# ELG4172 Digital Signal Processing

- Tutorial#9

Presented by: Hitham Jleed

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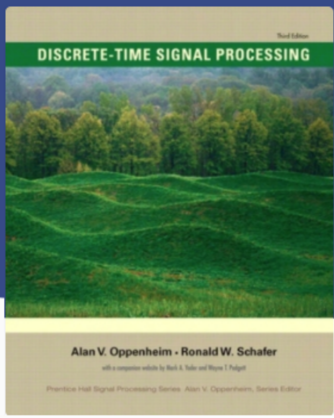


# Discrete-Time Signal Processing

Alan V Oppenheim; Ronald W. Schafer

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# Contents

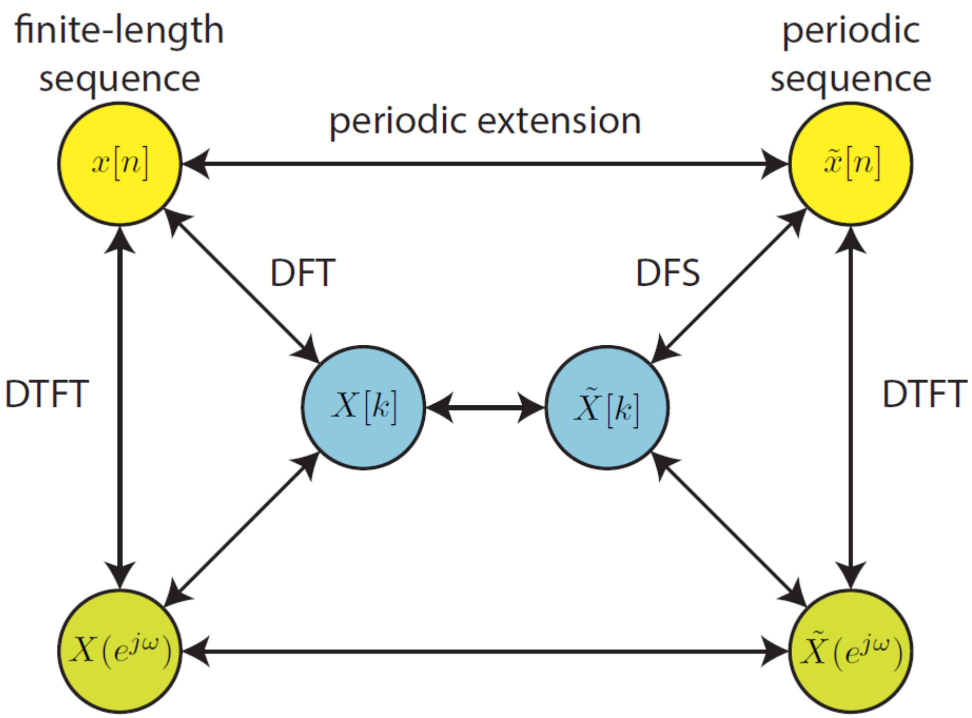
- N-point DFT
- DFS, DTFT, and DFT
- Circular Shift & convolution
- Relationship between DFT and H(Z)

Problems: 8.4, 8.6, 8.10, 8.24, 8.33, 8.51.

# Fourier Analysis- Tools

| Input Time Signal | | | Frequency spectrum | |
|---|---|---|---|---|
| Continuous | Periodic *(period T)* | FS | Discrete | $c_k = \frac{1}{T} \cdot \int_0^T x(t) \cdot e^{-jk\omega t} dt$ |
| Continuous | Aperiodic | FT | Continuous | $X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j2\pi f t} dt$ |
| Discrete | Periodic *(period N)* | DFS | Discrete | $\tilde{c}_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$ |
| Discrete | Aperiodic | DTFT | Continuous | $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n}$ |
| Discrete | Aperiodic | DFT | Discrete | $\tilde{X}[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$ |

| *time domain* | *frequency domain* | |
|---|---|---|
| continuous, period T | discrete | $x(t) = \sum_{k=-\infty}^{\infty} \hat{x}[k]\, e^{ik\frac{2\pi}{T}t}$ $\quad \hat{x}[k] = \frac{1}{T}\int_0^T x(t)\, e^{-ik\frac{2\pi}{T}t}\, dt$ |
| continuous | continuous | $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{x}(\omega) e^{i\omega t}\, d\omega \quad \hat{x}(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\, dt$ |
| discrete, period $N$ | discrete, period $N$ | $x[n] = \sum_{k=0}^{N-1} \hat{x}[k]\, e^{2\pi i \frac{nk}{N}} \quad \hat{x}[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-2\pi i \frac{kn}{N}}$ |
| discrete | continuous, period $2\pi$ | $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{x}(e^{i\omega}) e^{i\omega n}\, d\omega \quad \hat{x}(e^{i\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n}$ |




