ELG4172 Digital Signal Processing

- Tutorial#4

Presented by: Hitham Jleed

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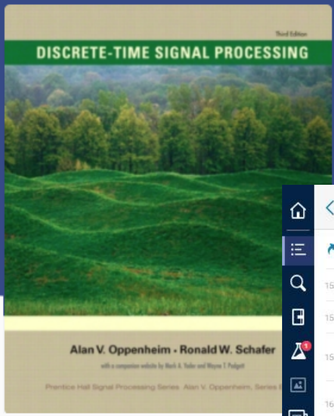

Discrete-Time Signal Processing

Alan V Oppenheim; Ronald W. Schafer

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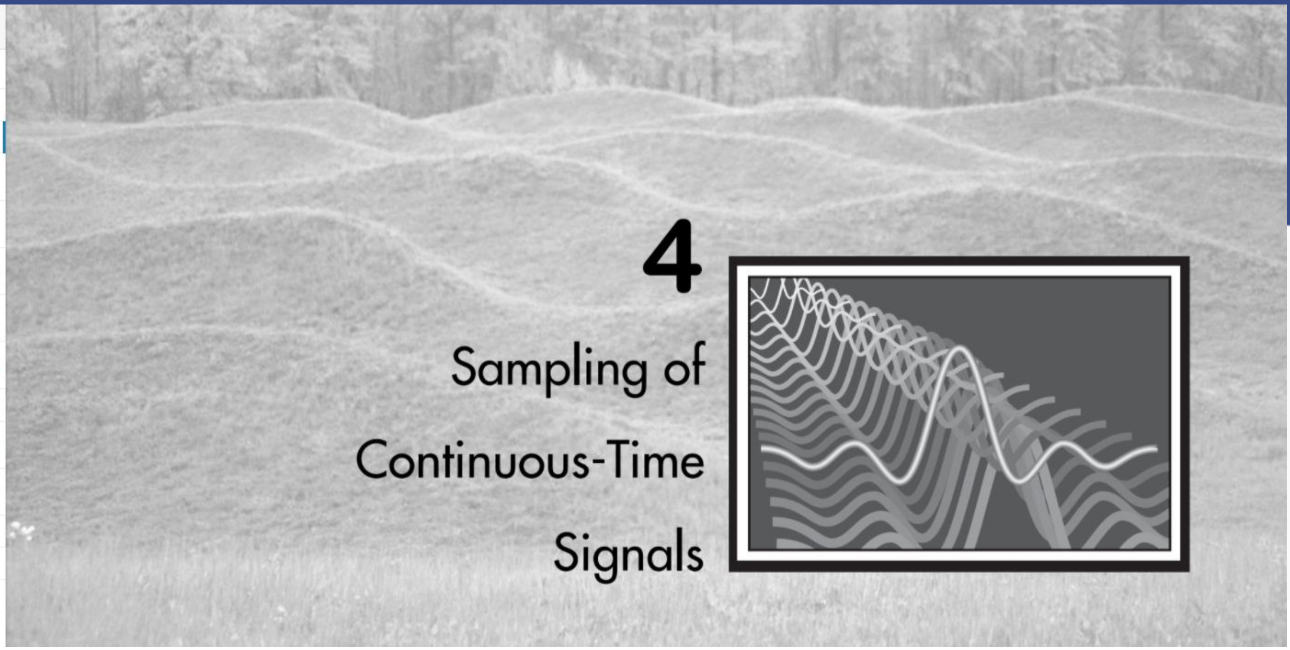

Contents

Problems: 4.3, 4.5, 4.10, 4.28, 4.23, 4.15.

4.3. The continuous-time signal

$$x_c(t) = \cos(4000\pi t)$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \cos\left(\frac{\pi n}{3}\right).$$

(a) Determine a choice for T consistent with this information.
(b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

Solution

4.3. (a) Since $x[n] = x_c(nT)$,

$$\frac{\pi n}{3} = 4000\pi n T \quad \Rightarrow \quad T = \frac{1}{12000}$$

(b) No. For example, since

$$\cos(\frac{\pi}{3}n) = \cos(\frac{7\pi}{3}n),$$

T can be 7/12000.

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4.5. Consider the system of Figure 4.10, with the discrete-time system an ideal lowpass filter with cutoff frequency $\pi/8$ radians/s.

(a) If $x_c(t)$ is bandlimited to 5 kHz, what is the maximum value of T that will avoid aliasing in the C/D converter?

(b) If $1/T = 10$ kHz, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat part (b) for $1/T = 20$ kHz.

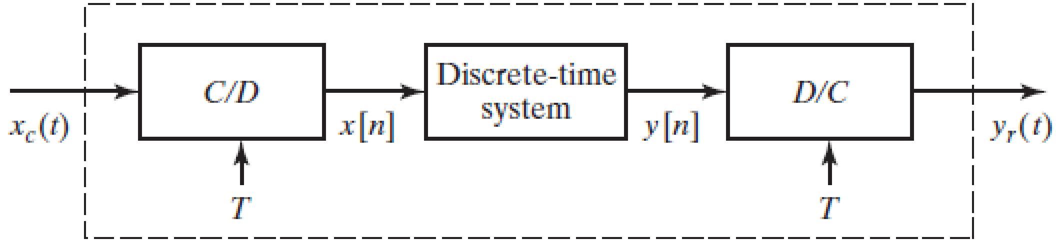
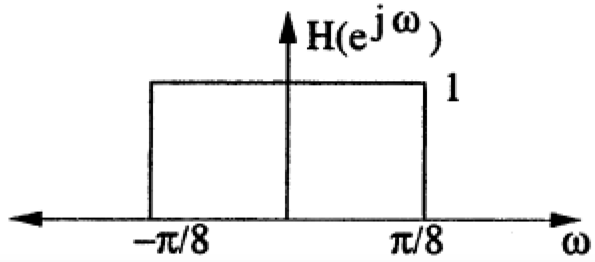


Figure 4.10 Discrete-time processing of continuous-time signals.

4.5. A plot of $H(e^{j\omega})$ appears below.



(a) $$x_c(t) = 0, \quad , |\Omega| \geq 2\pi \cdot 5000$$

The Nyquist rate is 2 times the highest frequency. $\Rightarrow T = \frac{1}{10{,}000}$ sec. This avoids all aliasing in the C/D converter.

(b)
$$\begin{aligned} \frac{1}{T} &= 10kHz \\ \omega &= T\Omega \\ \frac{\pi}{8} &= \frac{1}{10,000}\Omega_c \\ \Omega_c &= 2\pi \cdot 625 \text{rad/sec} \\ f_c &= 625Hz \end{aligned}$$

(c)
$$\begin{aligned} \frac{1}{T} &= 20kHz \\ \omega &= T\Omega \\ \frac{\pi}{8} &= \frac{1}{20,000}\Omega_c \\ \Omega_c &= 2\pi \cdot 1250 \text{rad/sec} \\ f_c &= 1250Hz \end{aligned}$$

4.10. Each of the following continuous-time signals is used as the input $x_c(t)$ for an ideal C/D converter as shown in Figure 4.1 with the sampling period T specified. In each case, find the resulting discrete-time signal $x[n]$.

