ELG4177 - DIGITAL SIGNAL PROCESSING
Tutorial 6

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Midterm Part2 Solution

Introduction to Lab#6

1. [4 marks] Consider an unknown LTI system. With an input x[n] = δ[n], the system's output was found to be: $y[n] = 3\delta[n-1] - \delta[n-2] + 7\delta[n-4] + 2\delta[n-6]$.

What is the system's output if the input is given by $x[n] = 3\delta[n] - 4\delta[n-1]$.

Q1: the system is LTI.

$x[n] = 3\delta[n] - 4\delta[n-1]$

when $x_1[n] = \delta[n]$, $y_1[n] = 3\delta[n-1] - \delta[n-2] + 7\delta[n-4] + 2\delta[n-6]$

when $x_2[n] = \delta[n-1]$,

$y_2[n] = y_1[n-1] = 3\delta[n-2] - \delta[n-3] + 7\delta[n-5] + 2\delta[n-7]$

$\therefore y[n] = 3y_1[n] - 4y_2[n]$

$= 9\delta[n-1] - 3\delta[n-2] + 21\delta[n-4] + 6\delta[n-6]$

$-12\delta[n-2] + 4\delta[n-3] - 28\delta[n-5] - 8\delta[n-7]$

$= 9\delta[n-1] - 15\delta[n-2] + 4\delta[n-3] + 21\delta[n-4]$

$-28\delta[n-5] + 6\delta[n-6] - 8\delta[n-7]$.

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Q1:- The system is LTI

$x[n] = \delta[n]$

$y[n] = h[n] = 3\delta[n-1] - \delta[n-2] + 7\delta[n-4]$
$+ 2\delta[n-6]$

$y[n] = x[n] \circledast h(n)$

$\delta[n] \circledast h(n) = h(n)$

$H(e^{jw}) = 3e^{-jw} - e^{-2jw} + 7e^{-4jw} + 2e^{-6jw}$

$Y(e^{jw}) = H(e^{jw})\, X(e^{jw})$

$= 9e^{-jw} - 12e^{-2j} - 3e^{-2jw}$

$+ 4e^{-3jw} + 21e^{-4jw} - 28e^{-5jw}$

$+ 6e^{-6jw} - 8e^{-7jw}$

$y[n] = 9\delta[n-1] - 15\delta[n-2] + 4\delta[n-3]$
$+ 21\delta[n-4] - 28\delta[n-5] + 6\delta[n-6]$
$- 8\delta[n-7]$

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2. [8 marks] Consider the discrete-time system described by: $y[n] = nx[n]$, where $x[n] = \delta[n - n_0]$. Use two different methods to calculate the Fourier transform of y[n], $Y(e^{j\omega})$.

Q2: $y[n] = n\delta[n-n_0]$

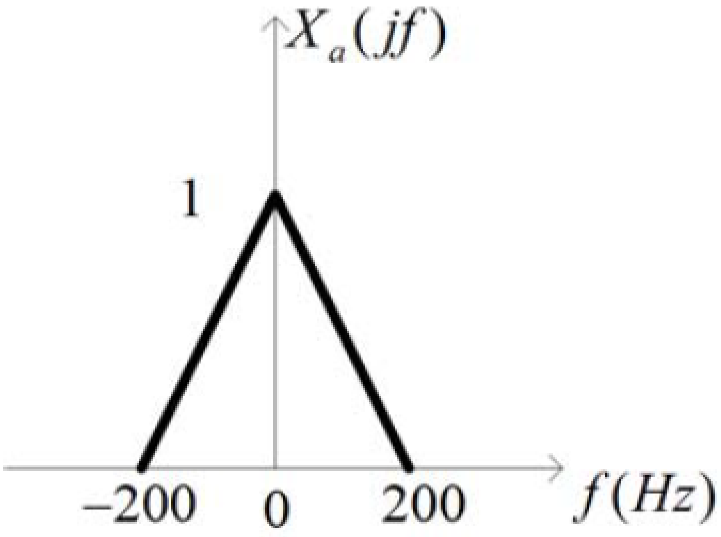
(1) $Y(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} n\delta(n-n_0) e^{-j\omega n} = n_0 e^{-j\omega n_0}$

(2) $\delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$

$n\delta[n-n_0] \longleftrightarrow j\frac{d}{d\omega}(e^{-j\omega n_0})$

$= j \cdot e^{-j\omega n_0} \cdot (-jn_0) = n_0 e^{-j\omega n_0}$.

