ELG4177 - DIGITAL SIGNAL PROCESSING
Lab5

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Assignment 05

FIR FILTER DESIGN

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There two methods for FIR Design

- **(A) Design with frequency sampling**
- **(B) Design with window functions**

Study objectives

- Understand how to set up specifications for design of discrete-time filters.
- Understand the conditions required to ensure linear phase in FIR filters and how to use them to design FIR filters by specifying their magnitude response.
- Design FIR filters with linear phase using the windowing method, the frequency sampling method, and the Parks–McClellan algorithm.
- Understand operation and use of the MATLAB filter design and analysis tool.

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FIR filter design

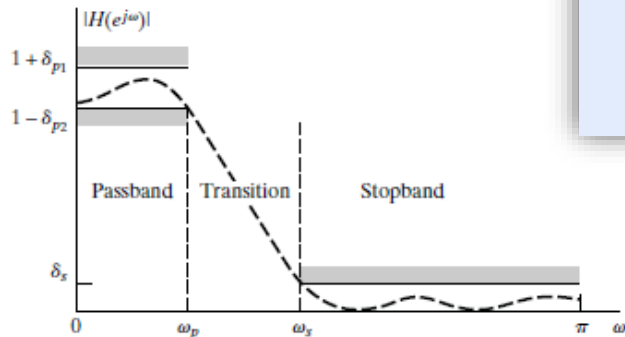|	IIR	FIR
	•Phase difficult to control & linear phase not possible for causal filters.	•Exactly linear phase always achievable.
	•Can be unstable.	•Always stable.
	•Fewer MADs.	•More MADs. (* polyphase?)
	•Analog history.	•No analog history other than antenna array design.
		•Can always be made causal.

Figure 7.1 Lowpass filter tolerance scheme.

Many of the filters used in practice are specified by a tolerance scheme similar to that which is presented below in Example 7.1, with no constraints on the phase response other than those imposed implicitly by requirements of stability and causality. For example, the poles of the system function for a causal and stable IIR filter must lie inside the unit circle. In designing FIR filters, we often impose the constraint of linear phase. This removes the phase of the signal from consideration in the design process.

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Part # A

OPTIMUM APPROXIMATIONS OF FIR FILTERS

Table 10.1 Properties of impulse response sequence $h[n]$ and frequency response function $H(e^{j\omega}) = A(e^{j\omega})e^{j\Psi(e^{j\omega})}$ of FIR filters with linear phase.

Type	$h[k]$	M	$A(e^{j\omega})$	$A(e^{j\omega})$	$\Psi(e^{j\omega})$
I	even	even	$\displaystyle\sum_{k=0}^{M/2} a[k]\cos\omega k$	even–no restriction	$-\dfrac{\omega M}{2}$
II	even	odd	$\displaystyle\sum_{k=1}^{\frac{M+1}{2}} b[k]\cos\left[\omega\left(k-\dfrac{1}{2}\right)\right]$	even $A(e^{j\pi})=0$	$-\dfrac{\omega M}{2}$
III	odd	even	$\displaystyle\sum_{k=1}^{M/2} c[k]\sin\omega k$	odd $A(e^{j0})=0$ $A(e^{j\pi})=0$	$\dfrac{\pi}{2}-\dfrac{\omega M}{2}$
IV	odd	odd	$\displaystyle\sum_{k=1}^{\frac{M+1}{2}} d[k]\sin\left[\omega\left(k-\dfrac{1}{2}\right)\right]$	odd $A(e^{j0})=0$	$\dfrac{\pi}{2}-\dfrac{\omega M}{2}$