(a) $x_c(t) = \cos(2\pi(1000)t)$, $\quad T = (1/3000)$ sec
(b) $x_c(t) = \sin(2\pi(1000)t)$, $\quad T = (1/1500)$ sec
(c) $x_c(t) = \sin(2\pi(1000)t) / (\pi t)$, $\quad T = (1/5000)$ sec

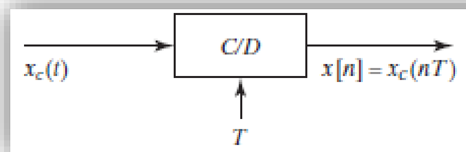


Figure 4.1 Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

4.10. Use $x[n] = x_c(nT)$, and simplify:

(a) $x[n] = \cos(2\pi n/3)$.

(b) $x[n] = \sin(4\pi n/3) = -\sin(2\pi n/3)$

(c) $x[n] = \dfrac{\sin(2\pi n/5)}{\pi n/5000}$

4.28. Consider the representation of the process of sampling followed by reconstruction shown in Figure P4.28.

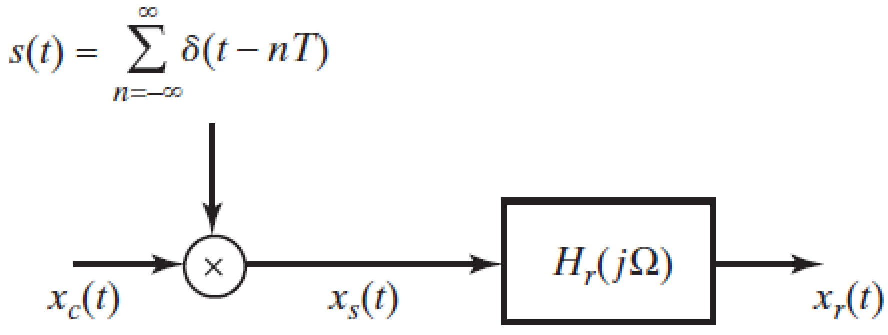


Figure P4.28

Assume that the input signal is

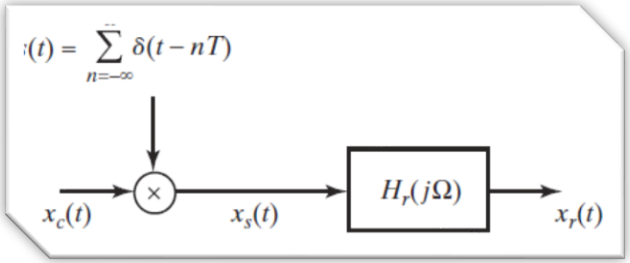
$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3) \qquad -\infty < t < \infty$$

The frequency response of the reconstruction filter is

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

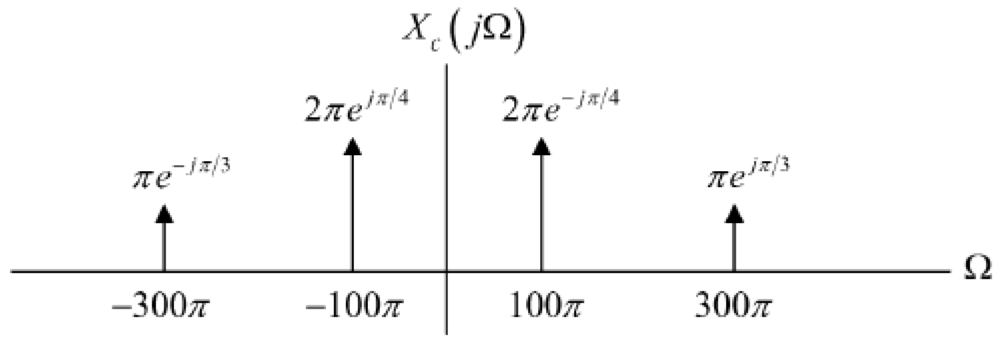
(a) Determine the continuous-time Fourier transform $X_c(j\Omega)$ and plot it as a function of Ω.

Solution



$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

$$X_c(j\Omega) = 2\pi e^{-j\pi/4}\delta(\Omega - 100\pi) + 2\pi e^{j\pi/4}\delta(\Omega + 100\pi) + \pi e^{j\pi/3}\delta(\Omega - 300\pi) + \pi e^{-j\pi/3}\delta(\Omega + 300\pi).$$



Ω denotes continuous-time frequency

ω denotes discrete-time frequency.

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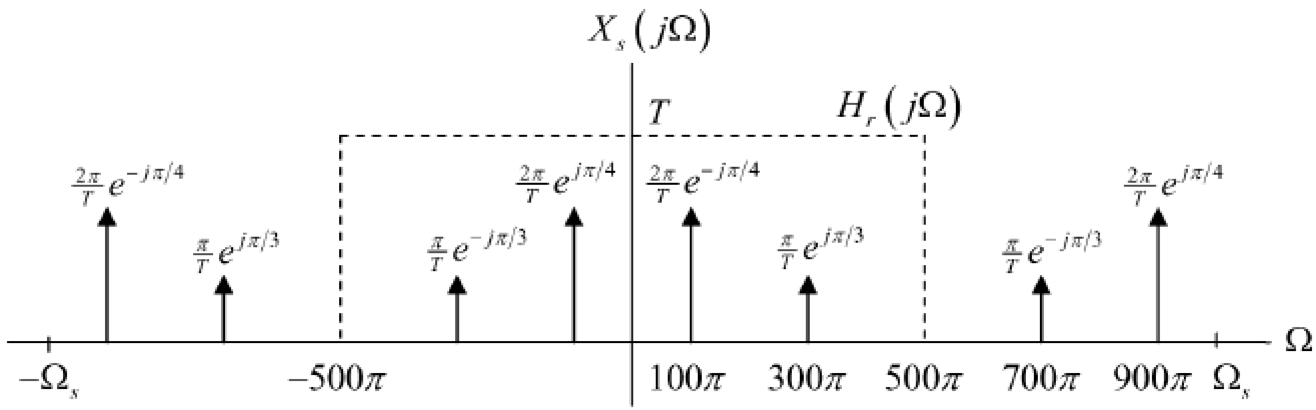
(b) Assume that $f_s = 1/T = 500$ samples/sec and plot the Fourier transform $X_s(j\Omega)$ as a function of Ω for $-2\pi/T \le \Omega \le 2\pi/T$. What is the output $x_r(t)$ in this case? (You should be able to give an exact equation for $x_r(t)$.)