**DFS and DFT**

First, the relationship between the DFS and DFT is quite clear – we merely apply the DFT to one period [0…$N$-1] of $\tilde{x}(n)$ and scale the output of the DFT by 1/N to get the DFS coefficients, e.g.

$$a_k = \frac{1}{N} X(k)$$

**DTFT and DFT**

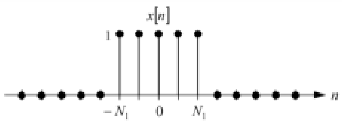
To relate the DFT and DTFT, we will need to truncate the DTFT to a finite range of $N$ samples.

- **Discrete Fourier Series (DFS) Pair for Periodic Signals**

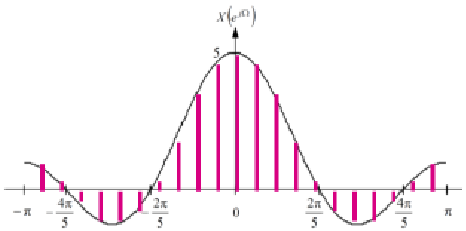
$$\tilde{x}[n]=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]W_N^{-kn} \qquad \tilde{X}[k]=\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{kn} \qquad W_N=e^{-j\frac{2\pi}{N}}$$

$$\tilde{x}[n] \overset{\text{DFS/IDFS}}{\longleftrightarrow} \tilde{X}[k]=\{Na_k\} \overset{\text{IDTFS/DTFS}}{\longleftrightarrow} \tilde{x}[n]$$

- **Discrete Fourier Transform (DFT) Pair**




$$x[n]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]W_N^{-kn} \qquad X[k]=\sum_{n=0}^{N-1}x[n]W_N^{kn}$$




  - $x[n]$ and $X[k]$ are one period of $\tilde{x}[n]$ and $\tilde{X}[k]$, respectively

- **DFT is DTFT sampled at N equally spaced frequencies between 0 and 2π:** $X[k]=X\left(e^{j\Omega}\right)\Big|_{\Omega=k\frac{2\pi}{N}},\quad 0\le k\le N-1$

$$\tilde{x}[n]=x[n]*\sum_k\delta(n-kN) \longleftrightarrow X[k]=X\left(e^{j\Omega}\right)\cdot\sum_k\delta(n-2\pi k/N)$$

**8.4.** Consider the sequence $x[n]$ given by $x[n] = \alpha^n u[n]$. Assume $|\alpha| < 1$. A periodic sequence $\tilde{x}[n]$ is constructed from $x[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n + rN].$$

**(a)** Determine the Fourier transform $X(e^{j\omega})$ of $x[n]$.
**(b)** Determine the DFS coefficients $\tilde{X}[k]$ for the sequence $\tilde{x}[n]$.
**(c)** How is $\tilde{X}[k]$ related to $X(e^{j\omega})$?

Q 8.4

$x[n] = \alpha^n u[n]$, $|\alpha| < 1$

Periodic sequence $\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n + rN]$, $|\alpha|<1$

(a) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n}$

$= \dfrac{1}{1 - \alpha e^{-j\omega}}$, $|\alpha| < 1$

(b) The DFS of $\tilde{x}[n]$:

$$\begin{aligned}
\tilde{X}[k] &= \sum_{n=0}^{N-1} \tilde{x}[n] W_N^{kn} \\
&= \sum_{n=0}^{N-1} \sum_{r=-\infty}^{\infty} x[n+rN] W_N^{kn} \\
&= \sum_{n=0}^{N-1} \sum_{r=-\infty}^{\infty} \alpha^{n+rN} u[n+rN] W_N^{kn} \\
&= \sum_{n=0}^{N-1} \sum_{r=0}^{\infty} \alpha^{n+rN} W_N^{kn}
\end{aligned}$$