3. [10 marks in total] Suppose that $x_a(t)$ is an analog signal whose spectrum is shown by the following figure. Sample $x_a(t)$ in the time domain using the Nyquist sampling period (T) and obtain $x[n] = x_a(nT)$.



a. [4 marks] Sketch $X(e^{j\omega})$ which is the Fourier Transform of $x[n]$.

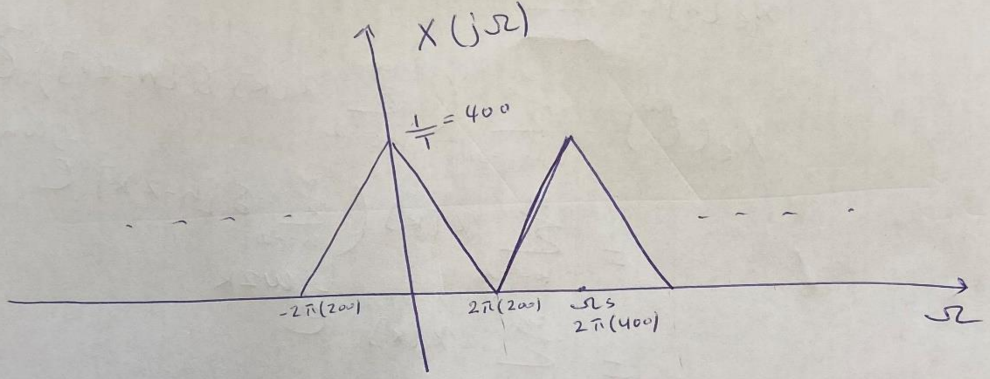
b. [6 marks] Increase the sampling rate by 2. Illustrate in the frequency domain the process of upsampling. In the illustration, draw the Fourier transform $X_e(e^{j\omega})$ of the expanded signal $x_e[n]$, the frequency response of the low-pass filter $H_i(e^{j\omega})$, and the Fourier transform $X_i(e^{j\omega})$ of the upsampled signal $x_i[n]$.

Q3 :-

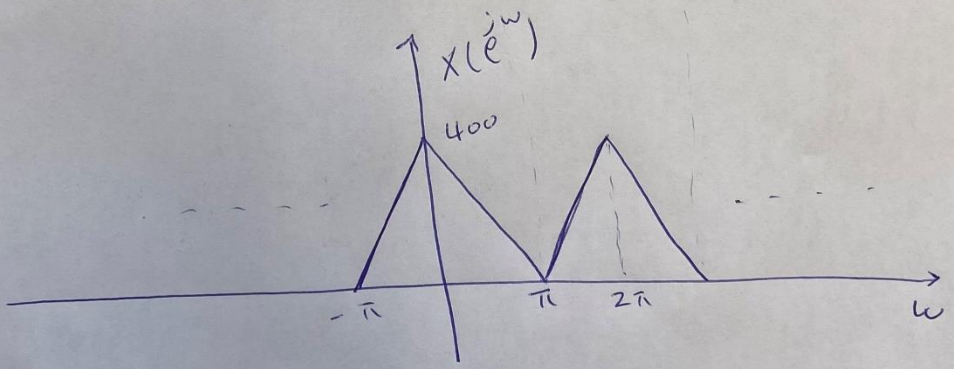
(a) Nyquist Sampling

$f_0 = 200\ HZ \Rightarrow f_s = 2 f_0 = 400\ HZ$

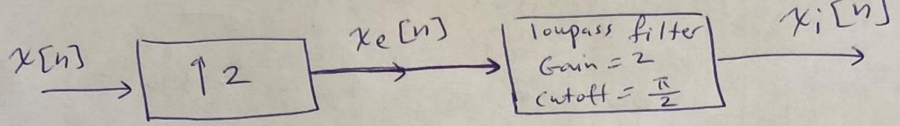
$T = \frac{1}{400}$



$\Omega T = \omega$



(b)