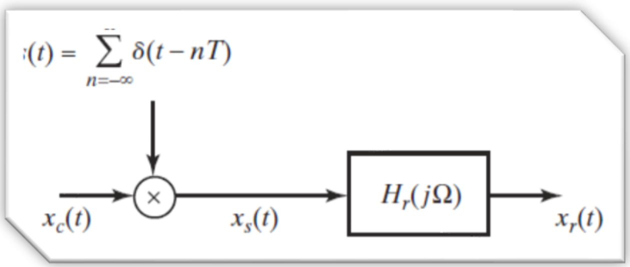
Solution

B. If $f_s = 1/T = 500$ samples/s then $\Omega_s = 2\pi/T = 1000\pi$ rad/s.



There is no aliasing, so $x_r(t) = x_c(t)$; that is,

$$x_r(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3).$$



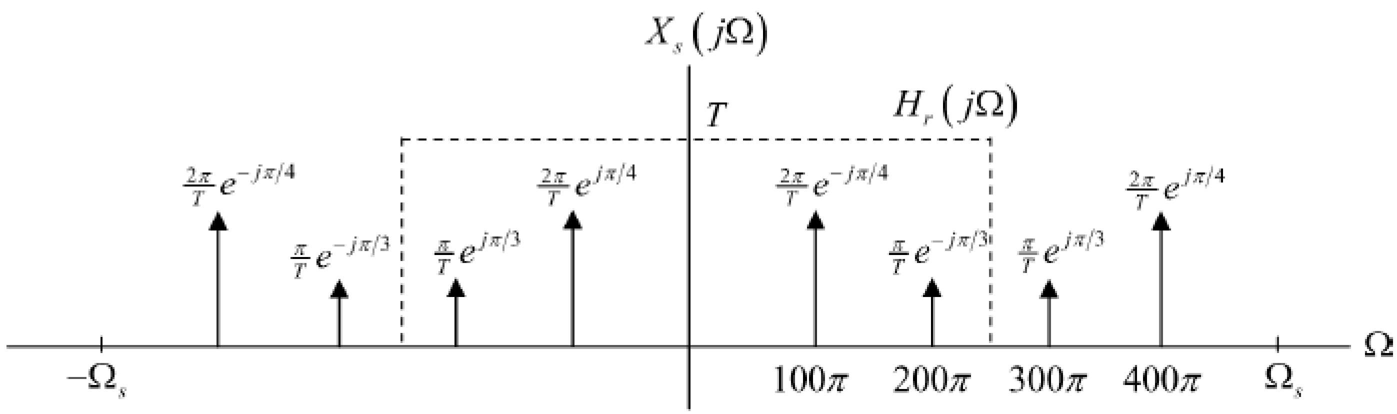
$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

(c) Now, assume that $f_s = 1/T = 250$ samples/sec. Repeat part (b) for this condition.

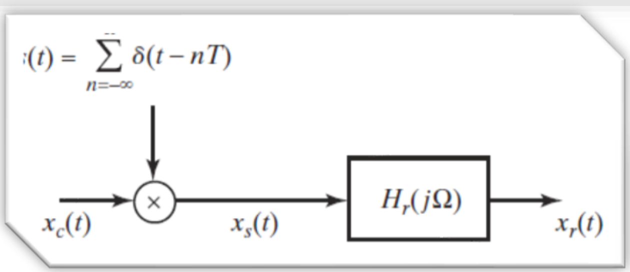
Solution

C. If $f_s = 1/T = 250$ samples/s then $\Omega_s = 2\pi/T = 500\pi$ rad/s.



Now there is aliasing and

$$x_r(t) = 2\cos(100\pi t - \pi/4) + \cos(200\pi t - \pi/3).$$



$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

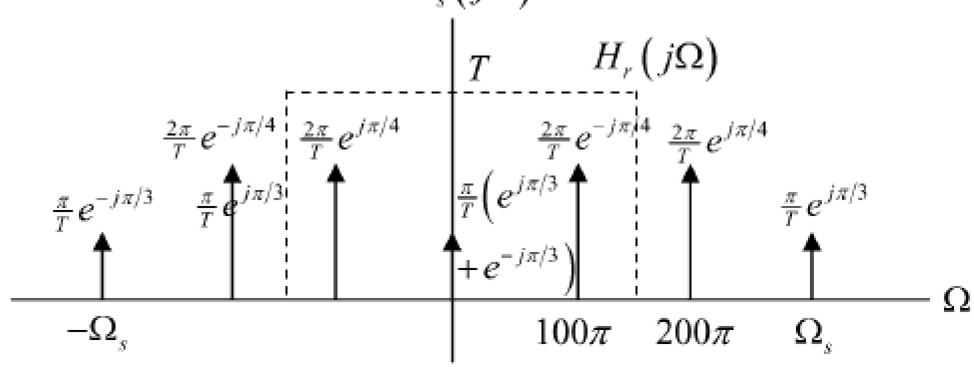
(d) Is it possible to choose the sampling rate so that

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant? If so, what is the sampling rate $f_s = 1/T$, and what is the numerical value of A?

Solution

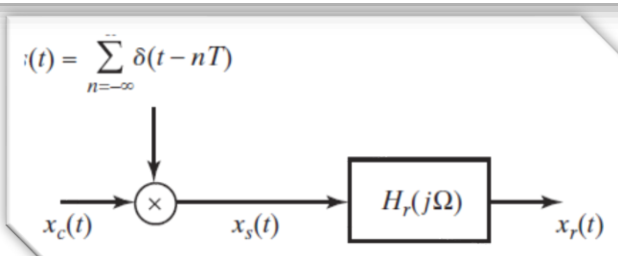
D. We want to sample the component at 300π rad/s exactly once per cycle, so that all the samples have the same value. At $\Omega_s = 300\pi$ rad/s we have



Now

$$x_r(t) = \cos(\pi/3) + 2\cos(100\pi - \pi/4) = 1/2 + 2\cos(100\pi - \pi/4).$$

We have $A = 1/2$.



$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

4.23. Figure P4.23-1 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \leq \omega \leq \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \leq \pi. \end{cases}$$

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

(i) $1/T_1 = 1/T_2 = 2 \times 10^4$
(ii) $1/T_1 = 4 \times 10^4, \quad 1/T_2 = 10^4$
(iii) $1/T_1 = 10^4, \quad 1/T_2 = 3 \times 10^4$.

(b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c, specify $H_c(j\Omega)$.

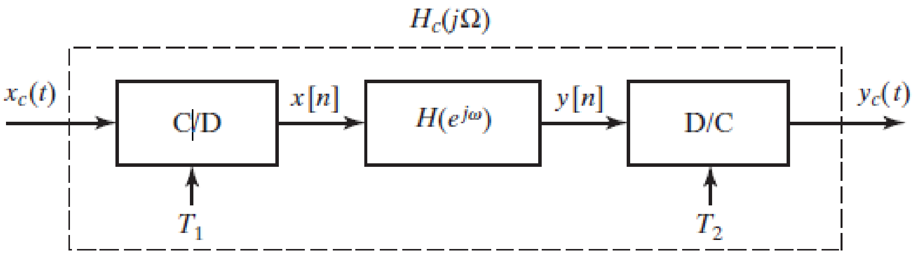


Figure P4.23-1

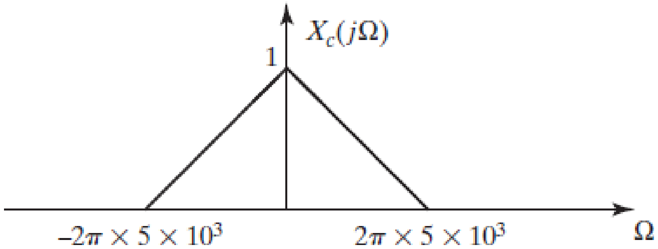


Figure P4.23-2

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

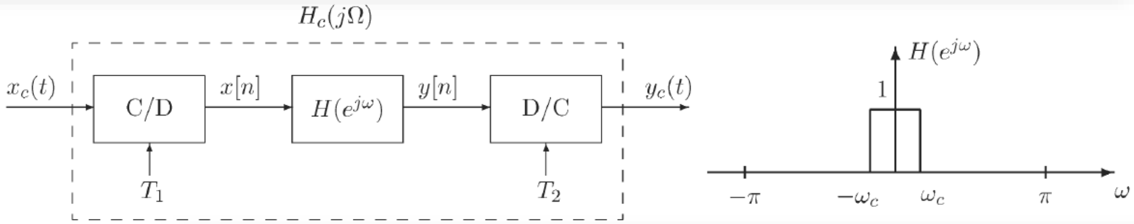
(i) $1/T_1 = 1/T_2 = 2 \times 10^4$

(ii) $1/T_1 = 4 \times 10^4$, $1/T_2 = 10^4$

(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$.