Rearranging the summations gives:

$$\begin{aligned}
\tilde{X}[k] &= \sum_{r=0}^{\infty} \alpha^{rN} \sum_{n=0}^{N-1} \alpha^{n} W_N^{kn} \\
&= \sum_{r=0}^{\infty} \alpha^{rN} \left( \frac{1-\alpha^N e^{-j2\pi k}}{1-\alpha e^{-j\frac{2\pi k}{N}}} \right), \ |\alpha| < 1 \\
&= \frac{1}{1-\alpha^N} \left( \frac{1-\alpha^N e^{-j2\pi k}}{1-\alpha e^{-j\frac{2\pi k}{N}}} \right), \ |\alpha| < 1 \\
\tilde{X}[k] &= \frac{1}{1-\alpha e^{-j(2\pi k/N)}}, \ |\alpha| < 1
\end{aligned}$$

(c) Comparing the results of part (a) and part (b):

$$\tilde{X}[k] = X(e^{j\omega})\big|_{\omega=2\pi k/N}.$$

**8.6.** Consider the complex sequence

$$x[n] = \begin{cases} e^{j\omega_0 n}, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$$

**(a)** Find the Fourier transform $X(e^{j\omega})$ of $x[n]$.
**(b)** Find the $N$-point DFT $X[k]$ of the finite-length sequence $x[n]$.
**(c)** Find the DFT of $x[n]$ for the case $\omega_0 = 2\pi k_0/N$, where $k_0$ is an integer.

(a) The Fourier transform of $x[n]$:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=0}^{N-1} e^{j\omega_0 n}e^{-j\omega n}$$

$$X(e^{j\omega}) = \frac{1-e^{-j(\omega-\omega_0)N}}{1-e^{-j(\omega-\omega_0)}} = \frac{e^{-j(\omega-\omega_0)(N/2)}}{e^{-j(\omega-\omega_0)/2}}\left(\frac{\sin[(\omega-\omega_0)(N/2)]}{\sin[(\omega-\omega_0)/2]}\right)$$

$$X(e^{j\omega}) = e^{-j(\omega-\omega_0)((N-1)/2)}\left(\frac{\sin[(\omega-\omega_0)(N/2)]}{\sin[(\omega-\omega_0)/2]}\right)$$

(b) N-point DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad 0 \le k \le (N-1)$$

$$= \sum_{n=0}^{N-1} e^{j\omega_0 n}W_N^{kn}$$

$$= \frac{1-e^{-j((2\pi k/N)-\omega_0)N}}{1-e^{-j((2\pi k/N)-\omega_0)}}$$

$$= e^{-j(\frac{2\pi k}{N}-\omega_0)(\frac{N-1}{2})}\frac{\sin\left[\left(\frac{2\pi k}{N}-\omega_0\right)\frac{N}{2}\right]}{\sin\left[\left(\frac{2\pi k}{N}-\omega_0\right)/2\right]}$$

(c) Suppose $\omega_0 = (2\pi k_0)/N$, where $k_0$ is an integer:

$$X[k] = \frac{1-e^{-j(k-k_0)2\pi}}{1-e^{-j(k-k_0)(2\pi)/N}}$$

$$= e^{-j(2\pi/N)(k-k_0)((N-1)/2)}\frac{\sin\pi(k-k_0)}{\sin(\pi(k-k_0)/N)}$$

Note that $X[k] = X(e^{j\omega})|_{\omega=(2\pi k)/N}$

**8.7.** Consider the finite-length sequence $x[n]$ in Figure P8.7. Let $X(z)$ be the $z$-transform of $x[n]$. If we sample $X(z)$ at $z = e^{j(2\pi/4)k}$, $k = 0, 1, 2, 3$, we obtain

$$X_1[k] = X(z)\big|_{z=e^{j(2\pi/4)k}}, \qquad k = 0, 1, 2, 3.$$

Sketch the sequence $x_1[n]$ obtained as the inverse DFT of $X_1[k]$.

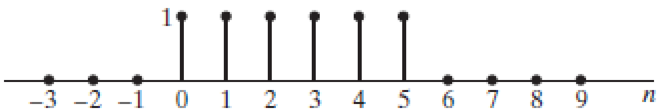
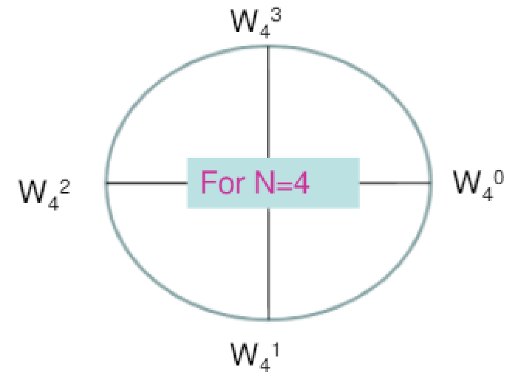



Figure P8.7




6-point sequence. $x[n]$ for $0 \leq n \leq 5$

Z-transform ⇒ at four equally-spaced points.

