ELG4172 Digital Signal Processing

- Tutorial-1

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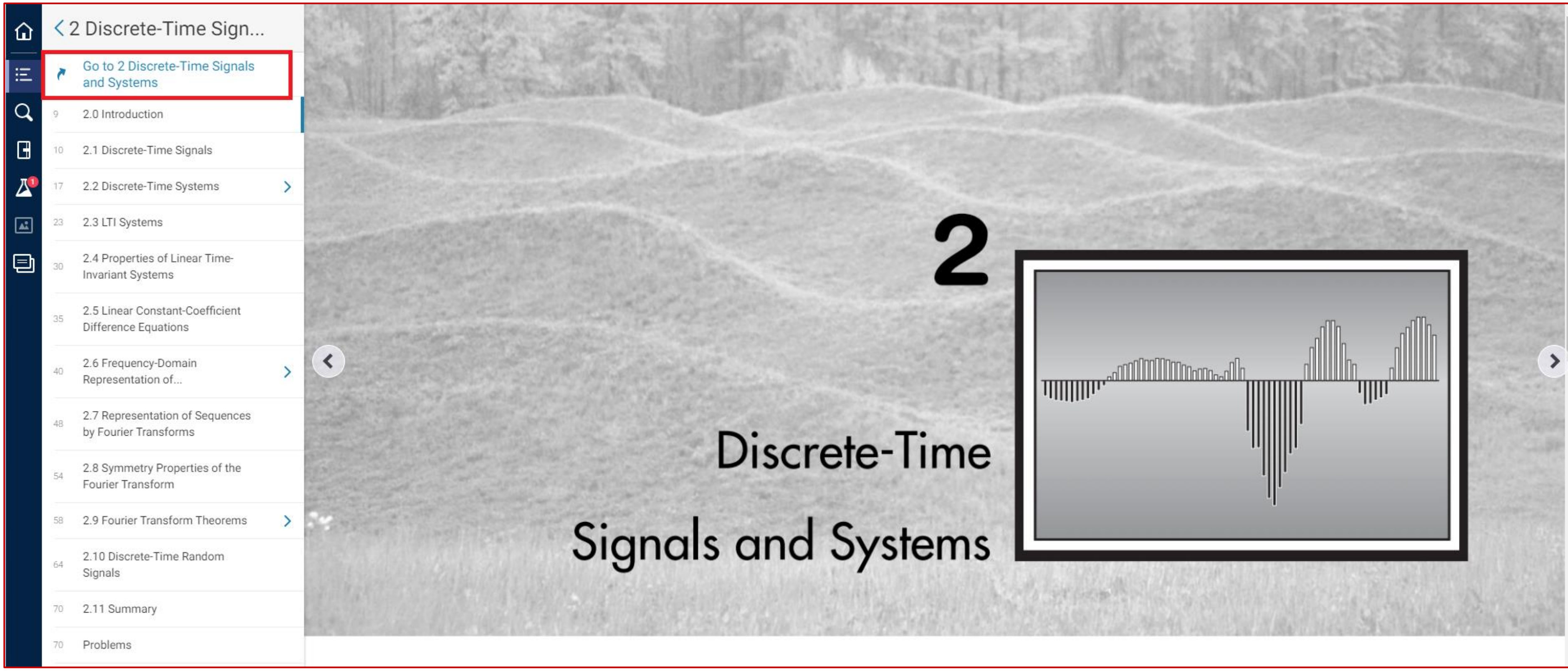

VitalSource

Exercises to judge whether a system is :

- Memory-Less
- Linear
- Time-invarient
- Causal
- Stable

2.2.1 Memoryless Systems

A system is referred to as memoryless if the output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n.