Type I: $h[n] = h[2\alpha - n]$ α an integer

Type II: $h[n] = h[2\alpha - n]$ α a ½-integer

Type III: $h[n] = -h[2\alpha - n]$ α an integer

Type IV: $h[n] = -h[2\alpha - n]$ α a ½-integer

group delay $= \alpha$

Types I & II: $H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega\alpha}$
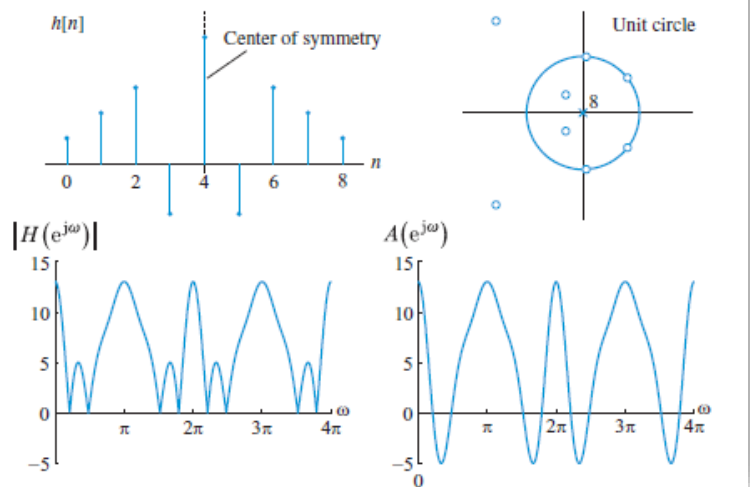
Types III & IV: $H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega\alpha}e^{j\pi/2}$

$$h_e[n] \xleftrightarrow{\text{DTFT}} A_e(e^{j\omega}) \quad \Rightarrow h_e[n-\alpha] \xleftrightarrow{\text{DTFT}} A_e(e^{j\omega})e^{-j\omega\alpha},$$
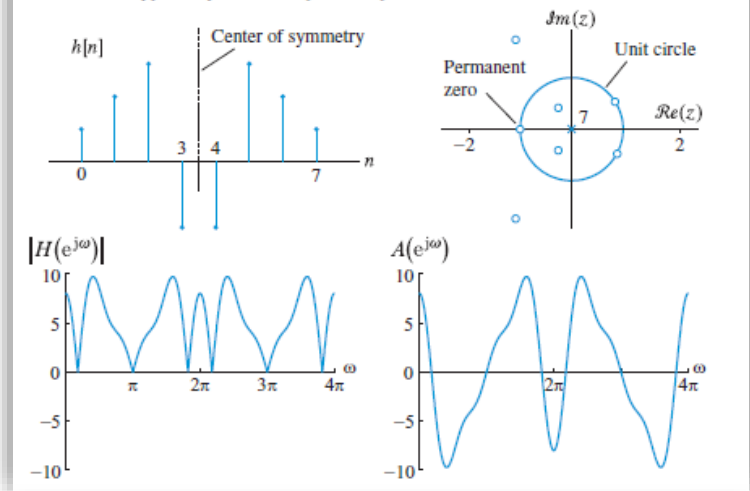
$$h_o[n] \xleftrightarrow{\text{DTFT}} jA_o(e^{j\omega}) \quad \Rightarrow h_o[n-\alpha] \xleftrightarrow{\text{DTFT}} A_o(e^{j\omega})e^{-j\omega\alpha+j\pi/2},$$
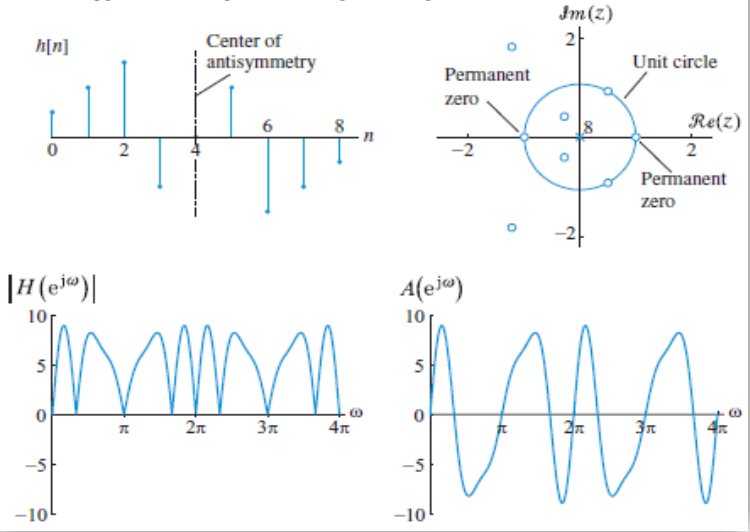
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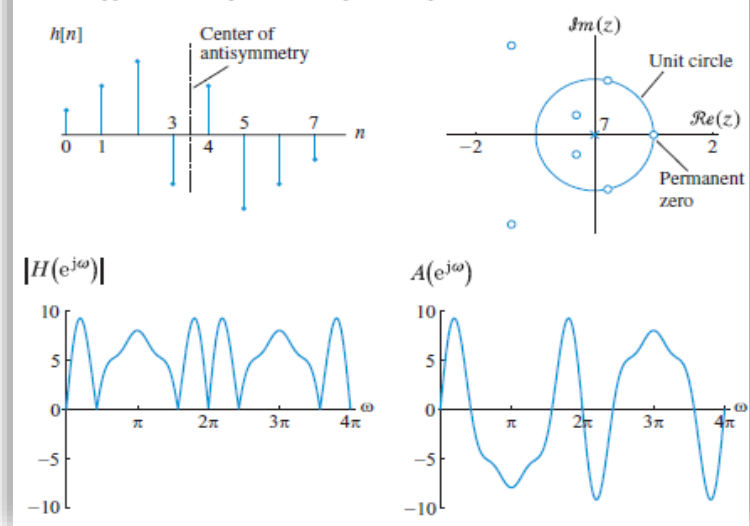Type I: Symmetric Impulse Response, Even Order $M = 8$