$\therefore X[k] = X(z)\big|_{z=e^{\frac{2\pi k}{4}}}$ where $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$\therefore X[k] = \sum_{n=0}^{5} x[n] W_4^{kn}$, $0 \leq k \leq 4$ ⇒ Given

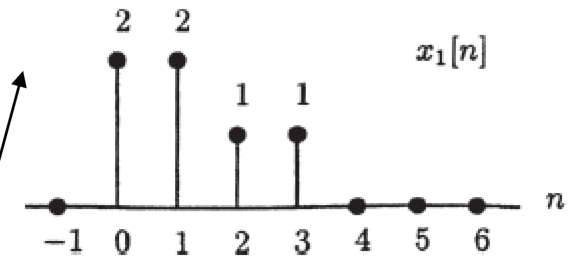
Note: We have taken a 4-point DFT. However, the original sequence was of length 6. As a result, we can expect some aliasing when we return to the time domain via inverse DFT.

$X[k] = W_4^{0k} + W_4^{k} + W_4^{2k} + W_4^{3k} + W_4^{4k} + W_4^{5k}$ $\quad 0 \leq k \leq 4$

Taking the inverse DFT by inspection, we note that there are six impulses (one for each value of $n$). However,

$$W_4^{4k} = W_4^{0k} \text{ and } W_4^{5k} = W_4^{k},$$

so two points are aliased.
The resulting time-domain signal is



# Circular Symmetries Properties

**Circular Symmetries of a Sequence.** As we have seen, the $N$-point DFT of a finite duration sequence $x(n)$, of length $L \leq N$, is equivalent to the $N$-point DFT of a periodic sequence $x_p(n)$, of period $N$, which is obtained by periodically extending $x(n)$, that is,

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

- Generally, the circular shift of the sequence can be represented as the index modulo $N$.

$$\grave{x}(n) = x(n-k, moduloN) \equiv x((n-k))_N$$

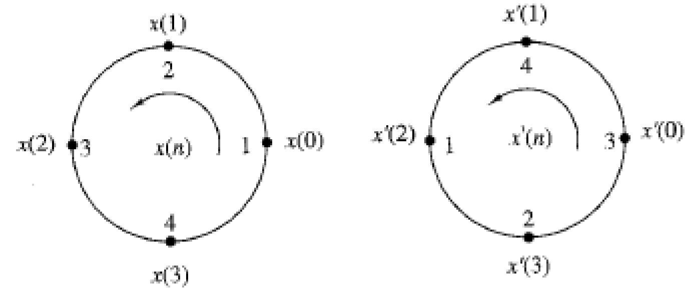
- Ex. if $k = 2$ and $N = 4$, we have

$$\grave{x}(0) = x(n-2, modulo4) \equiv x((n-2))_4$$

which implies that

$$\begin{aligned} \grave{x}(0) &= x((-2))_4 = x(2) \\ \grave{x}(1) &= x((-1))_4 = x(3) \\ \grave{x}(2) &= x((\ 0))_4 = x(0) \\ \grave{x}(3) &= x((\ 1))_4 = x(1) \end{aligned}$$

- $\grave{x}(n)$ is simply $x(n)$ circularly shifted by two units.




Circular shift of a sequence.

**8.10.** The two eight-point sequences $x_1[n]$ and $x_2[n]$ shown in Figure P8.10 have DFTs $X_1[k]$ and $X_2[k]$, respectively. Determine the relationship between $X_1[k]$ and $X_2[k]$.