2.2.2 Linear Systems

The class of linear systems is defined by the principle of superposition. If $y_1[n]$ and $y_2[n]$ are the responses of a system when $x_1[n]$ and $x_2[n]$ are the respective inputs, then the system is linear if and only if

Superposition

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n] \tag{2.23a}$$

and

homogeneity

$$T\{ax[n]\} = aT\{x[n]\} = ay[n], \tag{2.23b}$$

where a is an arbitrary constant. The first property is the *additivity property*, and the second the *homogeneity* or *scaling property*. These two properties together comprise the principle of superposition, stated as

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\} \tag{2.24}$$

2.2.3 Time-Invariant Systems

A time-invariant system (often referred to equivalently as a shift-invariant system)

A relaxed system $\mathcal{T}$ is *time invariant* or *shift invariant* if and only if

$$x(n) \xrightarrow{\mathcal{T}} y(n) \quad \text{implies that} \quad x(n-k) \xrightarrow{\mathcal{T}} y(n-k)$$

2.2.4 Causality

A system is causal if, for every choice of n_0, the output sequence value at the index $n = n_0$ depends only on the input sequence values for $n \leq n_0$.

2.2.5 Stability

A system is stable in the bounded-input, bounded-output (BIBO) sense if and only if every bounded input sequence produces a bounded output sequence. The input $x[n]$ is bounded if there exists a fixed positive finite value B_x such that

$$|x[n]| \leq B_x < \infty, \qquad \text{for all } n. \tag{2.43}$$

Stability requires that, for every bounded input, there exists a fixed positive finite value B_y such that

$$|y[n]| \leq B_y < \infty, \qquad \text{for all } n. \tag{2.44}$$

2 Discrete-Time Sign...

Go to 2 Discrete-Time Signals and Systems

9 2.0 Introduction
10 2.1 Discrete-Time Signals
17 2.2 Discrete-Time Systems
23 2.3 LTI Systems
30 2.4 Properties of Linear Time-Invariant Systems
35 2.5 Linear Constant-Coefficient Difference Equations
40 2.6 Frequency-Domain Representation of...
48 2.7 Representation of Sequences by Fourier Transforms
54 2.8 Symmetry Properties of the Fourier Transform
58 2.9 Fourier Transform Theorems
64 2.10 Discrete-Time Random Signals
70 2.11 Summary
70 Problems

2.18. For each of the following impulse responses of LTI systems, indicate whether or not the system is causal:

(a) $h[n] = (1/2)^n u[n]$
(b) $h[n] = (1/2)^n u[n-1]$
(c) $h[n] = (1/2)^{|n|}$
(d) $h[n] = u[n+2] - u[n-2]$
(e) $h[n] = (1/3)^n u[n] + 3^n u[-n-1]$.

2.19. For each of the following impulse responses of LTI systems, indicate whether or not the system is stable:

(a) $h[n] = 4^n u[n]$
(b) $h[n] = u[n] - u[n-10]$
(c) $h[n] = 3^n u[-n-1]$
(d) $h[n] = \sin(\pi n/3)u[n]$
(e) $h[n] = (3/4)^{|n|}\cos(\pi n/4 + \pi/4)$
(f) $h[n] = 2u[n+5] - u[n] - u[n-5]$.

2.20. Consider the difference equation representing a causal LTI system

$$y[n] + (1/a)y[n-1] = x[n-1].$$

Solution

2.18. $h[n]$ is causal if $h[n] = 0$ for $n < 0$. Hence, (a) and (b) are causal, while (c), (d), and (e) are not.

2.19. For each of the following impulse responses of LTI systems, indicate whether or not the system is stable:

(a) $h[n] = 4^n u[n]$
(b) $h[n] = u[n] - u[n-10]$
(c) $h[n] = 3^n u[-n-1]$
(d) $h[n] = \sin(\pi n/3)u[n]$
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2.20. Consider the difference equation representing a causal LTI system

$$y[n] + (1/a)y[n-1] = x[n-1].$$

a.) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 4^n = \infty$ [not stable]

b.) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{9} 1 = 10$ *[stable]*

c.) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{-1} 3^n = \sum_{n=1}^{\infty} (\frac{1}{3})^n = \frac{1}{2}$ [stable]

d) $\sum_{n=-\infty}^{\infty} |h[n]| = \infty$ [non stable]

Summing |h[n]| over all positive n grows to ∞

e) $\sum_{n=-\infty}^{\infty} |h[n]| =$ [stable] . *Note that* $|h[n]|$ *is upperbounded by* $(\frac{3}{4})^{|n|}$, *which is absolutely summable*

f). *If we add the three signal, we get,*

$$h[n] = \begin{cases} 2, & -5 \le n \le -1 \\ 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

So $\sum |h[n]| = 15$ [stable]

From(Proakis's book)

Q. Determine if the systems described by the following input–output equations are linear or nonlinear.

(a) $y(n) = nx(n)$ **(b)** $y(n) = x(n^2)$ **(c)** $y(n) = x^2(n)$

(d) $y(n) = Ax(n) + B$ **(e)** $y(n) = e^{x(n)}$

Solution.

