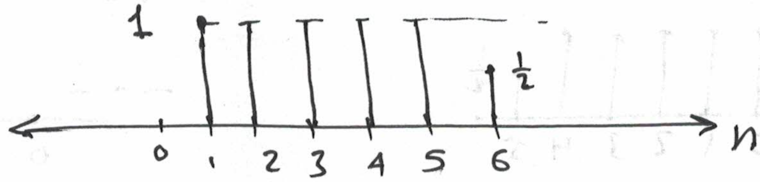
Tutorial 1

$\boxed{Q2.21}$

(a)

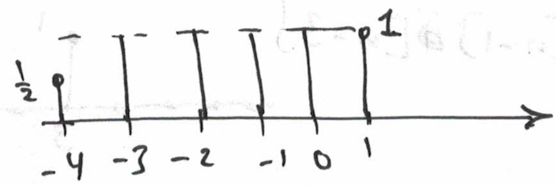
$x[n-2]$



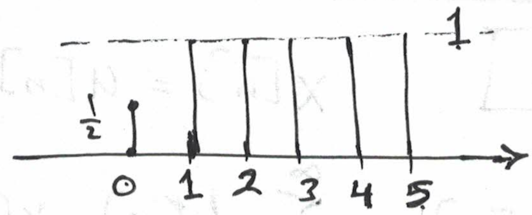
(b)

$x[4-n]$

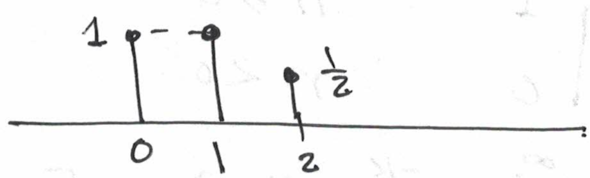
$x[-n]$



$x[4-n]$

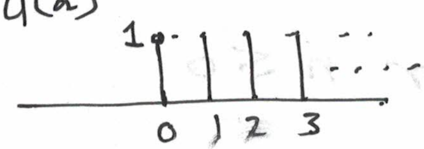


(c) $x[2n]$

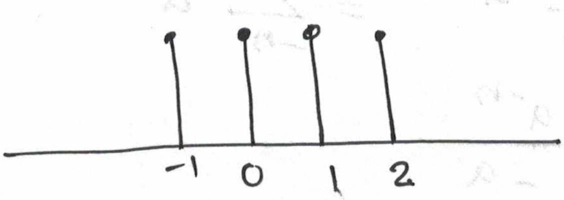


(d) $x[n]\,u[2-n]$

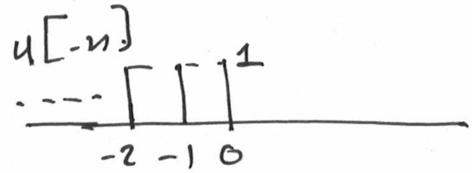
$u[n]$



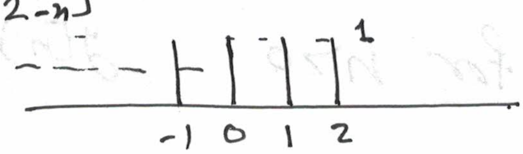
$x[n]\,u[2-n]$



$u[-n]$

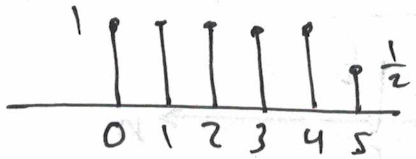


$u[2-n]$

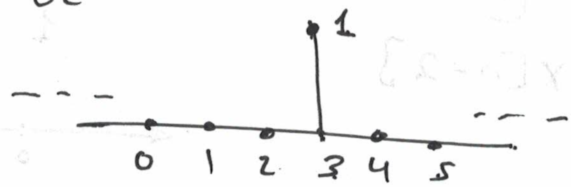


ⓔ $x[n-1]\ \delta[n-3]$

$x[n-1]$



$\delta[n-3]$



$x[n-1]\,\delta[n-3]$

$\boxed{Q2.3}$ $x[n] = u[n] \leftarrow$ unit step input

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$x[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k}\, u[-k] \,.\, u[n-k]$$

for $n \leq 0$

$$y[n] = \sum_{-\infty}^{\infty} a^{-k}\, u[-k]\, u[n-k]$$

$$y[n] = \sum_{-\infty}^{+n} a^{-k} = \sum_{-n}^{\infty} a^{k}$$

$$= \frac{a^{-n}}{1-a}$$

for $n > 0$ $\quad y(n) = \sum_{-\infty}^{0} a^{-k} = \sum_{0}^{\infty} a^{k} = \frac{1}{1-a}$