$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n - Lk]$, $L = 2$

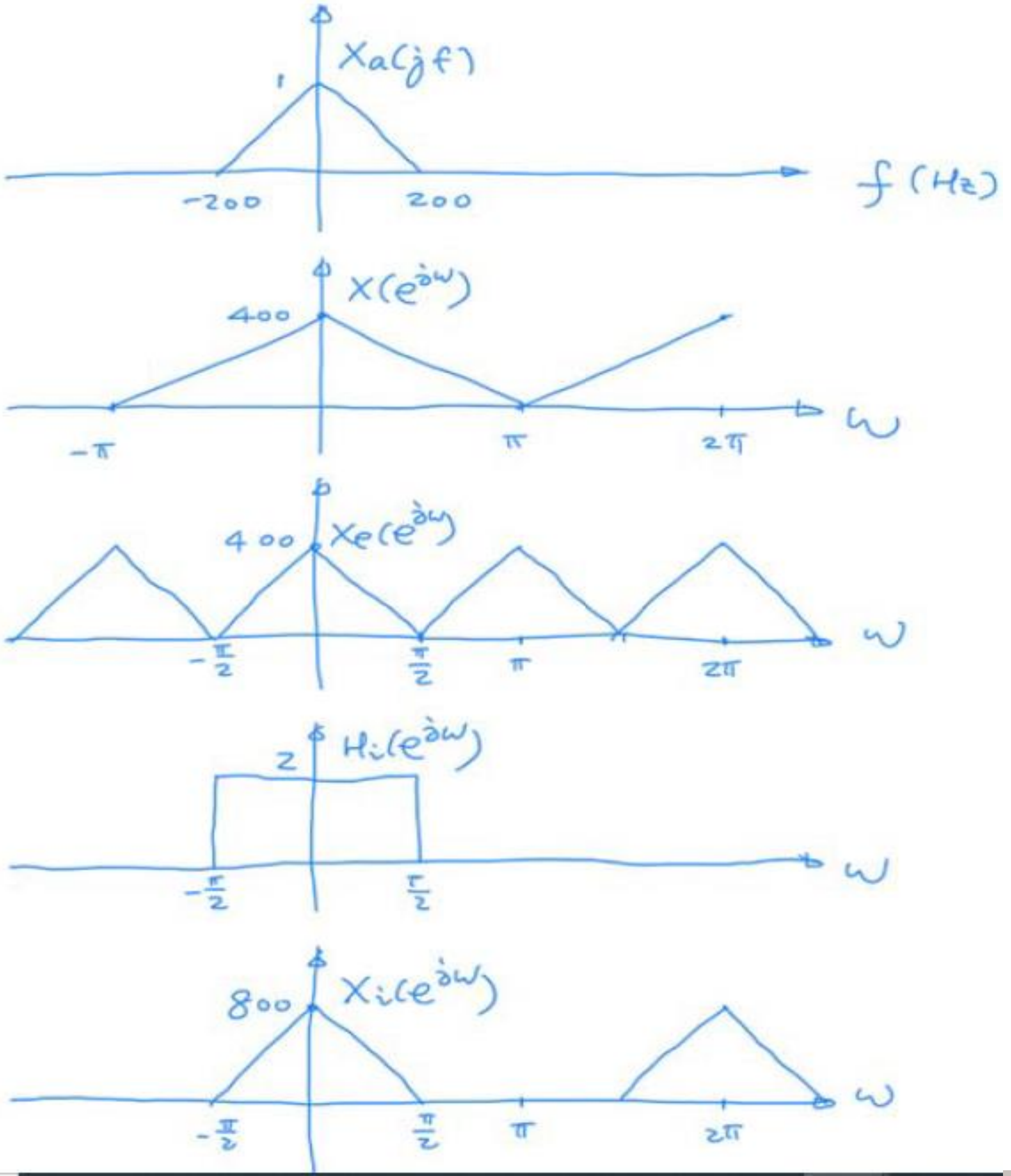
$X_e(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\, \delta[n-2k] \right) e^{-j\omega n}$