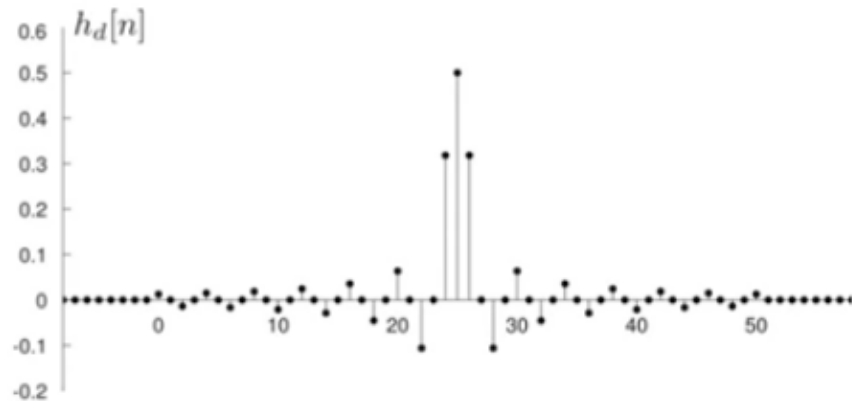Type II: Symmetric Impulse Response, Odd Order $M = 7$

Type III: Anti-Symmetric Impulse Response, Even Order $M = 8$

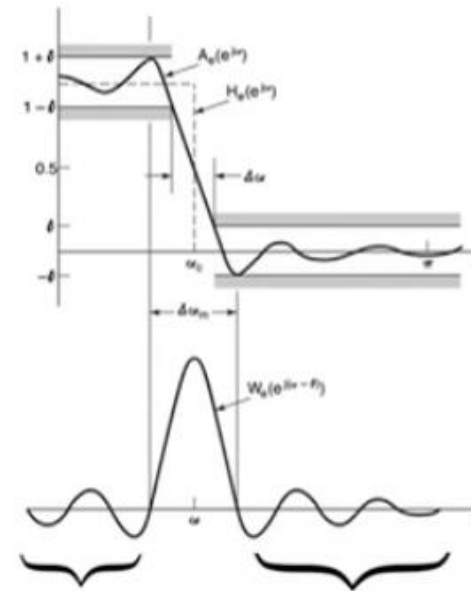Type IV: Anti-symmetric Impulse Response, Odd Order $M = 7$

Part # A

$h_d[n]$

Truncated impulse response of an ideal lowpass filter. (Delay is 25 samples, total length is 51 samples, and cutoff frequency is $\omega_c = \frac{\pi}{2}$)

$\implies \quad \dfrac{\theta(\omega)}{\omega} = \text{constant}$

$\implies \quad \theta(\omega) = K\,\omega$

$\implies \quad$ The phase is linear

The function $\theta(\omega)/\omega$ is called the *phase delay*. A linear phase filter therefore has constant phase delay.

- Main lobe controls transition band. $\Delta\omega \approx 2\pi(2/(M+1))$
- Side lobes control passband and stopband ripple.
- Passband and stopband ripple approximately equal over a wide range of frequencies.

Ideal Properties for $w[n]$

For a given window length, $W(e^{j\omega})$ "most like an impulse" $\implies$ narrow main lobe, low sidelobes.

