ELG4177 - DIGITAL SIGNAL PROCESSING
Lab4

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Assignment 04

WINDOWING

Useful Wiki References' links

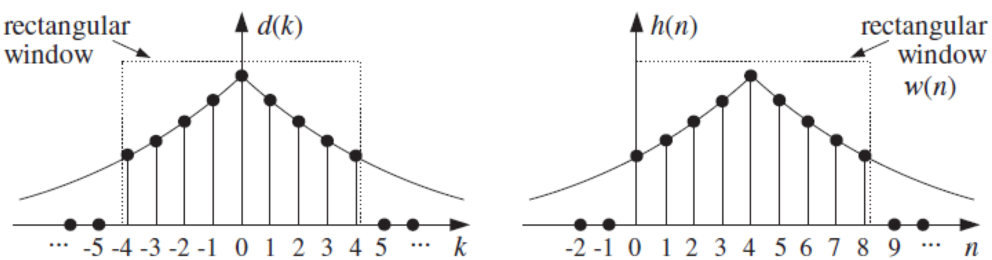
- The Rectangular Window
  - Side Lobes
  - Summary
- Generalized Hamming Window Family
  - Hann or Hanning or Raised Cosine
  - Matlab for the Hann Window
  - Hamming Window
  - Matlab for the Hamming Window
- Blackman-Harris Window Family
  - Blackman Window Family
  - Classic Blackman
  - Matlab for the Classic Blackman Window
- Kaiser Window
  - Kaiser Window Beta Parameter
  - Kaiser Windows and Transforms

- Windowing Functions to Eliminate Spectral Leakage (Matlab)

Rectangular Window

$$h(n) = d(n - M),\ n = 0, 1, \ldots, N - 1$$

- Pick an odd length $N = 2M + 1$, and let $M = (N - 1)/2$.
- Calculate the N coefficients $d(k)$ from Eq. (10.1.7), and
- Make them causal by the delay (10.1.10).

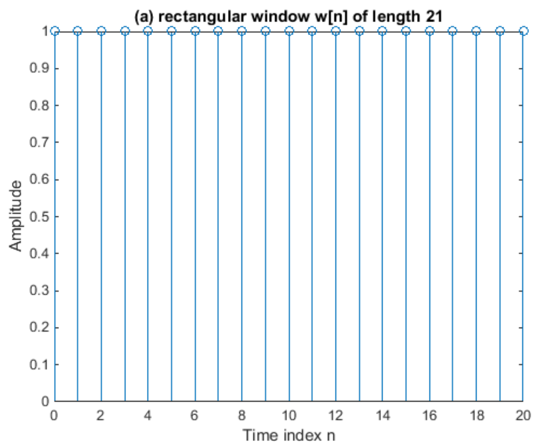


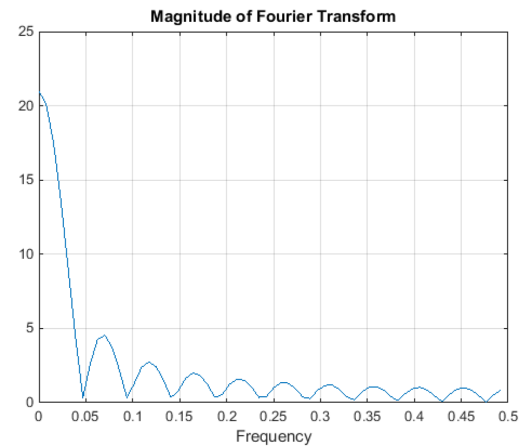
In Matlab w = boxcar(L);

$$h(n) = d(n - M) = \frac{\sin(\omega_c (n - M))}{\pi (n - M)},\quad n = 0, \ldots, M, \ldots, N - 1$$

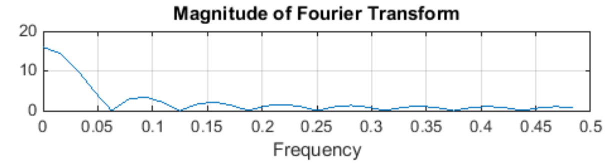
MATLAB

In Matlab w = boxcar(L);