$= \sum_{k=-\infty}^{\infty} x[k] \left(\sum_{n=-\infty}^{\infty} \delta[n-2k]\, e^{-j\omega n} \right)$

$= \sum_{k=-\infty}^{\infty} x[k]\, e^{-j\omega 2k}$

$= X(e^{j2\omega})$

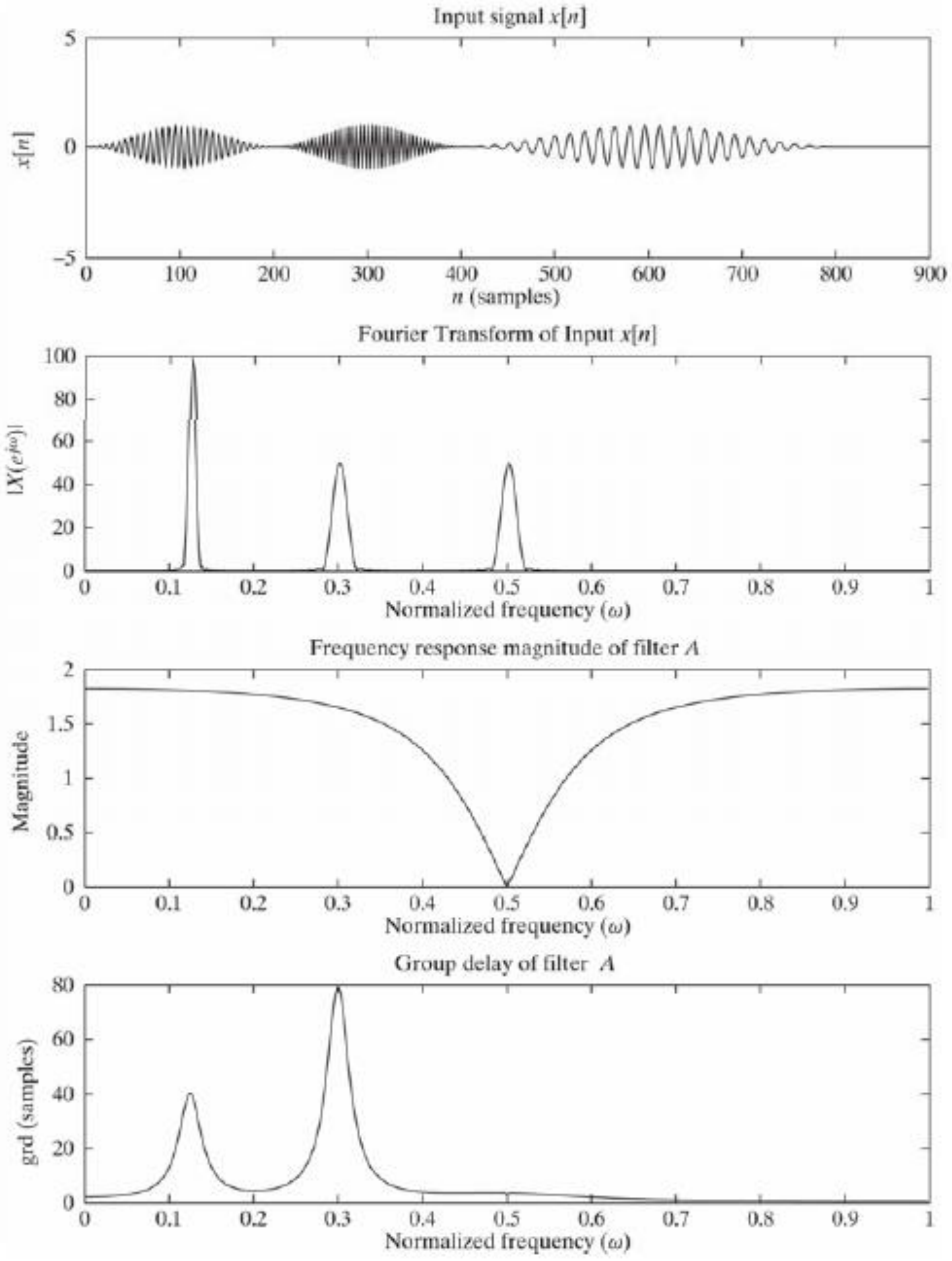
Q3.
(b).