- The main lobe as narrow as possible.
- The problem is not the height of the side lobes. The matter is how you accumulate the area as you integrate through it.
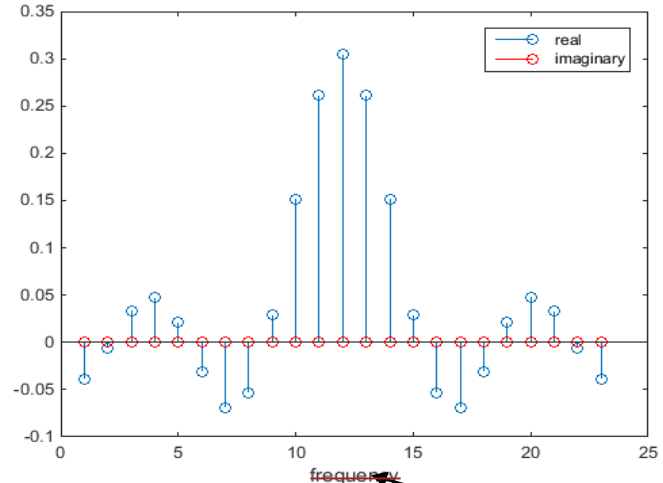
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Part # A
Illustration of (a) (b)&(c)

ideal filter
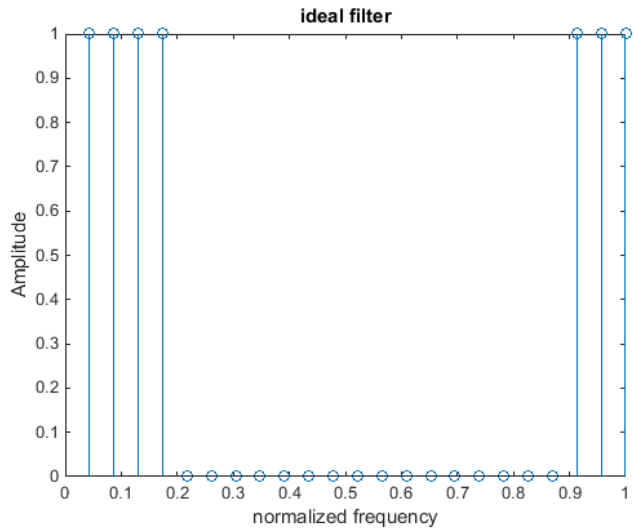

```matlab
wo=0.3*pi;          % Passband edge
len=23;             % Length of filter
pass_len=fix(wo*len/(2*pi))+1;   % Passband
length
Ad=[ones(1,pass_len),zeros(1,len-
2*pass_len+1),ones(1,pass_len-1)];
stem((1:len)/len,Ad);
xlabel('normalized frequency');
ylabel('Amplitude');
title('ideal filter')
```
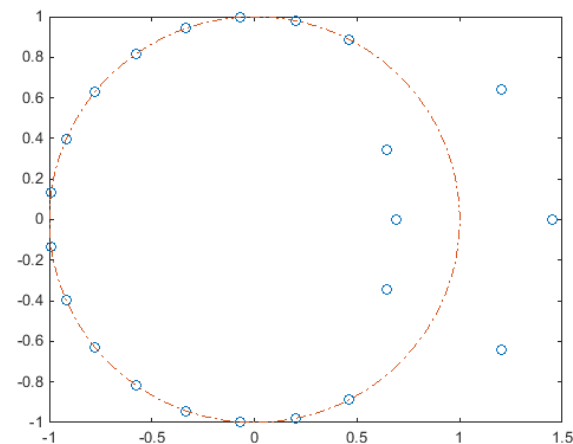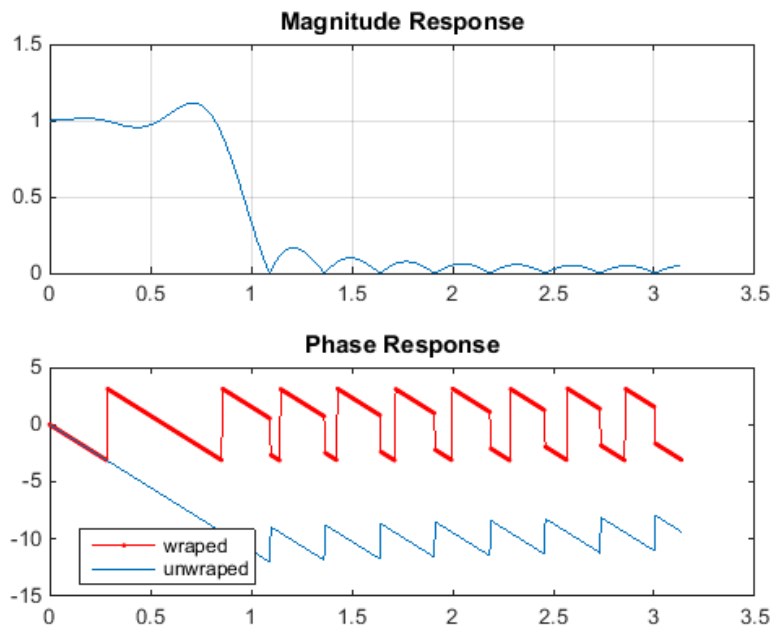