Ω denotes continuous-time frequency

ω denotes discrete-time frequency.



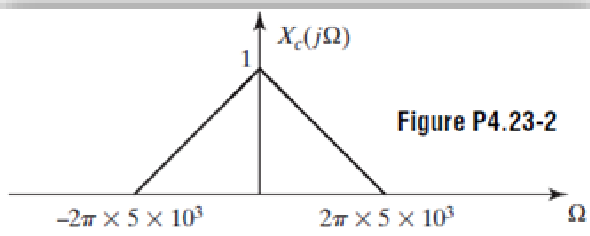
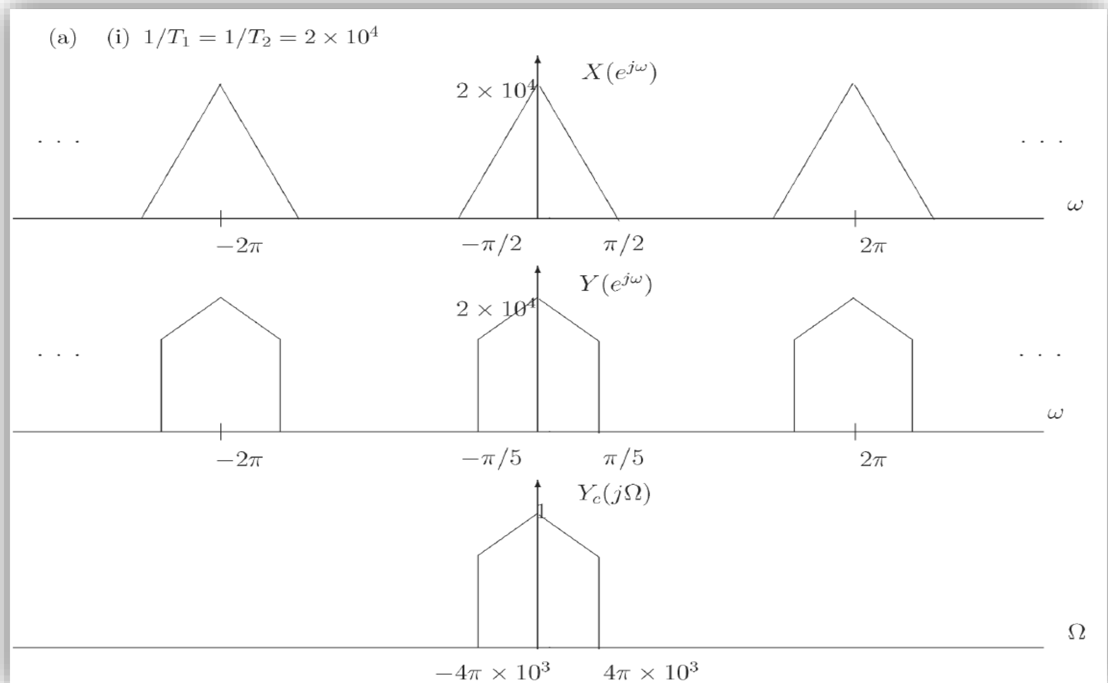


Figure P4.23-2



(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

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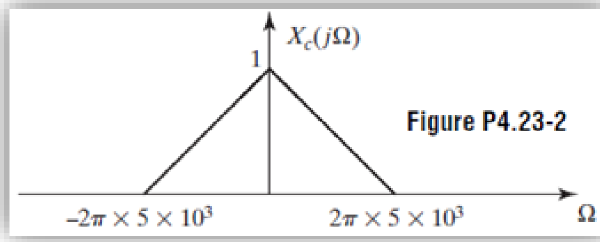
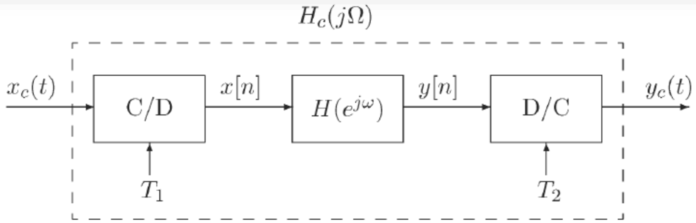
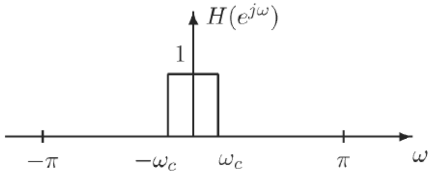
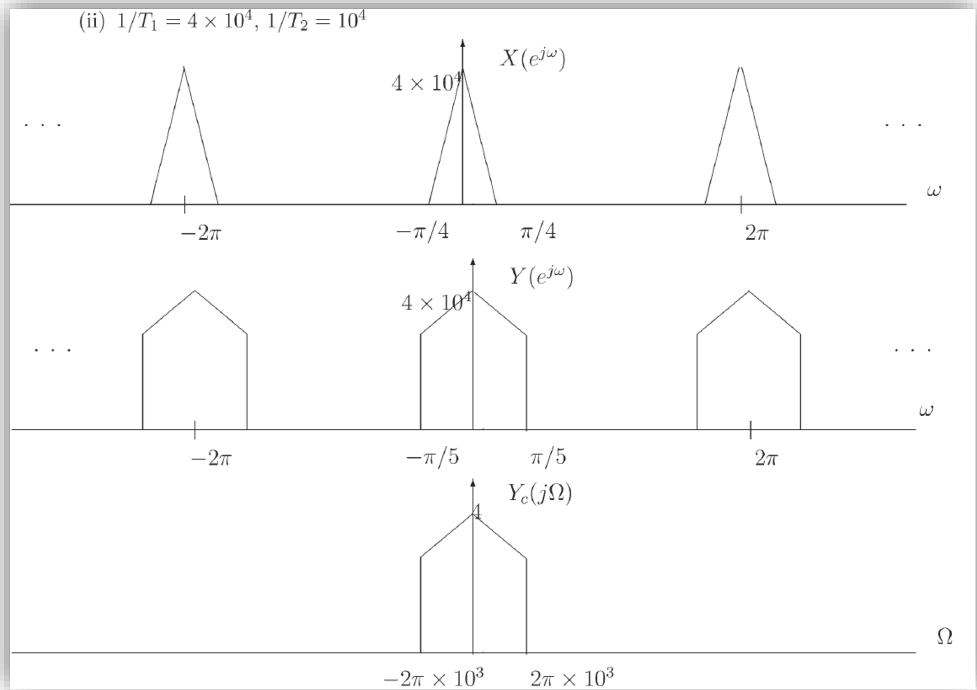