$\boxed{Q\,2.4}$

$$y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = 2x[n-1]$$

Take The Fourier Transform.

$$Y(e^{j\omega}) - \frac{3}{4} Y(e^{j\omega}) e^{-j\omega} + \frac{1}{8} Y(e^{j\omega}) e^{-2j\omega} = 2X(e^{j\omega}).$$

$$e^{-j\omega}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-2j\omega}}$$

Using partial fraction expansion:

$$H(e^{j\omega}) = \frac{-8}{1 - \frac{1}{4} e^{-j\omega}} + \frac{8}{1 - \frac{1}{2} e^{-j\omega}}$$

apply inverse fourier transform.

$$h[n] = -8 \left(\frac{+1}{4}\right)^n u[n] + 8\left(\frac{1}{2}\right)^n u[n]$$

<u>Note:</u> from fourier Table

$$a^n u[n] \longleftrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$\boxed{\text{Q } 2.20}$

$$y[n] + \left(\tfrac{1}{a}\right) y[n-1] = x[n-1]$$

(a) Assume $h[0] = 0$

$h[0] = 0$

$h[1] = x[0] = \delta[0] = 1$

$h[2] + \left(\tfrac{1}{a}\right) h[1] = \delta[1] \Rightarrow -1/a = h[2]$

$h[3] + \left(\tfrac{1}{a}\right)^2 h[1] = \delta[2]$

$h[3] = \left(\tfrac{-1}{a}\right)^2$

$\vdots$

$$h[n] = \left(\tfrac{-1}{a}\right)^{n-1} u[n-1]$$

<u>Another method:</u>

$$H(e^{jw}) + \left(\tfrac{1}{a}\right) H(e^{jw})\, e^{-jw} = 1 \cdot e^{-jw}$$

$$H(e^{jw}) = \frac{1}{1 + \left(\frac{1}{a}\right) e^{-jw}} \cdot e^{-jw}$$

$$\frac{1}{1 + \left(\frac{1}{a}\right) e^{-jw}} \longrightarrow \left(\tfrac{-1}{a}\right)^n u[n]$$

$e^{-jw} \longrightarrow$ shift in time domain by 1

$$\therefore H(e^{jw}) = \left(\tfrac{-1}{a}\right)^{n-1} u[n-1]$$

(b) $h[n]$ is absolutely summable if $\left|\frac{1}{a}\right| < 1$

or if $|a| > 1$

$\boxed{\text{Q } 2.6}$

(a) $y[n] - \frac{1}{2} y[n-1] = x[n] + 2x[n-1] + x[n-2]$

$$H(e^{j\omega}) - \frac{1}{2} H(e^{j\omega}) e^{-j\omega} = 1 + 2e^{-j\omega} + e^{-2j\omega}$$

$$H(e^{j\omega}) = \frac{1 + 2e^{-j\omega} + e^{-2j\omega}}{1 - \frac{1}{2} e^{-j\omega}}$$

(b) $$H(e^{j\omega}) = \frac{1 - \frac{1}{2} e^{-j\omega} + e^{-3j\omega}}{1 + \frac{1}{2} e^{-j\omega} + \frac{3}{4} e^{-2j\omega}}$$

$$H(e^{j\omega})\left[1 + \frac{1}{2} e^{-j\omega} + \frac{3}{4} e^{-2j\omega}\right] = 1 - \frac{1}{2} e^{-j\omega} + e^{-3j\omega}$$

$$h[n] + \frac{1}{2} h[n-1] + \frac{3}{4} h[n-2] = \delta[n] - \frac{1}{2}\delta[n-1] + \delta[n-3]$$

$$\therefore\; y[n] + \frac{1}{2} y[n-1] + \frac{3}{4} y[n-2] = x[n] - \frac{1}{2} x[n-1] + x[n-3]$$

$\boxed{Q2.7}$

(a) $x[n]$ is periodic with period N if $x[n] = x[n+N]$ for some integer N.

$$x[n] = e^{j(\pi n/6)}$$

$$e^{j(\pi n/6)} = e^{j\left(\frac{\pi}{6}(n+N)\right)} = e^{j\frac{\pi n}{6}} e^{j\frac{\pi N}{6}}$$

$$e^{j\frac{\pi N}{6}} = 1 = e^{j2\pi k}$$

$$\frac{\pi N}{6} = 2\pi k \Rightarrow N = 12k \Rightarrow \boxed{N=12}$$

(b) $$e^{j\left(\frac{3\pi n}{4}\right)} = e^{j\left(\frac{3\pi}{4}(n+N)\right)}$$

$$\frac{3\pi N}{4} = 2\pi k \Rightarrow N = \frac{8}{3}k$$

choose min $k=3 \Rightarrow \boxed{N=8}$

(c) $$x[n] = \frac{\sin\left(\frac{\pi n}{5}\right)}{\pi n}$$

not periodic as the denominator is linear in n.