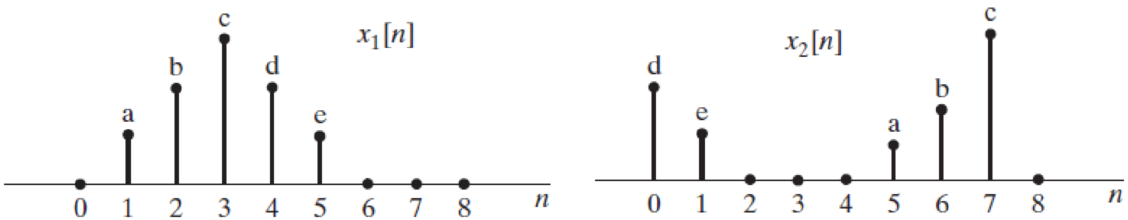
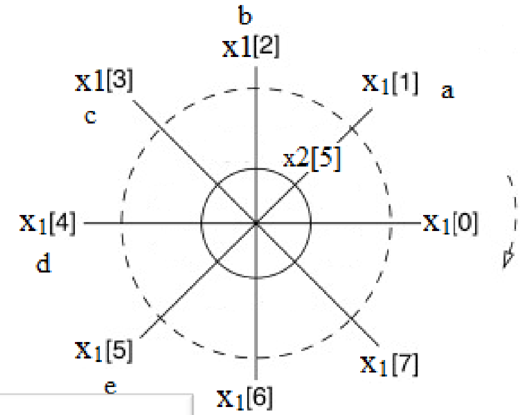



Figure P8.10




From Fig P8.10 , the two 8-pt sequences are related through a circular shift. Specifically,

$$x_2[n] = x_1[((n-4))_8]$$

From property 5 in Table 8.2,

$$\text{DFT}\{x_1[((n-4))_8]\} = W_8^{4k} X_1[k]$$

Thus,

$$X_2[k] = W_8^{4k} X_1[k] = e^{-j\pi k} X_1[k] \qquad X_2[k] = (-1)^k X_1[k]$$

```
>> x1=['0' 'a' 'b' 'c' 'd' 'e' '0' '0' '0' ]
x1 =
   '0abcde000'
>> x2 = circshift(x1,-4)
x2 =
   'de0000abc'
```

**TABLE 8.2 SUMMARY OF PROPERTIES OF THE DFT**

| | Finite-Length Sequence (Length $N$) | $N$-point DFT (Length $N$) |
|---|---|---|
| 1. | $x[n]$ | $X[k]$ |
| 2. | $x_1[n], x_2[n]$ | $X_1[k], X_2[k]$ |
| 3. | $ax_1[n] + bx_2[n]$ | $aX_1[k] + bX_2[k]$ |
| 4. | $X[n]$ | $Nx[((-k))_N]$ |
| 5. | $x[((n-m))_N]$ | $W_N^{km} X[k]$ |
| 6. | $W_N^{-\ell n} x[n]$ | $X[((k-\ell))_N]$ |
| 7. | $\sum_{m=0}^{N-1} x_1[m]x_2[((n-m))_N]$ | $X_1[k]X_2[k]$ |
| 8. | $x_1[n]x_2[n]$ | $\frac{1}{N}\sum_{\ell=0}^{N-1} X_1[\ell]X_2[((k-\ell))_N]$ |
| 9. | $x^*[n]$ | $X^*[((-k))_N]$ |
| 10. | $x^*[((-n))_N]$ | $X^*[k]$ |
| 11. | $\mathcal{R}e\{x[n]\}$ | $X_{ep}[k] = \frac{1}{2}\{X[((k))_N] + X^*[((-k))_N]\}$ |
| 12. | $j\mathcal{I}m\{x[n]\}$ | $X_{op}[k] = \frac{1}{2}\{X[((k))_N] - X^*[((-k))_N]\}$ |
| 13. | $x_{ep}[n] = \frac{1}{2}\{x[n] + x^*[((-n))_N]\}$ | $\mathcal{R}e\{X[k]\}$ |
| 14. | $x_{op}[n] = \frac{1}{2}\{x[n] - x^*[((-n))_N]\}$ | $j\mathcal{I}m\{X[k]\}$ |
| | Properties 15–17 apply only when $x[n]$ is real. | |
| 15. | Symmetry properties | $\begin{cases} X[k] = X^*[((-k))_N] \\ \mathcal{R}e\{X[k]\} = \mathcal{R}e\{X[((-k))_N]\} \\ \mathcal{I}m\{X[k]\} = -\mathcal{I}m\{X[((-k))_N]\} \\ \|X[k]\| = \|X[((-k))_N]\| \\ \angle\{X[k]\} = -\angle\{X[((-k))_N]\} \end{cases}$ |
| 16. | $x_{ep}[n] = \frac{1}{2}\{x[n] + x[((-n))_N]\}$ | $\mathcal{R}e\{X[k]\}$ |
| 17. | $x_{op}[n] = \frac{1}{2}\{x[n] - x[((-n))_N]\}$ | $j\mathcal{I}m\{X[k]\}$ |