Figure P4.23-2







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(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

(i) $1/T_1 = 1/T_2 = 2 \times 10^4$

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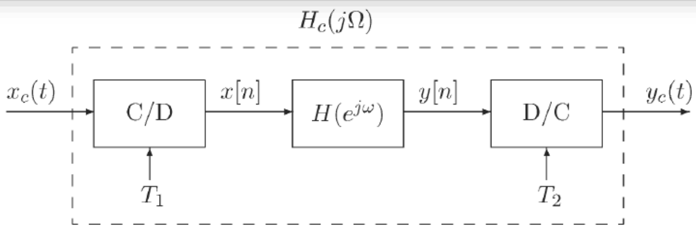
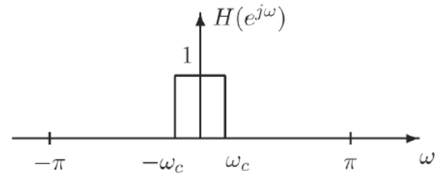




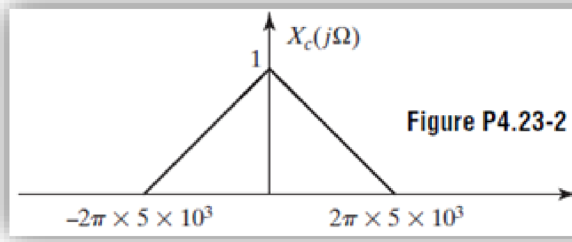
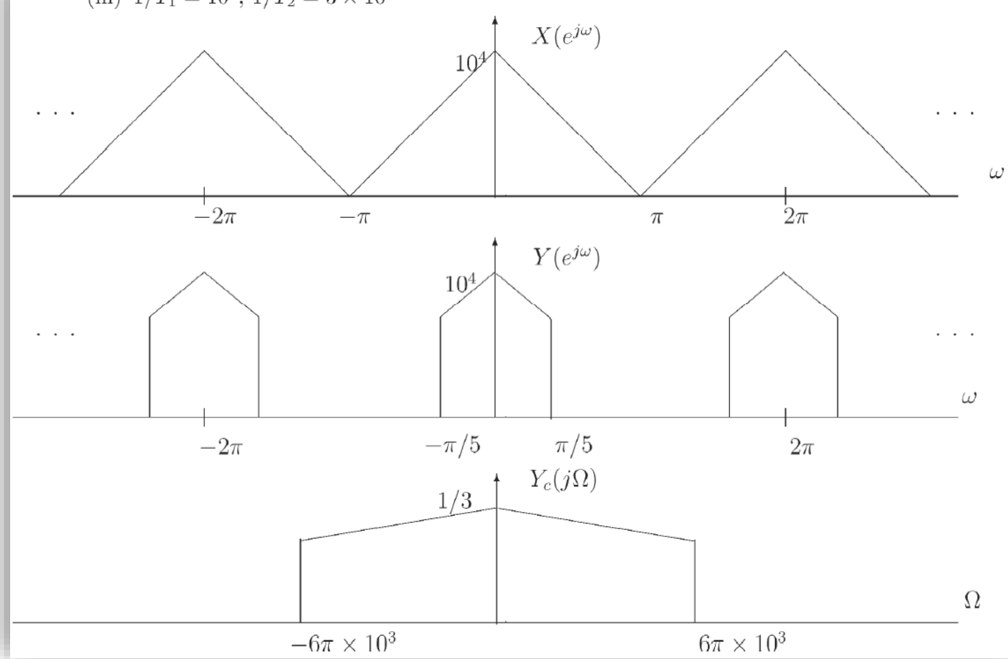


Figure P4.23-2

(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$



(b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c, specify $H_c(j\Omega)$.

Ω denotes continuous-time frequency

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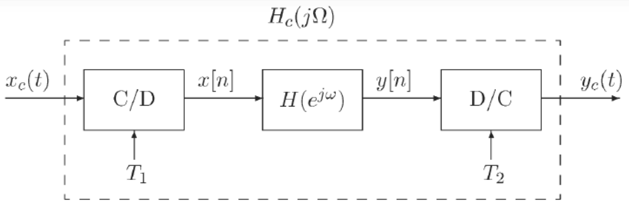
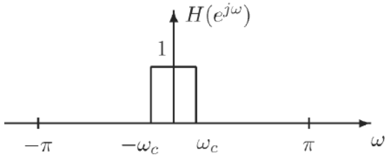




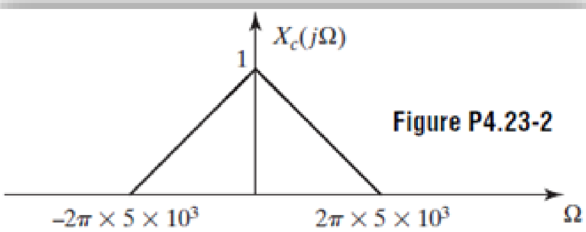
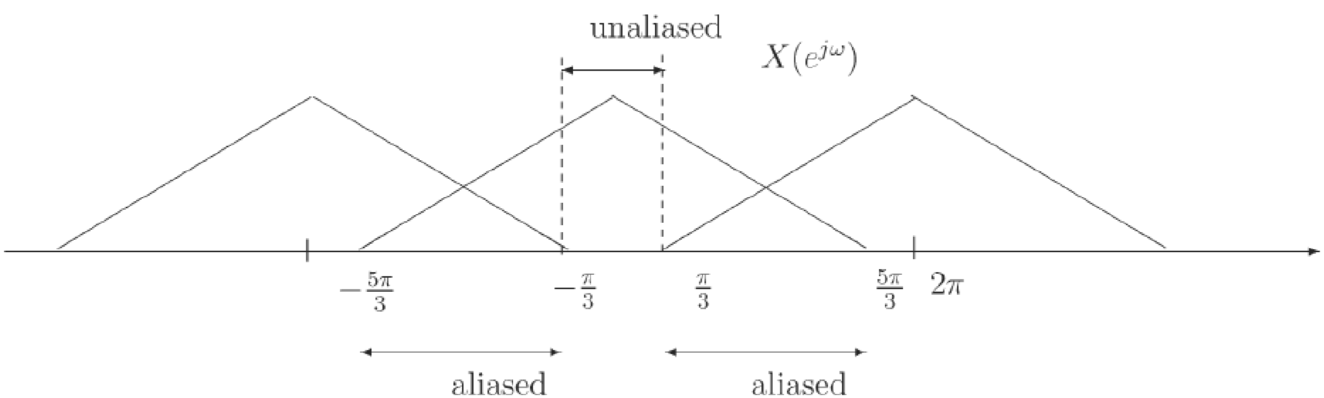