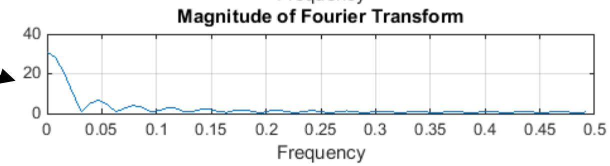




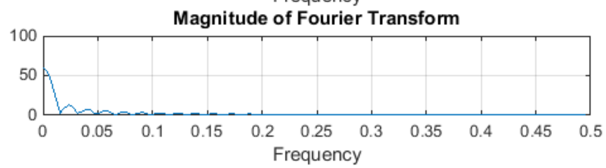
16 coefficients



31 coefficients



61 coefficients



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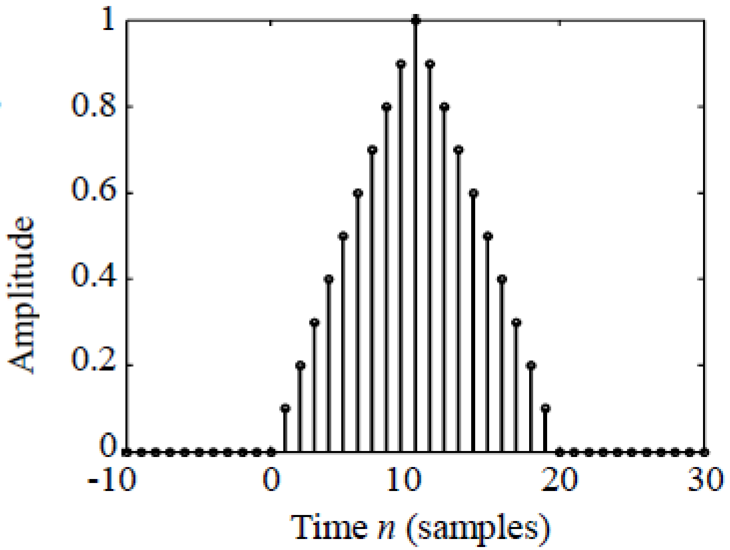
triangular window (Bartlett window)

Bartlett or triangular window:

$$w[n]=\begin{cases} 2n/M, & 0\le n\le M/2, \\ 2-2n/M, & M/2<n\le M, \\ 0, & \text{otherwise} \end{cases}$$



GNU Octave/MATLAB:

```
w=bartlett(M+1);
```

or nearly equivalently

```
w=triang(M+1);
```

Hamming, Hann & blackman Windows

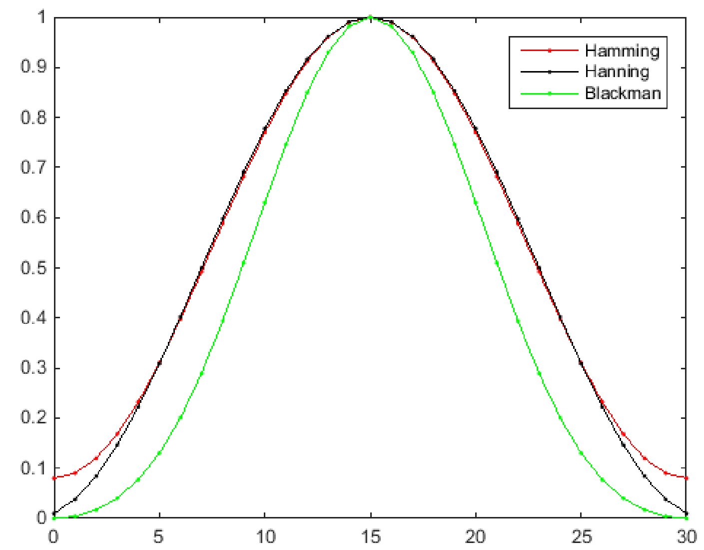
$$w[n] = \begin{cases} \alpha - (1-\alpha)\cos\left(\frac{2n\pi}{M}\right), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

If $\alpha = 0.54$ it is a *Hamming window*.

If $\alpha = 0.5$ it is a *von Hann* or *raised cosine window*.

Blackman window:

$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$



GNU Octave/ **MATLAB:**

```
w=hamming(M+1);
w=hann(M+1);
w=blackman(M+1);
```

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Kaiser Window

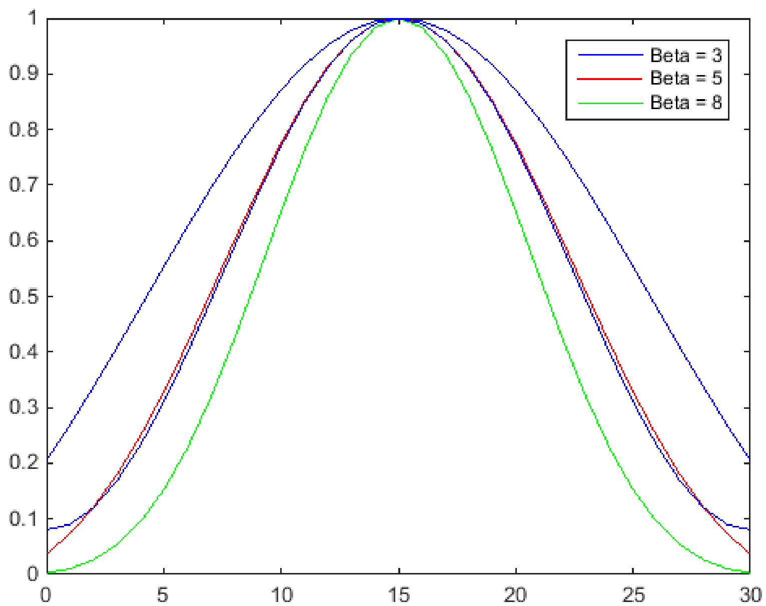
Kaiser window:

$$w[n]=\begin{cases}\dfrac{I_0\left[\beta\sqrt{1-((2n-M)/M)^2}\right]}{I_0(\beta)} & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$$

where $I_0(x)$ is the 0^{th}-order modified Bessel function of the first kind.

GNU Octave/MATLAB:

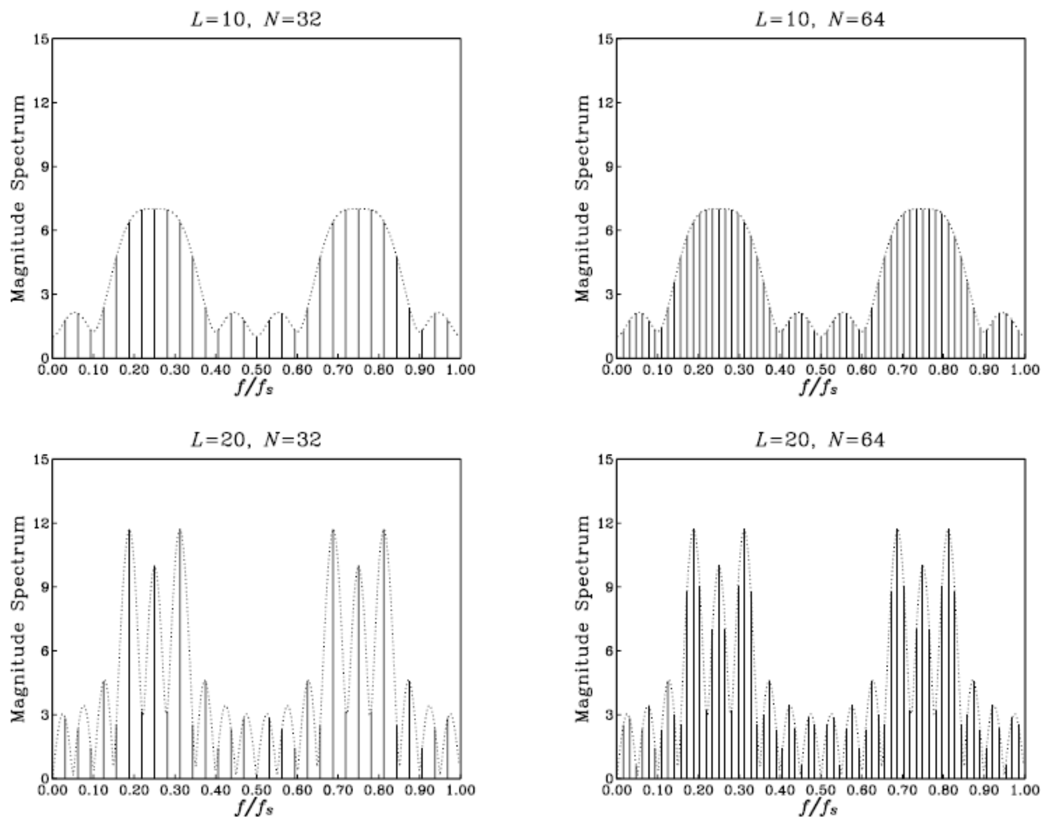
```
w=kaiser(M+1,beta);
```



Physical Resolution: **Effect of Windowing**
Computational Resolution: **Effect of Spectral Sampling**

-Physical Resolution is caused by a windowing operation (L)