n   Shifted by M

```matlab
M=(len-1)/2;    % Type 1
k=0:(len-1);
pd=exp(2*pi*j*(-M)*k/len);
Hd=Ad.*pd;
h=real(ifft(Hd));
h2=imag(ifft(Hd));
figure;  stem(h);hold on;
stem(h2, 'r');
legend('real','imaginary'); xlabel('n');
```

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Part # A cont.

Magnitude Response

Phase Response

legend:
- wraped
- unwraped

$$\theta(\omega) = -M\omega$$

$$M = \frac{N-1}{2}.$$

M=11

ringing $\overset{\mathcal{F}}{\longleftrightarrow}$ truncation   (Gibbs in time-domain)

truncation $\overset{\mathcal{F}}{\longleftrightarrow}$ ringing   ($H(\omega)$ pass/stopband rippling)

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Part # A

Illustration of (d) &(e)

(d) Design a 23 coeffs. FIR filter as in a) but this time with a phase response of zero (i.e pd=ones(1,len)). **Find the resulting impulse response** h[n], and the **magnitude and phase frequency response** of this filter h[n]. Is h[n] **symetrical/anti-symetrical** ? How is the magnitude response ? Is the phase response linear phase ? Why ?

(e) Design a 23 coeffs. FIR filter as in a) but this time with value of a M=5. **Find the resulting impulse response** h[n], and the **magnitude and phase frequency response** of this filter h[n]. Is h[n] symetrical/anti-symetrical ? How is the magnitude response ? Is the phase response linear phase ?
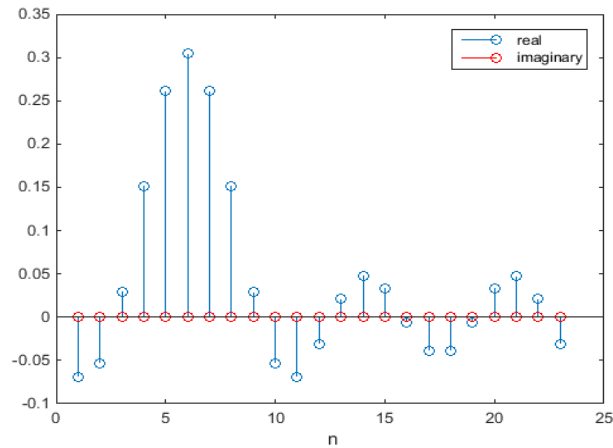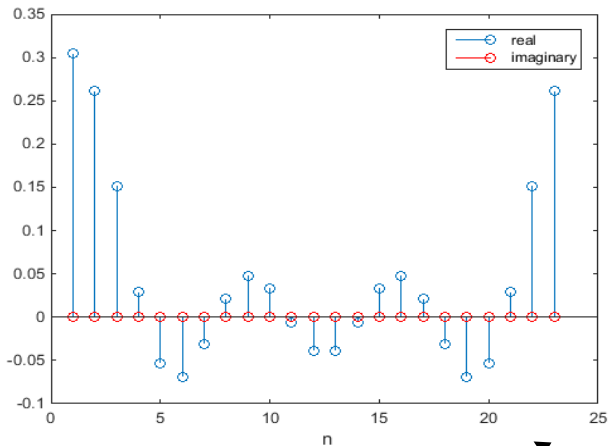

It should look like this

Part # A

Illustration of (f) &(g)

(f)From a), d), and e), what is the effect of M and why should it be set to M=(len-1)/2 ?