Figure P4.23-2

(b) From the figure below, it can be seen that the only portion of the spectrum which remains unaffected by the aliasing is $|\omega| < \pi/3$. So if we choose $\omega_c < \pi/3$, the overall system is LTI with a frequency response of

$$H_c(j\Omega) = \begin{cases} 1 & \text{for } |\Omega| < \omega_c \times 6 \times 10^3 \\ 0 & \text{otherwise.} \end{cases}$$



4.6.4 Changing the Sampling Rate by a Non-integer Factor

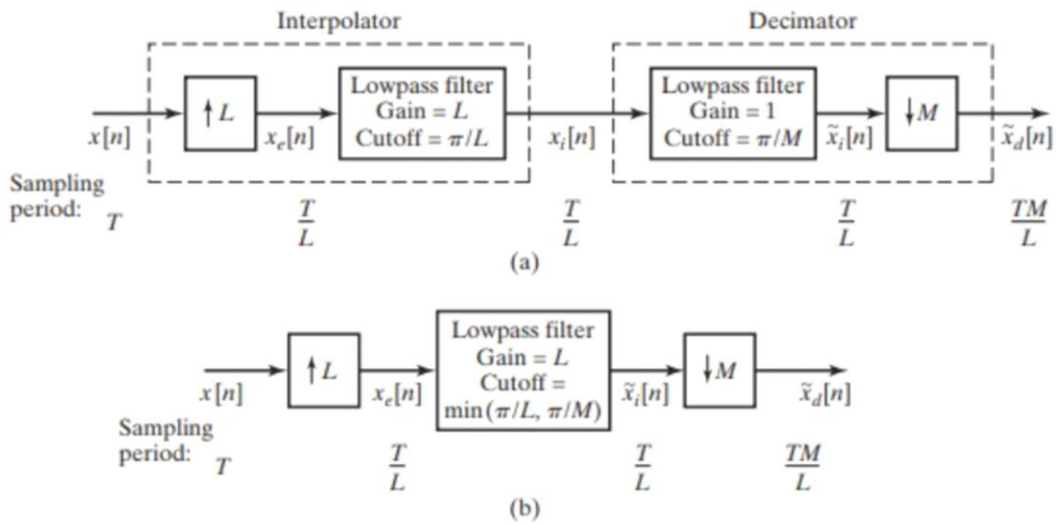


Figure 4.29 (a) System for changing the sampling rate by a noninteger factor. (b) Simplified system in which the decimation and interpolation filters are combined.

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To Read more from other books:
2) Mitra, 3/e: Section 13.1-13.3
1) Proakis& Manolakis 4/e: Section 11.1-11.4
3) Orfanidis: Section 12.1-12.3

4.15. Consider the system shown in Figure P4.15. For each of the following input signals $x[n]$, indicate whether the output $x_r[n] = x[n]$.

(a) $x[n] = \cos(\pi n/4)$

(b) $x[n] = \cos(\pi n/2)$

(c) $x[n] = \left[\dfrac{\sin(\pi n/8)}{\pi n}\right]^2$

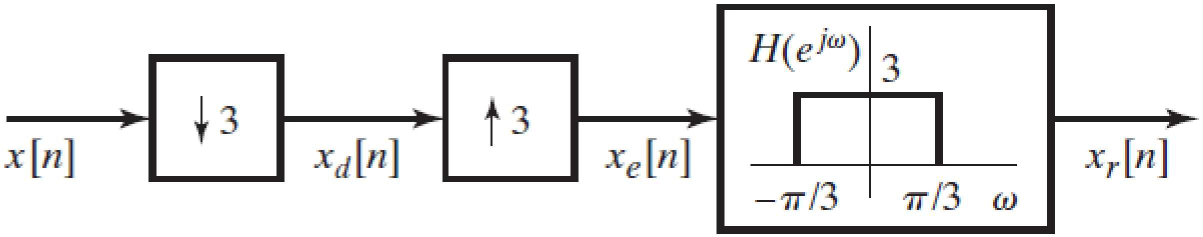


Figure P4.15

Hint: Use the modulation property of the Fourier transform to find $X(e^{j\omega})$.

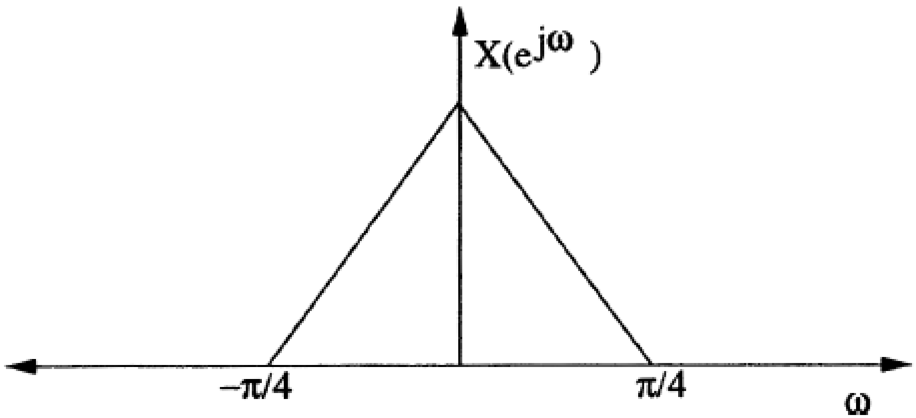
Solution

4.15. The output $x_r[n] = x[n]$ if no aliasing occurs as result of downsampling. That is, $X(e^{j\omega}) = 0$ for $\pi/3 \leq |\omega| \leq \pi$.

(a) $x[n] = \cos(\pi n/4)$. $X(e^{j\omega})$ has impulses at $\omega = \pm\pi/4$, so there is no aliasing. $x_r[n] = x[n]$.

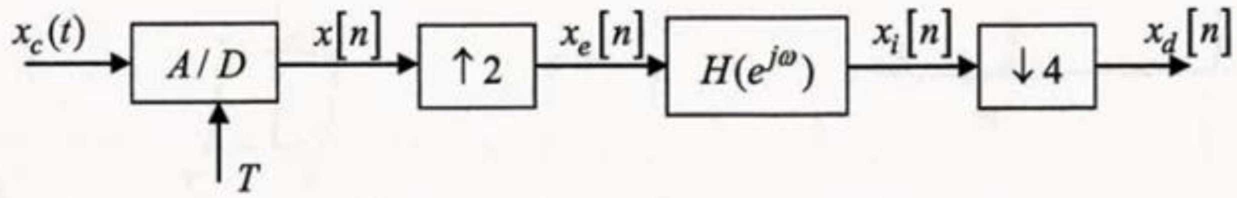
(b) $x[n] = \cos(\pi n/2)$. $X(e^{j\omega})$ has impulses at $\omega = \pm\pi/2$, so there is aliasing. $x_r[n] \neq x[n]$.

(c) A sketch of $X(e^{j\omega})$ is shown below. Clearly there will be no aliasing and $x_r[n] = x[n]$.