(a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = nx_1(n)$$

$$y_2(n) = nx_2(n)$$

A linear combination of the two input sequences results in the output

$$y_3(n) = \mathcal{T}[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)]$$

$$= a_1nx_1(n) + a_2nx_2(n)$$

On the other hand, a linear combination of the two outputs

$$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n)$$

Since the right-hand sides are identical, the system is linear.

(b) As in part (a), we find the response of the system to two separate input signals $x_1(n)$ and $x_2(n)$. The result is

$$y_1(n) = x_1(n^2)$$

$$y_2(n) = x_2(n^2)$$

The output of the system to a linear combination of $x_1(n)$ and $x_2(n)$ is

$$y_3(n) = \mathcal{T}[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(n^2) + a_2 x_2(n^2)$$

Finally, a linear combination of the two outputs in yields

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1(n^2) + a_2 x_2(n^2)$$

we conclude that the system is linear

(c) The output of the system is the square of the input. (Electronic devices that have such an input–output characteristic are called square-law devices.) From our previous discussion it is clear that such a system is memoryless. We now illustrate that this system is nonlinear.

The responses of the system to two separate input signals are

$$y_1(n) = x_1^2(n) \qquad y_2(n) = x_2^2(n)$$

The response of the system to a linear combination of these two input signals is

$$y_3(n) = \mathcal{T}[a_1 x_1(n) + a_2 x_2(n)] = [a_1 x_1(n) + a_2 x_2(n)]^2$$
$$= a_1^2 x_1^2(n) + 2a_1 a_2 x_1(n) x_2(n) + a_2^2 x_2^2(n)$$

On the other hand,

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1^2(n) + a_2 x_2^2(n)$$

system is nonlinear.

(d) Assuming that the system is excited by $x_1(n)$ and $x_2(n)$ separately, we obtain the corresponding outputs

$$y_1(n) = Ax_1(n) + B$$

$$y_2(n) = Ax_2(n) + B$$

A linear combination of $x_1(n)$ and $x_2(n)$ produces the output

$$y_3(n) = \mathcal{T}[a_1x_1(n) + a_2x_2(n)]$$

$$= A[a_1x_1(n) + a_2x_2(n)] + B$$

$$= Aa_1x_1(n) + a_2Ax_2(n) + B$$

On the other hand, if the system were linear, its output to the linear combination of $x_1(n)$ and $x_2(n)$ would be a linear combination of $y_1(n)$ and $y_2(n)$, that is,

$$a_1y_1(n) + a_2y_2(n) = a_1Ax_1(n) + a_1B + a_2Ax_2(n) + a_2B$$

The reason that this system fails to satisfy the linearity test is not that the system is nonlinear (in fact, the system is described by a linear equation) but the presence of the constant B. Consequently, the output depends on both the input excitation and on the parameter $B \neq 0$. Hence, for $B \neq 0$, the system is not relaxed. If we set $B = 0$, the system is now relaxed and the linearity test is satisfied.

(e) Note that the system described by the input–output equation

$$y(n) = e^{x(n)}$$

is non-relaxed. If $x(n) = 0$, we find that $y(n) = 1$. This is an indication that the system is nonlinear. This, in fact, is the conclusion reached when the linearity test is applied.

From (Hayes's book)

Q. Determine if the systems are time invariant or time variant.

(a) $y(n) = x(n) - x(n-1)$ "Differentiator"

(b) $y(n) = nx(n)$ "Time" multiplier

(c) $y(n) = x(-n)$ "Folder"

(d) $y(n) = x(n) \cos \omega_0 n$ Modulator

Solution.