(g) Now design an even length filter of length 22, using the same approach as in a). Find the resulting impulse response h[n], and the magnitude and phase frequency response of this filter h[n]. Note that an symmetric even-length linear-phase FIR filter always have a zero at z=pi Compare the amount of overshoot near the band edge with the design for len=23.
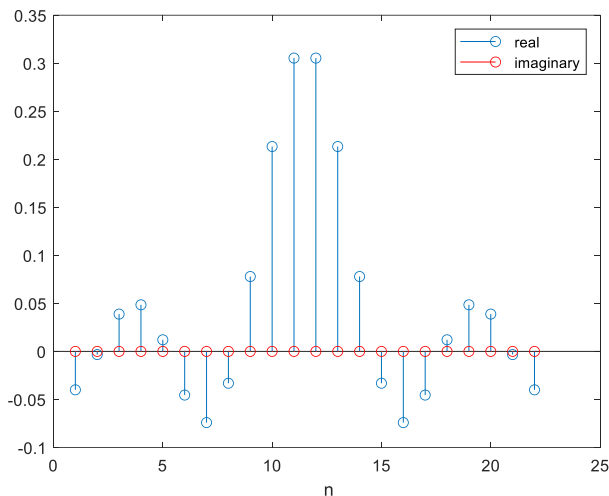
Hint: [See slide #6]
Plot Z-plane to clarify which type the filter is

One way:

```
plot(roots(h),'o');
ang = linspace(0, 2*pi, 100);  mag = ones(1,100);
[x y] = pol2cart(ang, mag);hold on; plot(x,y, '-.');
```

The simple way:

```
zplane(h,[1])
```

The ideal Filter:

```
Ad=[ones(1,pass_len),zeros(1,len-2*pass_len+1),-1*ones(1,pass_len-1)]; % Must use negative !!!

h=real(ifft(Hd))+1i.*imag(ifft(Hd));
```

(Part B) Design with window functions

Part # B

Window Design Method

- Practically, one uses **truncated** & **delayed** impulse response

$$\tilde{h}(n) = \hat{h}\left(n - \frac{M-1}{2}\right) \text{ where } \hat{h}(n) = \frac{\omega_c}{\pi}\frac{\sin(\omega_c n)}{\omega_c n} 1_{\{-\frac{M-1}{2},\ldots,\frac{M-1}{2}\}}(n)$$

where M is the *filter length* & $N = M - 1$ is known as *filter order*.

- Delaying operation $\rightarrow$ introduce linear phase term.

$\Rightarrow$ The resulting filter is causal and has a linear phase.

$\Rightarrow$ Transition region gets smaller but the ripple remains.

$\Rightarrow$ PROBLEM: Sharp discontinuity of rectangular windows!

Solution to Sharp Discontinuity of Rectangular Window ➜ use Other windows. Windows with no abrupt discontinuity can be used to reduce Gibbs oscillations (e.g. Hanning, Hamming, Blackman)

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Part # B

h) Design a length-23 linear-phase FIR low-pass filter with a passband edge of $\omega_0 = 0.3\pi$. radians/sample using a window approach. **Do not use a frequency sampling technique as in a).** Use the windowing approach with the following windows: rectangular, **Hanning, Hamming** and **Kaiser** with $\beta = 6$. Find the frequency response of the resulting filters, and compare with the filter found in a).

Windowed Coeffecients
- rectangular
- Hann
- Hamming
- Kaiser
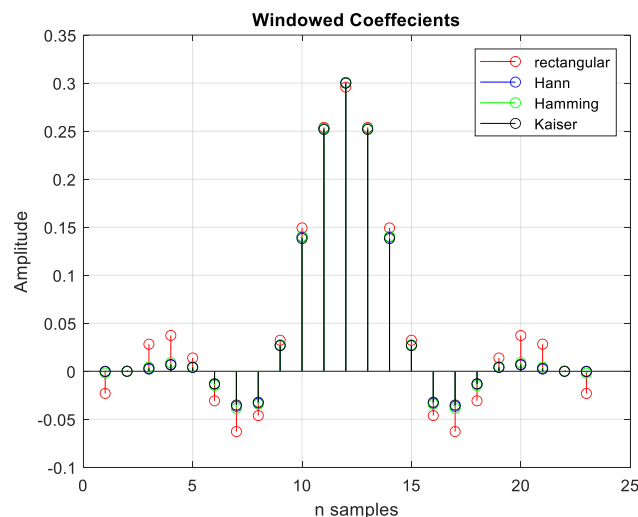
The cut-off frequency "wn" must be normalized between $0 < Wn < 1.0$,

```
wo=0.3*pi;        % Passband edge
len=23;M=len-1;   % order=length-1
wn=wo/pi;
h = fir1(M,wn,rectwin(M+1));
```