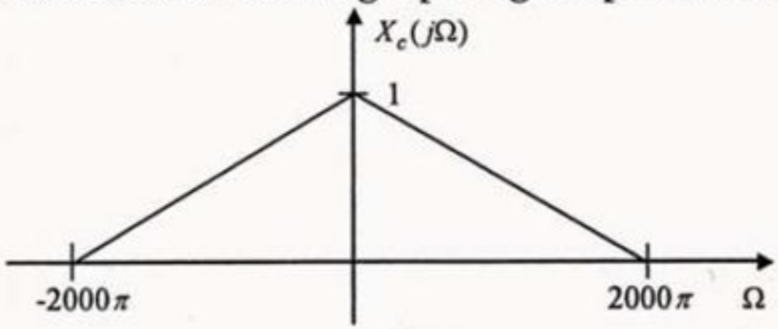


Question 6 Sample rate conversion

From Old Exam ELG 4177



Consider the following input signal spectrum and sampling rate :



$T = 1/2000$ sec. ($F_s = 2000$ Hz, $\Omega_s = 4000\pi$ rad./sec.), and the following lowpass digital filter :

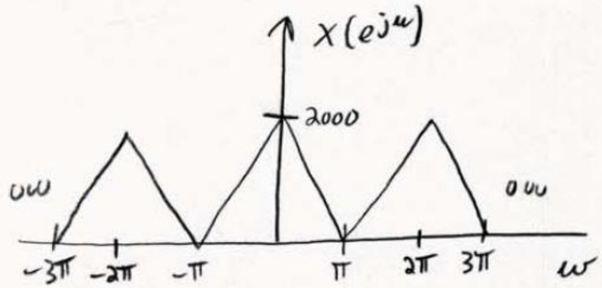
$$\left.\begin{array}{ll} H(e^{j\omega}) = 1 & |\omega| < \pi/4 \\ H(e^{j\omega}) = 0 & |\omega| > \pi/4 \end{array}\right\}$$ over an interval $-\pi \le \omega < \pi$, and periodic 2π.

Draw carefully the spectrum of the different discrete time signals (i.e. draw $X(e^{j\omega})$, $X_e(e^{j\omega})$, $X_i(e^{j\omega})$ and $X_d(e^{j\omega})$, with appropriate levels and frequencies).

Solution

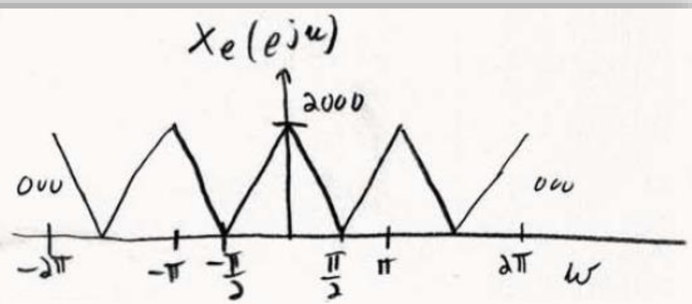
(1) Sampling

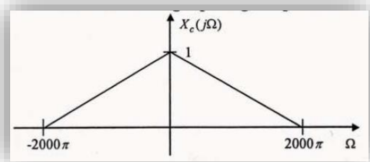
$$X(e^{j\omega}) = F_s \sum_{k=-\infty}^{+\infty} X_c\left(j\frac{\omega}{F_s} - k\Omega_s\right)$$

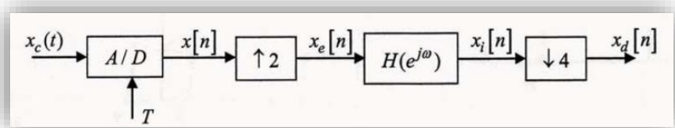


(2) Up-Sampling (*2)

$$X_e(e^{j\omega}) = X(e^{j\omega L}) = X(e^{j2\omega})$$

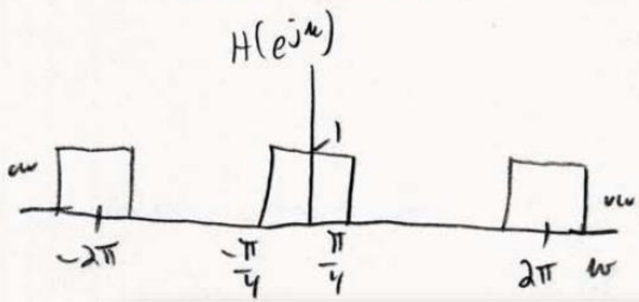


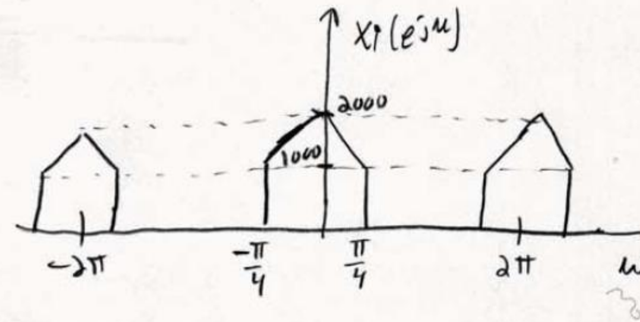




(3) Filter

$$X_i(e^{j\omega}) = X_e(e^{j\omega}) \cdot H(e^{j\omega})$$





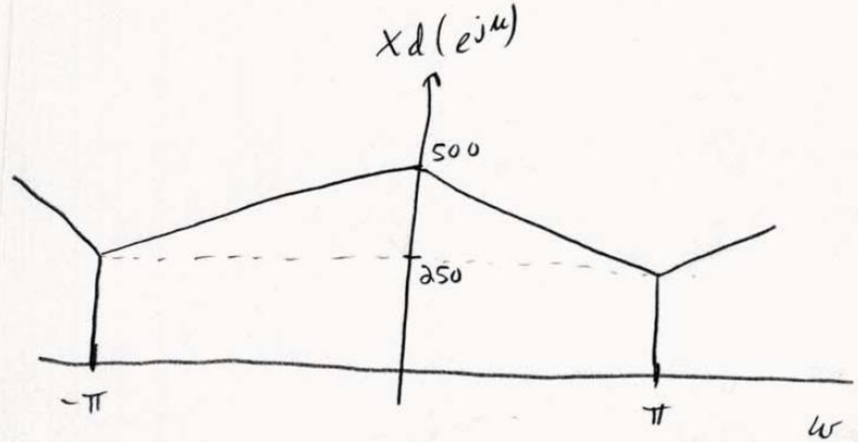
(4) Down-Sampling (/4)

$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X_i\left(e^{j\left(\frac{\omega}{M} - k\frac{2\pi}{M}\right)}\right) = \frac{1}{4}\sum_{k=0}^{3} X_i\left(e^{j\left(\frac{\omega}{4} - k\frac{\pi}{2}\right)}\right)$$

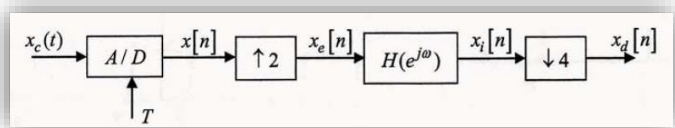
$\omega_{max} = \frac{\pi}{4}$ $\omega_{max} \le \frac{\pi}{M} = \frac{\pi}{4}$ NO SPECTRAL ALIASING, THEN

$X_d(e^{j\omega}) = \frac{1}{M} X_i(e^{j\frac{\omega}{4}})$

OVER $-\pi \le \omega \le \pi$



$$H(e^{j\omega}) = 1 \quad |\omega| < \pi/4$$
$$H(e^{j\omega}) = 0 \quad |\omega| > \pi/4$$