(a) This system is described by the input–output equations

$$y(n) = \mathcal{T}[x(n)] = x(n) - x(n-1)$$

Now if the input is delayed by k units in time and applied to the system, it is clear from the block diagram that the output will be

$$y(n, k) = x(n-k) - x(n-k-1)$$

Delay at the input

On the other hand, we note that if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1)$$

Delay at the output

Since the right-hand sides are identical, it follows that $y(n, k) = y(n-k)$. Therefore, the system is time invariant.

(b) The input–output equation for this system is

$$y(n) = \mathcal{T}[x(n)] = nx(n)$$

The response of this system to $x(n-k)$ is

$$y(n, k) = nx(n-k)$$

Delay at the input

Now if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = (n-k)x(n-k)$$

Delay at the output

$$= nx(n-k) - kx(n-k)$$

This system is time variant, since $y(n, k) \neq y(n-k)$.

(c) This system is described by the input–output relation

$$y(n) = \mathcal{T}[x(n)] = x(-n)$$

The response of this system to $x(n-k)$ is

$$y(n, k) = \mathcal{T}[x(n-k)] = x(-n-k)$$

Now, if we delay the output $y(n)$ by k units in time, the result will be

$$y(n-k) = x(-n+k)$$

Or x(-(n-k))

Since $y(n, k) \neq y(n-k)$, the system is time variant.

(d) The input–output equation for this system is

$$y(n) = x(n) \cos \omega_0 n$$

The response of this system to $x(n-k)$ is

$$y(n, k) = x(n-k) \cos \omega_0 n$$

it is evident that the system is time variant.

Lab Preparation

Lab #1

Please Review these Functions

- **zplane**
- **impz**
- **freqz**
- **filter**
- **stepz**
- **Stem**
- **fft**

Transient Vs Steady State Responses

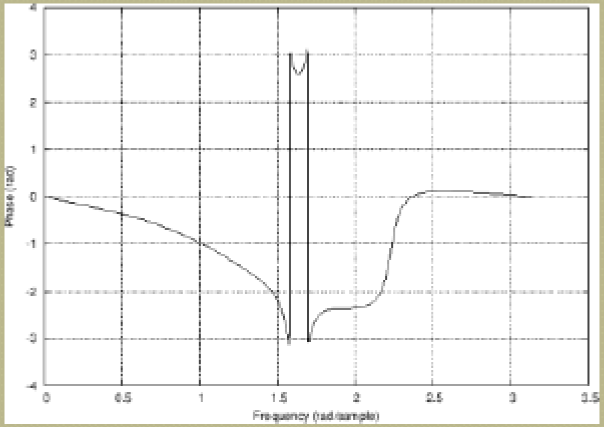
$$y[n] = y_{ss}[n] + y_t[n]$$

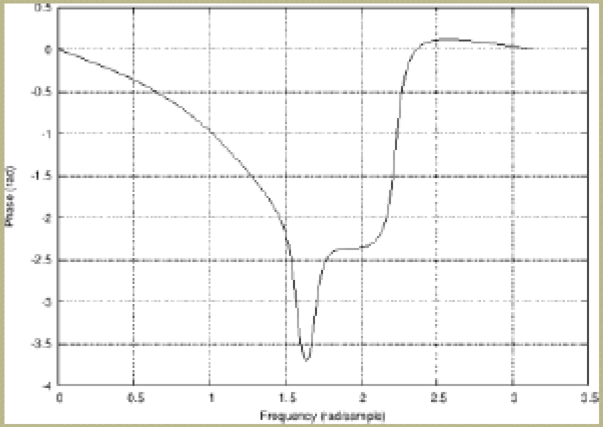
Steady State Response

Transient Response

Wrapping vs unwrapping phase

- $\angle X(e^{j\omega})$ not uniquely specified
  - ARG[$X(e^{j\omega})$]: Principal value between -π and +π
  - arg[$X(e^{j\omega})$]: Continuous function (not modulo 2π)
  - Phase unwrapping (MATLAB: unwrap)





Source: Smith, J.O. Introduction to Digital Filters with Audio Applications, http://ccrma.stanford.edu/~jos/filters/, online book, accessed Jan 15, 2012

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