**8.24.** Figure P8.24 shows a finite-length sequence $x[n]$. Sketch the sequences

$$x_1[n] = x[((n-2))_4], \qquad 0 \le n \le 3,$$

and

$$x_2[n] = x[((-n))_4], \qquad 0 \le n \le 3.$$

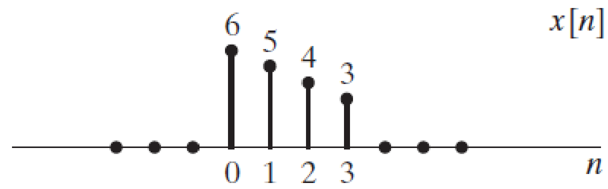
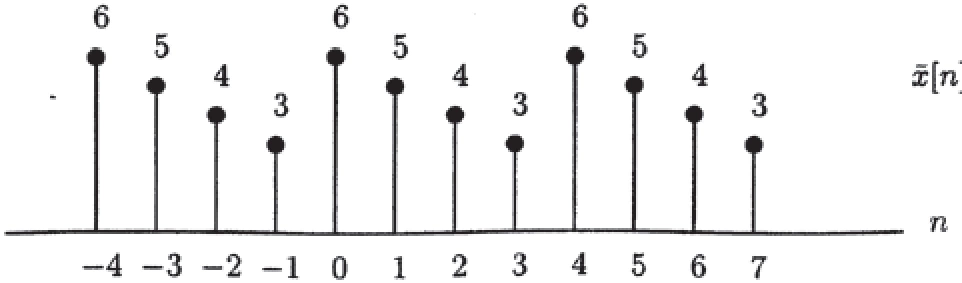
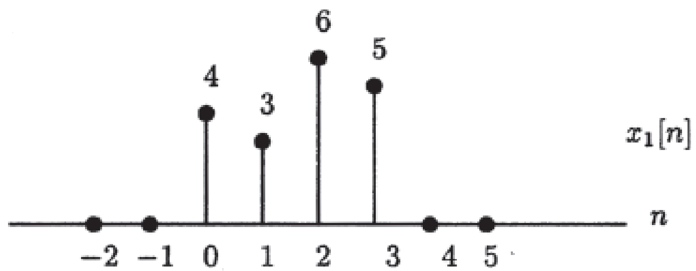


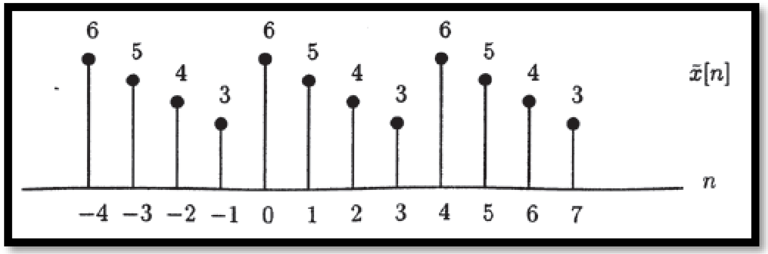

**Figure P8.24**

We may approach this problem in two ways. First, the notion of modulo arithmetic may be simplified if we utilize the implied periodic extension. That is, we redraw the original signal as if it were periodic with period $N = 4$. A few periods are sufficient:

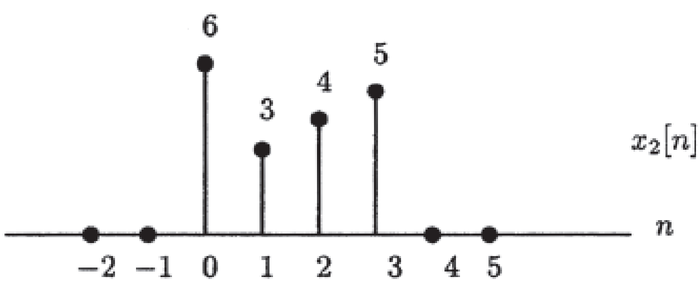


To obtain $x_1[n] = x[((n-2))_4]$, we shift by two (to the right) and only keep those points which lie in the original domain of the signal (i.e. $0 \le n \le 3$):




To obtain $x_2[n] = x[((-n))_4]$, we fold the pseudo-periodic version of $x[n]$ over the origin (time-reversal), and again we set all points outside $0 \le n \le 3$ equal to zero. Hence,




Note that $x[((0))_4] = x[0]$, etc.

In the second approach, we work with the given signal. The signal is confined to $0 \le n \le 3$; therefore, the circular nature must be maintained by picturing the signal on the circumference of a cylinder.