(d) assume $x[n]$ is periodic for some N

$$e^{j\pi n/\sqrt{2}} = e^{j\frac{\pi}{\sqrt{2}}(n+N)}$$

$$\frac{\pi N}{\sqrt{2}} = 2\pi K \Rightarrow N = 2\sqrt{2}K$$

There is no integer K for which N is integer

$\therefore$ $x[n]$ is not periodic.

$\boxed{Q\ 2.8}$

$$y[n] = h[n] * x[n]$$

FT

$$Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{5}{1+\frac{1}{2}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}} \cdot \frac{5}{1+\frac{1}{2}e^{-j\omega}}$$

$$= \frac{3}{1+\frac{1}{2}e^{-j\omega}} + \frac{2}{1-\frac{1}{3}e^{-j\omega}}$$

$$y[n] = 2\left(\tfrac{1}{3}\right)^n u[n] + 3\left(-\tfrac{1}{2}\right)^n u[n]$$

$\boxed{Q\ 2.15}$

$y[n] = v[n]\ u[n]$

ⓐ let $x_1[n] = \delta[n] \longrightarrow y_1[n] = \left(\frac{1}{4}\right)^n u[n]$

$x_2[n] = \delta[n-1] \longrightarrow y_2[n] = \left(\frac{1}{4}\right)^{n-1} u[n]$

even that $x_2[n] = x_1[n-1]$,

$y_2[n] \neq y_1[n-1]$

hence, it is not an LTI system.

ⓑ for $x_2[n]$ & $y_2[n]$ in part a:

$x_2[n] = 0$ for $n < 1$, but $y_2[0] = 4 \neq 0$

$\therefore$ The system is not causal.

ⓒ The system is stable, since $h[n]$ is absolutely summable, and multiplication by $u[n]$ does not cause any unbounded sequence.

Q2.22

for LTI system, we use the convolution:

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

for $n = m + N$

$$y[m+N] = \sum_{k} x[m+N-k]\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} x[(m-k)+N]\, h[k]$$

$x[n]$ is periodic $\Rightarrow$ $x[n] = x[n + rN]$, r is integer

$$y[m+N] = \sum_{k=-\infty}^{\infty} x[m-k]\, h[k]$$

$$= y[m]$$

$\therefore$ The output must be periodic with period N.