$X_a(jf)$
1
-200
200
f (Hz)
$X(e^{j\omega})$
400
$-\pi$
π
2π
ω
$X_e(e^{j\omega})$
400
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
2π
ω
$H_i(e^{j\omega})$
2
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
ω
$X_i(e^{j\omega})$
800
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
2π
ω

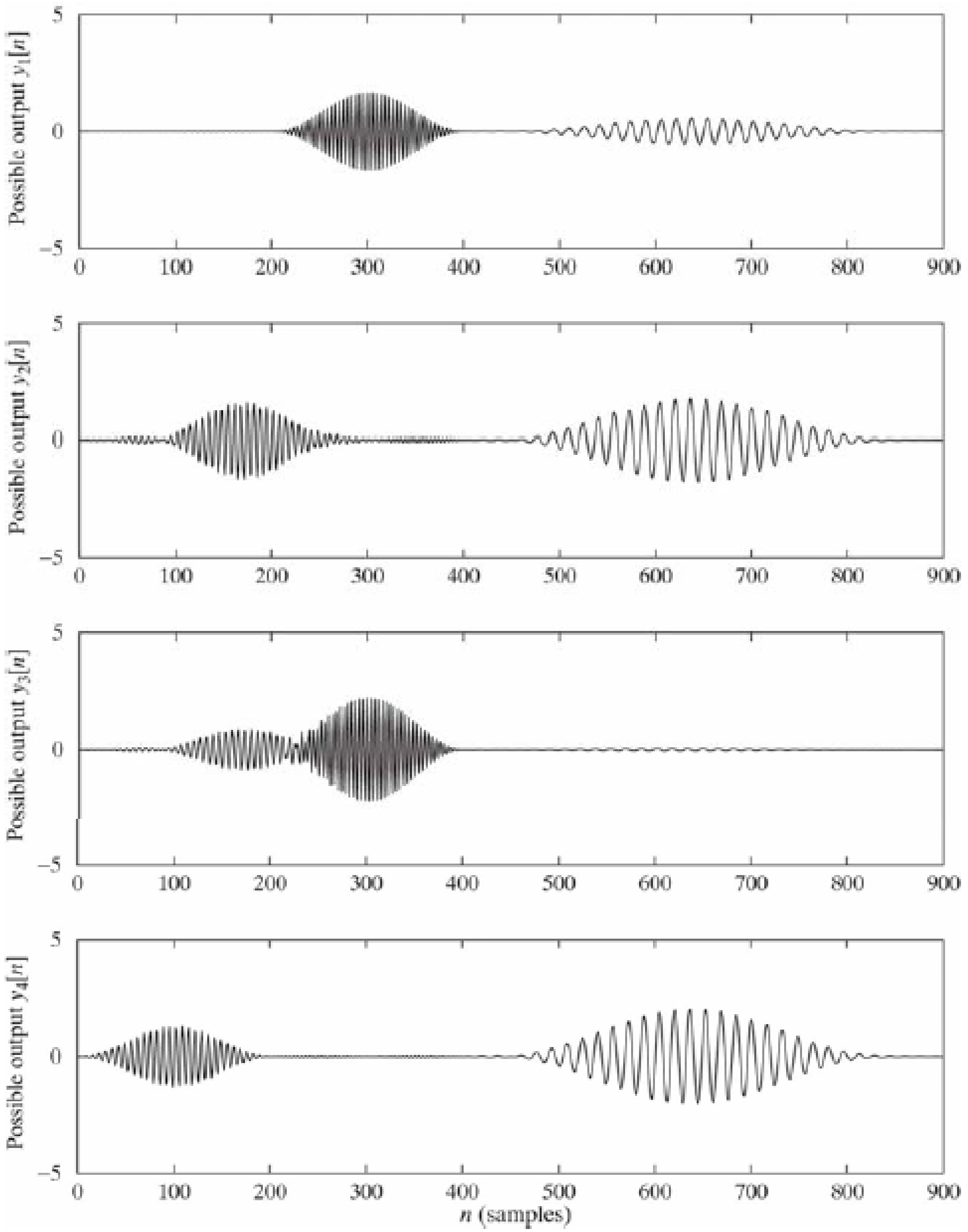
4. [8 marks] A discrete-time LTI system with input x[n] and output y[n] has the frequency response magnitude and group delay functions shown in the first figure (which contains x[n], $|X(e^{j\omega})|$, Magnitude, and grd). The signal x[n] is the sum of three narrowband pulses. In particular, the first figure contains the following plots:

- x[n], the input signal.



- $|X(e^{j\omega})|$, the Fourier transform magnitude of the particular input x[n]
- Frequency response magnitude plot for the system
- Group delay plot for the system

In the second figure you are given four (4) possible output signals, $y_i[n]$ i=1,2,3,4. Determine which one of the possible output signals is the output of the system when the input is x[n]. Provide a justification for your choice.



Q4:

- look at the magnitude response, at $\omega = 0.5\pi$, the magnitude $= 0$. This means that the frequency $\omega = 0.5\pi$ will be eliminated, So $y_2[n]$ & $y_4[n]$ are the candidates.
- group delay for $\omega = 0.3\pi$ is 80 samples. Group delay for $\omega = 0.1\pi$ is 60 samples.

So $y_2[n]$ is the answer.

5. [10 marks in total] Consider a causal LTI system satisfying the following difference equation relating $x[n]$ and $y[n]$:

$$y[n] = -\frac{1}{8}(2y[n-1] - 3y[n-2]) + x[n] + 2x[n-1] + x[n-2]$$

4 2 2

a. Determine the system function $H(z)$. Give the zeros and poles. Sketch the pole-zero plot and specify the region of convergence in the z-plane by shading. [8 marks]

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b. Is this system stable? [2 marks]

Q5:

(a) $y[n] = -\frac{1}{8}(2y[n-1] - 3y[n-2]) + x[n] + 2x[n-1] + x[n-2]$

$$Y(z) = -\frac{1}{8}(2Y(z)z^{-1} - 3Y(z)z^{-2}) + X(z) + 2X(z)z^{-1} + X(z)z^{-2}$$

$$Y(z)\left[1 + \frac{1}{8}(2z^{-1} - 3z^{-2})\right] = X(z)\left[1 + 2z^{-1} + z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 + \frac{1}{8}(2z^{-1} - 3z^{-2})}$$

$$= \frac{(1+z^{-1})^2 \cdot 8}{8 + 2z^{-1} - 3z^{-2}} = \frac{(1+z^{-1})^2 \cdot 8}{(2-z^{-1})(4+3z^{-1})}$$

$$= \frac{(1+z^{-1})^2}{(1-\frac{1}{2}z^{-1})(1+\frac{3}{4}z^{-1})}$$

(b) stable.