Consider the finite-length sequence $x[n]$ in Fig. 1 below. The five-point DFT of $x[n]$ is denoted by $X[k]$. Plot the sequence $y[n]$ whose DFT is

$$Y[k] = W_5^{-2k} X[k], \text{ where } W_5 = e^{-j\frac{2\pi}{5}}.$$

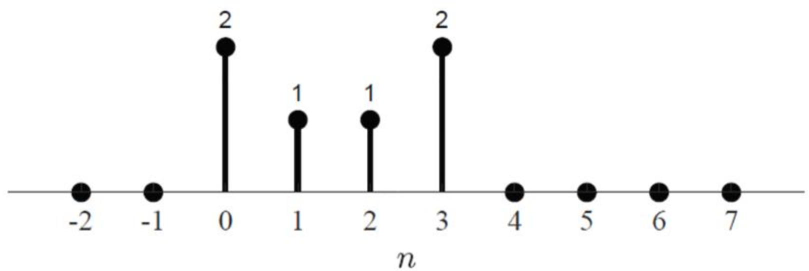
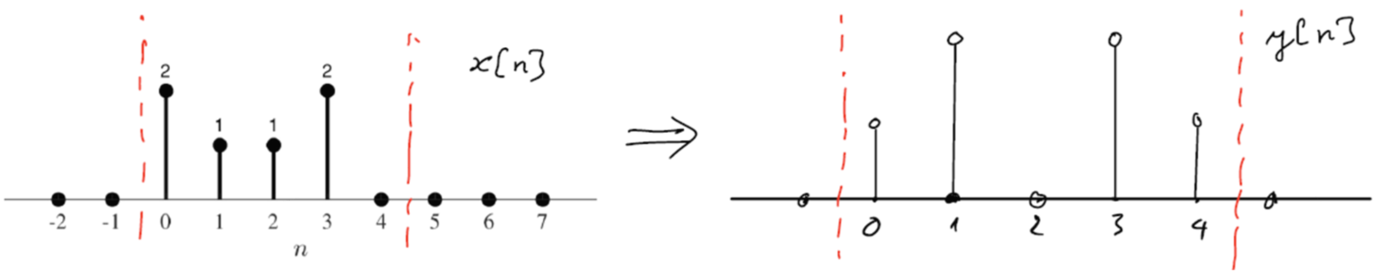



Figure 1: Sequence $x[n]$

From the property of the DFT, $y[n]$ is equal to $x[n]$ circularly shifted by 2 to the left, i.e.,

$$y[((n))_5] = \tilde{y}[n] = \tilde{x}[n+2] = x[((n+2))_5]$$



**8.33.** An FIR filter has a 10-point impulse response, i.e.,

$$h[n] = 0 \qquad \text{for } n < 0 \text{ and for } n > 9.$$

Given that the 10-point DFT of $h[n]$ is given by

$$H[k] = \frac{1}{5}\delta[k-1] + \frac{1}{3}\delta[k-7],$$

find $H(e^{j\omega})$, the DTFT of $h[n]$.

---

By computing the IDFT we find

$$h[n] = \frac{1}{10}\left(\frac{1}{5}e^{j(2\pi/10)n} + \frac{1}{3}e^{j(2\pi/10)7n}\right) \qquad \text{for } n = 0, 1, \ldots, 9.$$

Now

$$\begin{aligned}
H(e^{j\omega}) &= \sum_{n=0}^{9} h[n]e^{j\omega n} = \frac{1}{10}\sum_{n=0}^{9}\left(\frac{1}{5}e^{j(2\pi/10)n} + \frac{1}{3}e^{j(2\pi/10)7n}\right)e^{-j\omega n} \\
&= \frac{1}{10}\left(\frac{1}{5}\sum_{n=0}^{9} e^{j((2\pi/10)-\omega)n} + \frac{1}{3}\sum_{n=0}^{9} e^{j((2\pi/10)7-\omega)n}\right) \\
&= \frac{1}{10}\left(\frac{1}{5}\cdot\frac{1-(e^{j((2\pi/10)-\omega)})^{10}}{1-e^{j((2\pi/10)-\omega)}} + \frac{1}{3}\cdot\frac{1-(e^{j((2\pi/10)7-\omega)})^{10}}{1-e^{j((2\pi/10)7-\omega)}}\right) \\
&= \frac{1}{10}\left(\frac{1}{5}\cdot\frac{1-e^{-j10\omega}}{1-e^{j((2\pi/10)-\omega)}} + \frac{1}{3}\cdot\frac{1-e^{-j10\omega}}{1-e^{j((2\pi/10)7-\omega)}}\right).
\end{aligned}$$

(Sampling $H(e^{j\omega})$ at $\omega_k = \frac{2\pi k}{10}$ gives $H[k]$.)