Q 2.29

(a) $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=0}^{\infty} h[n-k]$$

$$y[0] = \sum_{k=0}^{\infty} h[-k] = h[0] = 1$$

$$y[1] = \sum_{k=0}^{\infty} h[1-k] = h[1] + h[0] = 2$$

$$y[2] = \sum_{k=0}^{\infty} h[2-k] = h[2] + h[1] + h[0] = 3$$

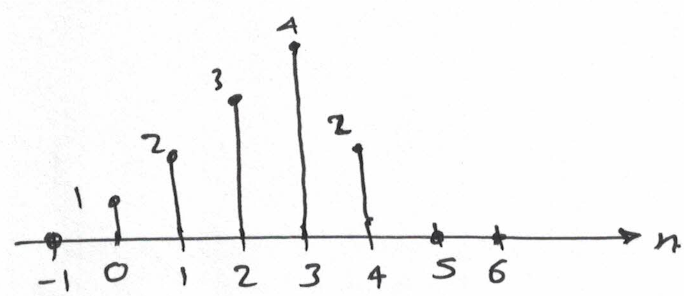
$$y[3] = \sum h[3-k] = 4$$

$$y[4] = \sum_{k=0}^{\infty} h[4-k] = h[4] + h[3] + h[2] + h[1] + h[0]$$

$$= -2 + 1 + 1 + 1 + 1 = 2$$

$$y[5] = \sum_{k=0}^{\infty} h[5-k] = 0$$

$$y[6] = 0$$

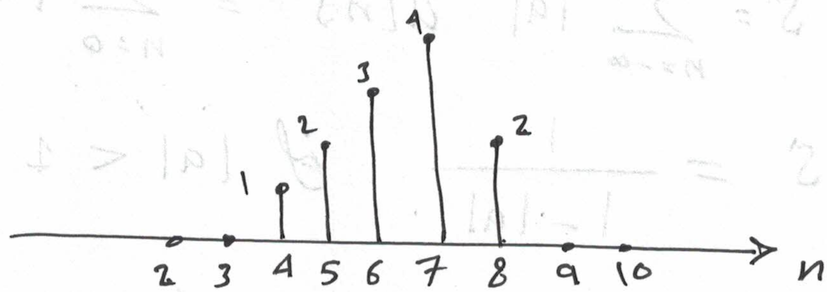


(b) The system is an LTI

$x[n] = u[n] \longrightarrow y[n]$ in part a

$x_1[n] = u[n-4] \longrightarrow$ shift the input in time

$y_1[n] = y[n-4]$ shifts the output in time

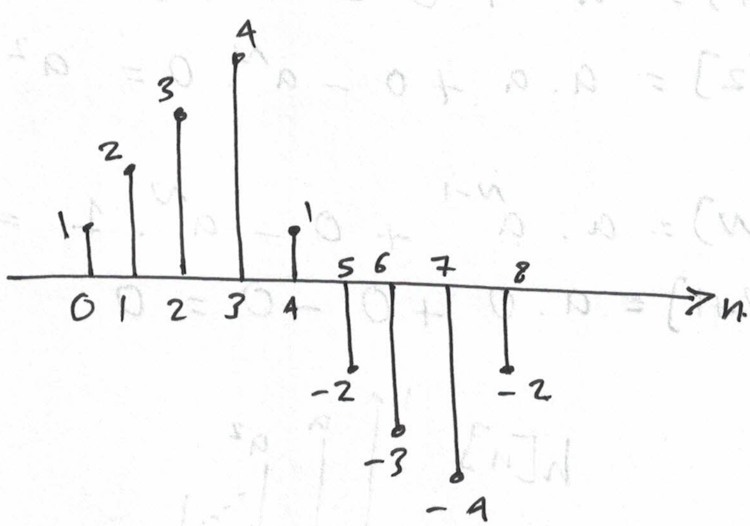


(c) using linearity property:

$y_2[n] = (u[n] - u[n-4]) * h[n]$

$= u[n] * h[n] - u[n-4] * h[n]$

$= y[n) - y_1[n]$



Q2.31

(a) The system is stable if $h[n]$ is absolutely summable $\sum_{-\infty}^{\infty} |h[n]| < \infty$

$$S' = \sum_{n=-\infty}^{\infty} |a|^n u[n] = \sum_{n=0}^{\infty} |a|^n$$

$$S' = \frac{1}{1-|a|} \quad \text{if } |a| < 1$$

$\therefore$ The system is stable if $|a| < 1$

(b)

$$y[n] = a\,y[n-1] + x[n] - a^N x[n-N]$$

$$h[n] = a\,h[n-1] + \delta[n] - a^N \delta[n-N]$$

The system is causal, then $h[-1] = 0$

$$h[0] = a.0 + 1 - a^N.0 = 1$$

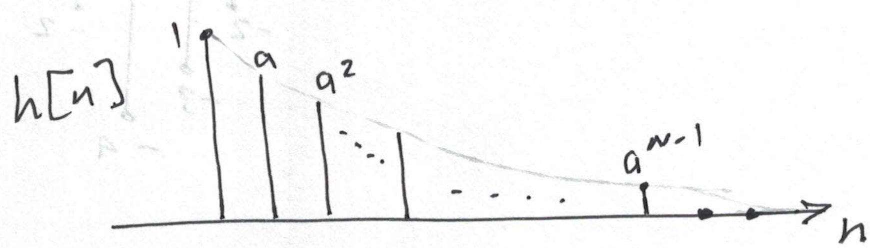
$$h[1] = a + 0 - a^N.0 = a$$

$$h[2] = a.a + 0 - a^N.0 = a^2$$

$\vdots$

$$h[N] = a.a^{N-1} + 0 - a^N.1 = a^N - a^N = 0$$

$$h[N+1] = a.0 + 0 - 0 = 0$$



(c) Even though the system has a feedback (recursive system), its impulse response is finite. Hence, the system is FIR

(d) FIR system are always stable because $h[n]$ is absolutely summable (i.e., $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$) due to summation of finite number of terms.