MATLAB

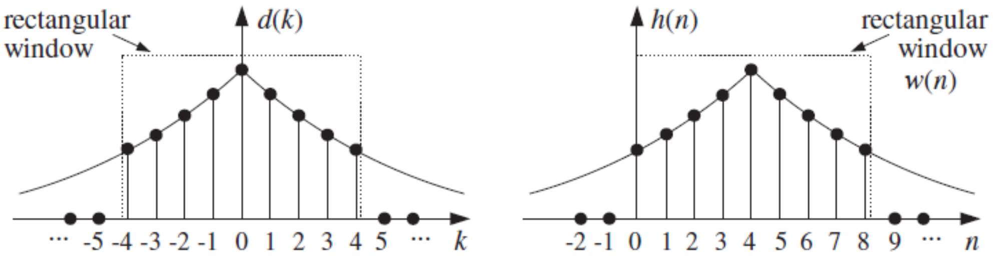
Introduction to Lab#4

WINDOWING

Rectangular Window

$$h(n) = d(n - M),\ n = 0, 1, \ldots, N - 1$$

- Pick an odd length $N = 2M + 1$, and let $M = (N - 1)/2$.
- Calculate the N coefficients $d(k)$ from Eq. (10.1.7), and
- Make them causal by the delay (10.1.10).



In Matlab w = boxcar(L);

$$h(n) = d(n - M) = \frac{\sin(\omega_c (n - M))}{\pi (n - M)},\ n = 0, \ldots, M, \ldots, N - 1$$

Triangular window (Bartlett window)

Bartlett or triangular window:

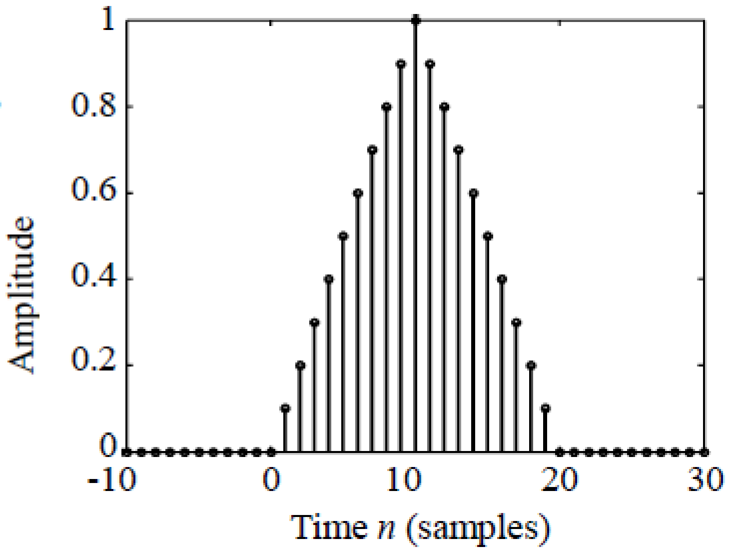
$$w[n]=\begin{cases} 2n/M, & 0\le n\le M/2, \\ 2-2n/M, & M/2<n\le M, \\ 0, & \text{otherwise} \end{cases}$$

GNU Octave/MATLAB:

```
w=bartlett(M+1);
```

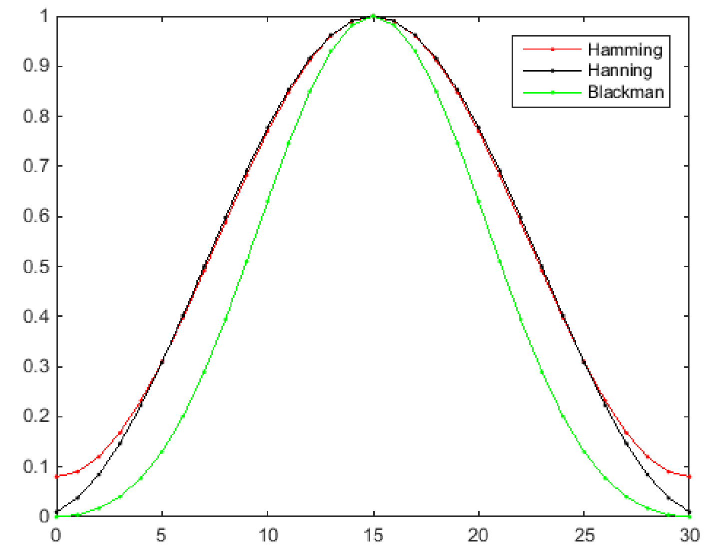
or nearly equivalently

```
w=triang(M+1);
```



Hamming, Hann & blackman Windows

$$w[n]=\begin{cases}\alpha-(1-\alpha)\cos\left(\frac{2n\pi}{M}\right), & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$$



If $\alpha = 0.54$ it is a *Hamming window.*

If $\alpha = 0.5$ it is a *von Hann* or *raised cosine window.*

Blackman window:

$$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2\pi n}{M}\right)+0.08\cos\left(\frac{4\pi n}{M}\right) & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$$

GNU Octave/ **MATLAB:**

```
w=hamming(M+1);
w=hann(M+1);
w=blackman(M+1);
```

Kaiser Window

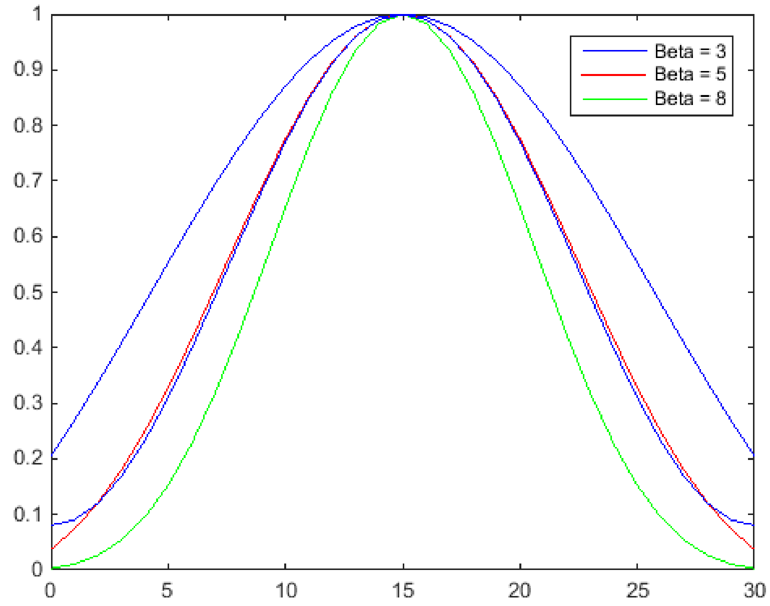
Kaiser window:

$$w[n]=\begin{cases}\dfrac{I_0\left[\beta\sqrt{1-((2n-M)/M)^2}\right]}{I_0(\beta)} & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$$

where $I_0(x)$ is the 0^{th}-order modified Bessel function of the first kind.

GNU Octave/MATLAB:

```
w=kaiser(M+1,beta);
```



THE END