# From Final 2010 Uottawa-ELG4177

Question 3 DFT (/5)

A sequence $x[n] = 4,5,1,2,3,7 \quad n = 0,1,2,3,4,5$ has a discrete time Fourier transform $X(e^{j\omega})$.

a) Find $X(e^{j\omega})$.

b) A DFT $X(k)$ of size $N = 4$ is obtained by computing samples of $X(e^{j\omega})$ at frequencies $\omega = k\frac{2\pi}{N} \quad 0 \le k \le N-1$. Find an expression for $X(k)$.

c) What is the time domain sequence $x_1[n]$ obtained by computing a size $N = 4$ inverse DFT from $X(k)$ in b) ? (no complicated computations required here)

d) How can we compute the linear convolution of $x[n] = 4,5,1,2,3,7 \quad n = 0,1,2,3,4,5$ and $h[n] = 1,3,1,4 \quad n = 0,1,2,3$ using the DFT domain ? (you don't need to provide the convolution result but you do need to provide the details of the process).

a) Find $X(e^{j\omega})$. $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = 4 + 5e^{-j\omega} + e^{-j2\omega} + 2e^{-j3\omega} + 3e^{-j4\omega} + 7e^{-j5\omega}$

b) A DFT $X(k)$ of size $N = 4$ is obtained by computing samples of $X(e^{j\omega})$ at frequencies $\omega = k\frac{2\pi}{N} \quad 0 \le k \le N-1$. Find an expression for $X(k)$.

$$X(k) = X(e^{j\omega})\Big|_{\omega = k\frac{2\pi}{4}} = 4 + 5e^{-jk\frac{2\pi}{4}} + e^{-j2k\frac{2\pi}{4}} + 2e^{-j3k\frac{2\pi}{4}} + 3e^{-j4k\frac{2\pi}{4}} + 7e^{-j5k\frac{2\pi}{4}}$$

c) What is the time domain sequence $x_1[n]$ obtained by computing a size $N = 4$ inverse DFT from $X(k)$ in b) ? (no complicated computations required here)

SAMPLING $X(e^{j\omega})$ CAN BE SEEN AS PRODUCING A PERIODIC SEQUENCE

$\hat{x}_1[n] = \sum_{k=-\infty}^{+\infty} x[n-kN]$ WHICH CAN CREATE TIME ALIASING IN THE TIME DOMAIN,

$x_1[n]$ IS $\hat{x}_1[n]$ CONSIDERED OVER ONE PERIOD, IN THE INTERVAL $0 \leq n \leq N-1$.

FOR $N=4$, THE SAMPLES $\{3,7\}$ WILL BE ALIASED OVER SAMPLES $\{4,5\}$,

LEADING TO $x_1[n] = 7, 12, 1, 2 \quad n = 0,1,2,3$. THIS COULD BE OBTAINED BY COMPUTING $x_1[n] = IDFT(X_1(k))$, BUT IT TAKES LONGER...

d) How can we compute the linear convolution of $x[n] = 4,5,1,2,3,7 \quad n = 0,1,2,3,4,5$ and $h[n] = 1,3,1,4 \quad n = 0,1,2,3$ using the DFT domain ? (you don't need to provide the convolution result but you do need to provide the details of the process).

$x[n]$ SIZE $L_1 = 6$

$h[n]$ SIZE $L_2 = 4$

$y[n] = x[n] * h[n]$ (LINEAR CONVOLUTION) WILL BE OF SIZE $N = 6+4-1 = 9$

WE NEED 9 SAMPLES OF $Y(e^{j\omega})$ IN ITS DFT $Y(k)$ TO FULLY REPRESENT $y[n]$, WITHOUT TIME ALIASING. $X(k)$ AND $H(k)$ MUST ALSO BE OF SIZE $N=9$:

$X(k) = DFT(\underbrace{x[n]}_{6} + \underbrace{\text{ZERO PADDING}}_{3}) \qquad H(k) = DFT(\underbrace{h[n]}_{4} + \underbrace{\text{ZERO PADDING}}_{5})$

$Y(k) = X(k)\,H(k) \qquad y[n] = IDFT(Y(k))$

# THE END