h=fir1(M,f,A) designs an Mth order linear-phase FIR digital filter with the frequency response specified by arrays f and A, and returns the filter coefficients in length $M + 1$ array h. The array f is a normalized frequency array where f=1 corresponds to π radians (or half the sampling frequency). The first and last elements of f must equal 0 and 1, respectively. The array A contains frequency response magnitude values specified at locations in f.

Part # B

(i) Design a length-23 linear-phase FIR low-pass filter with a passband edge of $\omega_0 = 0.3\pi$ radians/sample and a stopband edge of $\omega_0 = 0.35\pi$ radians/sample, using the **remez/firpm** Matlab function (you can also use the **remezord/firpmord** function). What is the particular characteristic of the magnitude of the filter frequency response? How does the frequency response compare with the responses from b) and h)?

>>h = firpm(N,f,a)

filter designed by firpm
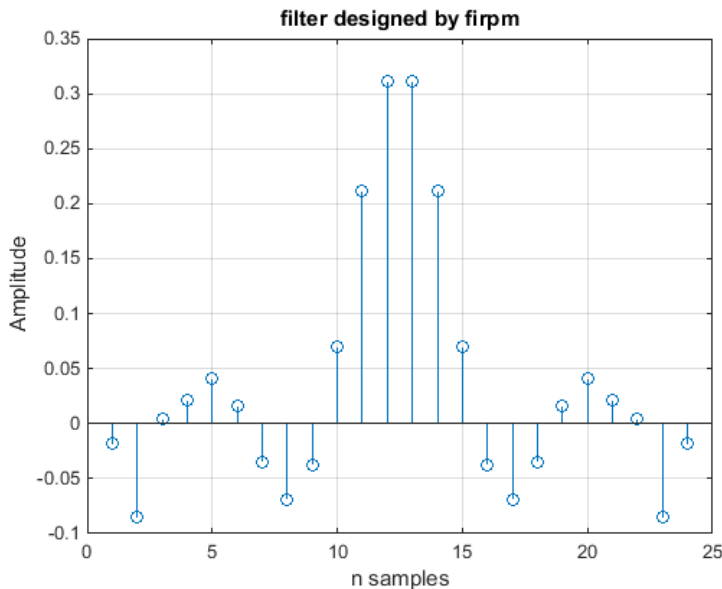
N: is the filter order.
f: is a vector of pairs of normalized frequency points, specified in the range between 0 and 1
a: is a vector containing the desired amplitudes at the points specified in f
f and a are the same length. This length must be an even number.
>> help firpm
>> help firpmord

firpm is a new version function of **remez**
firpmord is a new version function of **remezord**

Hint: see this example.
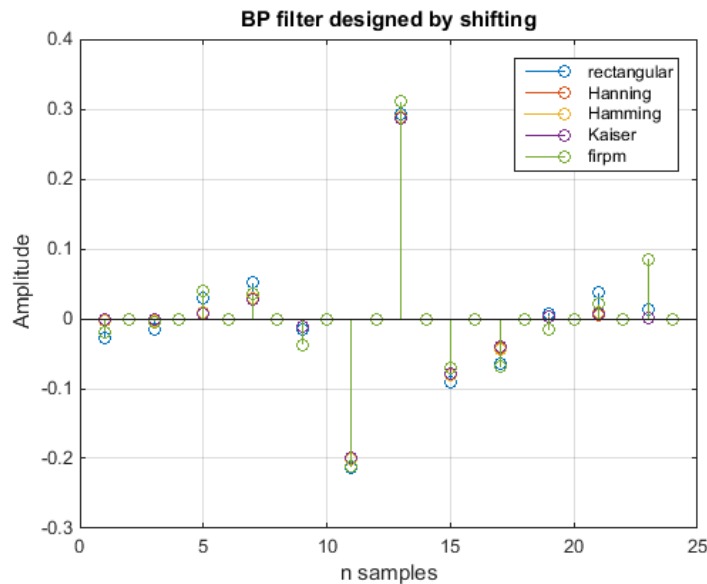https://www.mathworks.com/help/signal/ug/fir-filter-design.html