-Computational Resolution is caused by the DFT/FFT sampling of the DTFT (N)



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Effect of Windowing (Physical Resolution)

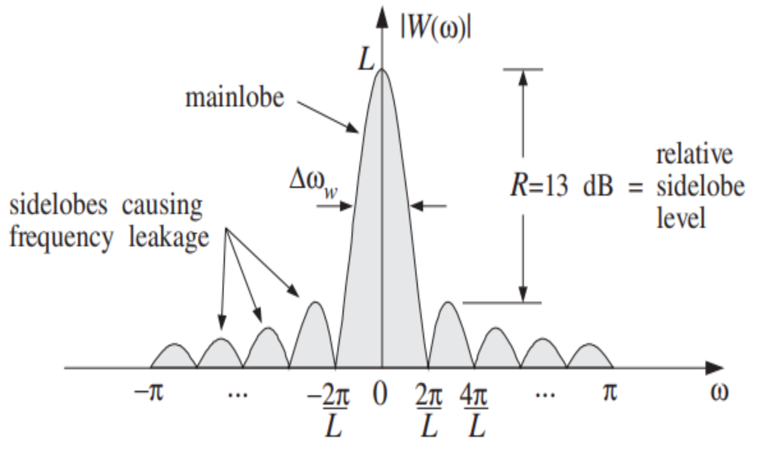
The sidelobes are between the zeros of $W(\omega)$, which are the zeros of the numerator $\sin(\omega L/2) = 0$, that is, $\omega = 2\pi k/L$, for $k = \pm 1, \pm 2, \ldots$ (with $k = 0$ excluded).

The mainlobe peak at DC dominates the spectrum, because $w(n)$ is essentially a DC signal, except when it cuts off at its endpoints. The higher frequency components that have "leaked" away from DC and lie under the sidelobes represent the sharp transitions of $w(n)$ at the endpoints.

The *width* of the mainlobe can be defined in different ways. For example, we may take it to be the width of the base, $4\pi/L$, or, take it to be the 3-dB width, that is, where $|W(\omega)|^2$ drops by 1/2. For simplicity, we will define it to be *half* the base width, that is, in units of radians per sample:

$$\Delta\omega_w = \frac{2\pi}{L} \quad \text{(rectangular window width)}$$

In units of Hz, it is defined through $\Delta\omega_w = 2\pi\Delta f_w/f_s$.



$$\Delta\omega \geq \Delta\omega_w = \frac{2\pi}{L}$$

These equations can be rewritten to give the *minimum number* of samples required to achieve a desired frequency resolution Δf. The smaller the desired separation, the longer the data record:

$$L \geq \frac{f_s}{\Delta f} = \frac{2\pi}{\Delta\omega}$$

Window Shape	Relative peak	Approx. mainlobe width
	sidelobe magnitude	(in frequency)
Rectangular/boxcar	-13 dB	2/M
Bartlett (triangle)	-26 dB	4/M
Hanning (raised cosine)	-31 dB	4/M
Hamming (rasied cosine on pedestal	-42 dB	4/M
Blackman	-58 dB	6/M

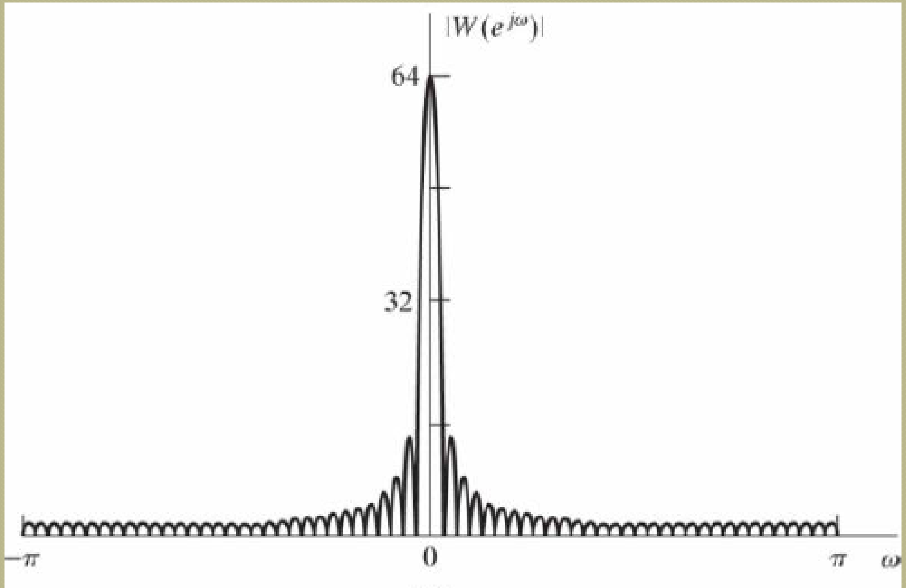
http://web.mit.edu/ruggles/SpectralAnalysis/reference.html

