Q 2.21

2.21. A discrete-time signal $x[n]$ is shown in Figure P2.21.

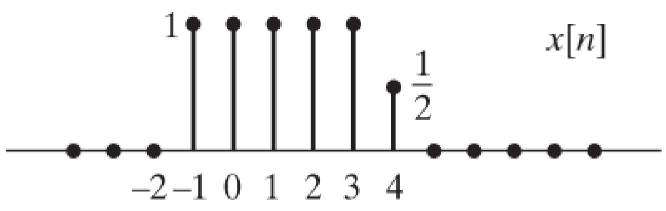

Figure P2.21

Sketch and label carefully each of the following signals:

(a) $x[n-2]$
(b) $x[4-n]$
(c) $x[2n]$
(d) $x[n]u[2-n]$
(e) $x[n-1]\delta[n-3]$.

Q 2.3

3. By direct evaluation of the convolution sum, determine the unit step response ($x[n] = u[n]$) of an LTI system whose impulse response is

$$h[n] = a^{-n}u[-n], \qquad 0 < a < 1.$$

Q 2.4

4. Consider the linear constant-coefficient difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1].$$

Determine $y[n]$ for $n \geq 0$ when $x[n] = \delta[n]$ and $y[n] = 0, n < 0$.

Q 2.20

2.20. Consider the difference equation representing a causal LTI system

$$y[n] + (1/a)y[n-1] = x[n-1].$$

(a) Find the impulse response of the system, $h[n]$, as a function of the constant a.
(b) For what range of values of a will the system be stable?

Q 2.6

6. **(a)** Determine the frequency response $H(e^{j\omega})$ of the LTI system whose input and output satisfy the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2].$$

(b) Write a difference equation that characterizes a system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}.$$

Q 2.7

7. Determine whether each of the following signals is periodic. If the signal is periodic, state its period.

(a) $x[n] = e^{j(\pi n/6)}$

(b) $x[n] = e^{j(3\pi n/4)}$

(c) $x[n] = [\sin(\pi n/5)]/(\pi n)$

(d) $x[n] = e^{j\pi n/\sqrt{2}}$.

Q 2.8

8. An LTI system has impulse response $h[n] = 5(-1/2)^n u[n]$. Use the Fourier transform to find the output of this system when the input is $x[n] = (1/3)^n u[n]$.

Q 2.15

15. Consider the system illustrated in Figure P15. The output of an LTI system with an impulse response $h[n] = \left(\frac{1}{4}\right)^n u[n+10]$ is multiplied by a unit step function $u[n]$ to yield the output of the overall system. Answer each of the following questions, and briefly justify your answers:

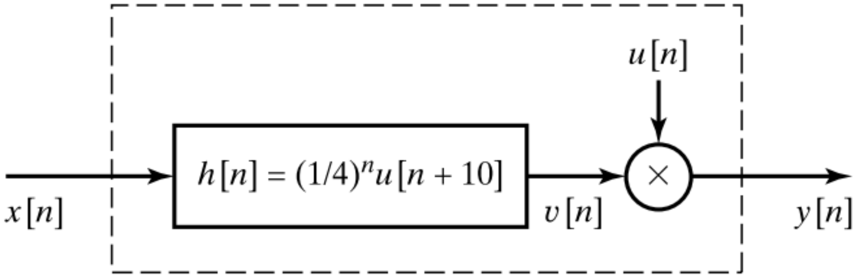

Figure P15

(a) Is the overall system LTI?
(b) Is the overall system causal?
(c) Is the overall system stable in the BIBO sense?

Q 2.22

22. Consider a discrete-time LTI system with impulse response $h[n]$. If the input $x[n]$ is a periodic sequence with period N (i.e., if $x[n] = x[n + N]$), show that the output $y[n]$ is also a periodic sequence with period N.

Q 2.29

29. An LTI system has impulse response defined by

$$h[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0, 1, 2, 3 \\ -2 & n = 4, 5 \\ 0 & n > 5 \end{cases}$$

Determine and plot the output $y[n]$ when the input $x[n]$ is:

(a) $u[n]$
(b) $u[n-4]$
(c) $u[n] - u[n-4]$.

Q 2.31

31. If the input and output of a causal LTI system satisfy the difference equation

$$y[n] = ay[n-1] + x[n],$$

then the impulse response of the system must be $h[n] = a^n u[n]$.

(a) For what values of a is this system stable?

(b) Consider a causal LTI system for which the input and output are related by the difference equation

$$y[n] = ay[n-1] + x[n] - a^N x[n-N],$$

where N is a positive integer. Determine and sketch the impulse response of this system. *Hint*: Use linearity and time-invariance to simplify the solution.

(c) Is the system in part (b) an FIR or an IIR system? Explain.

(d) For what values of a is the system in part (b) stable? Explain.