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Part # B

Modulation of filters

j) Transform one of the low-pass filters that you have designed in this assignment to a band-pass filter. Hint : use the frequency shifting property or the modulation property of the Fourier transform.

$$h_{BP}[n] = h[n].e^{jn\omega}$$

use: $\omega = \pi/2$

BP filter designed by shifting

Part # B

Extra

Compute FIR Order

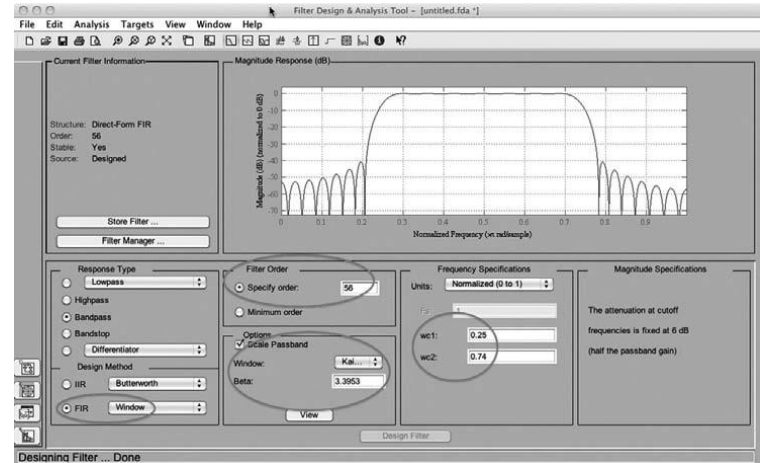$$dev = [\delta_p \quad \delta_s];$$

f = [fp   fc];

a = [1   0];
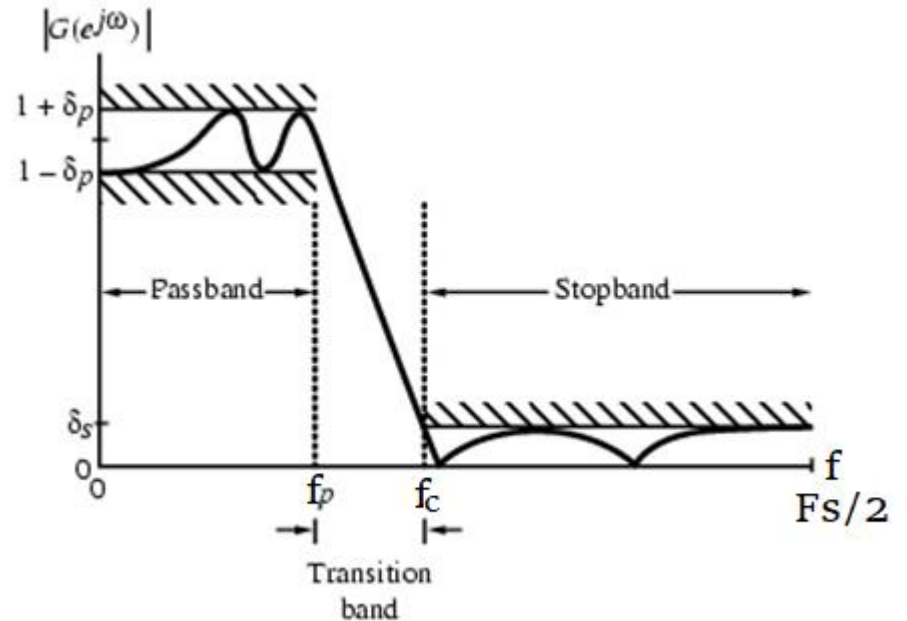
[n,fo,ao,w] = firpmord(f, a, dev, fs);

h = firpm(n,fo,ao,w);

fdatool

FDATool for window design The SP toolbox in MATLAB contains a GUI-based tool for designing FIR and IIR digital filters that makes designing them a convenient and straightforward task. You can use this tool to: Design filters, Analyze filters, and Modify existing filter designs

Finish the lab and submit your report

The END

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