Effect of Windowing

- Windowing has 2 undesirable effects
  - Loss of frequency resolution
    - Smoothing of spectrum (peaks and discontinuities)
    - Unable to resolve 2 close components
    - Caused by width of mainlobe
    - Rectangular window is good here
  - Spectral leakage
    - Component at one frequency leaks into vicinity
    - Unable to detect a weak frequency component
    - Caused by amplitude of sidelobes
    - Non-rectangular window is better here

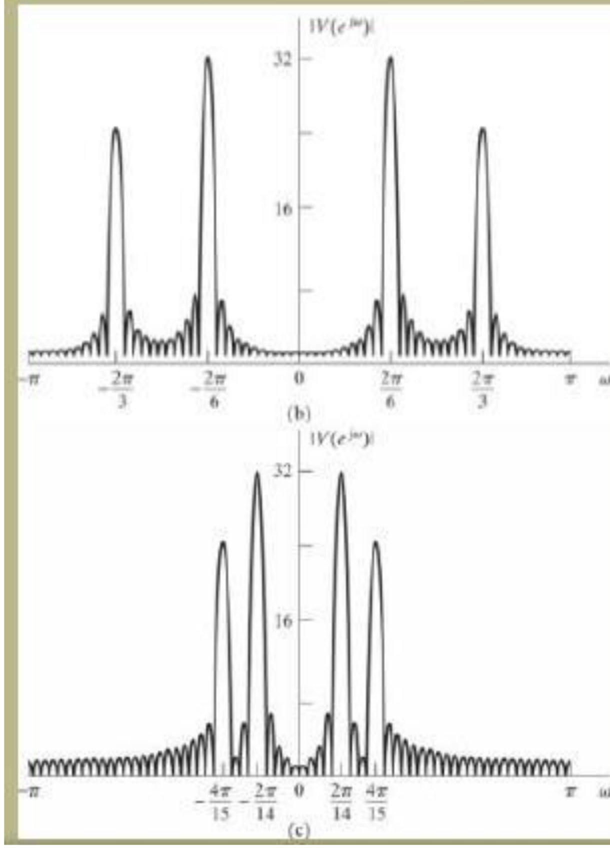
Effect of windowing

- Rectangular window
  - Size $L = 64$ samples



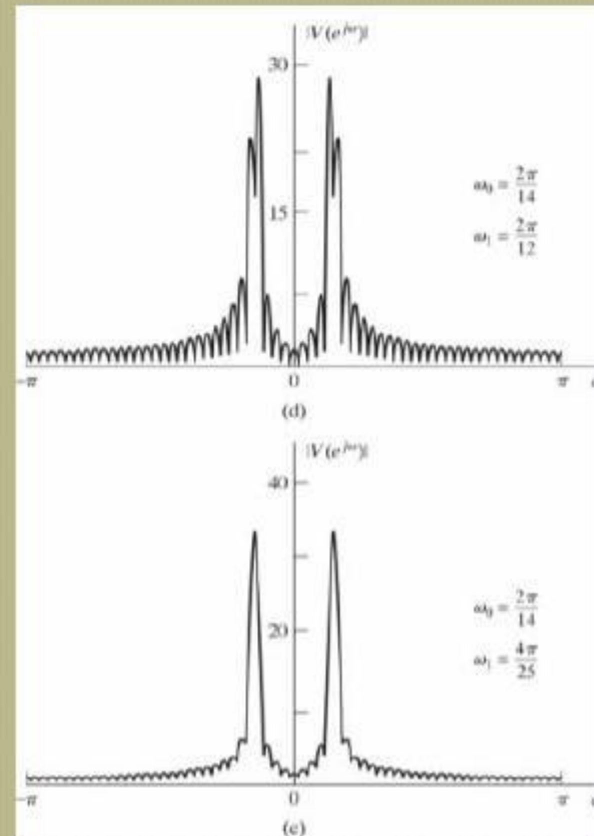
(a)

Windowed signal



$A_0 = 1$

$A_1 = 0.75$



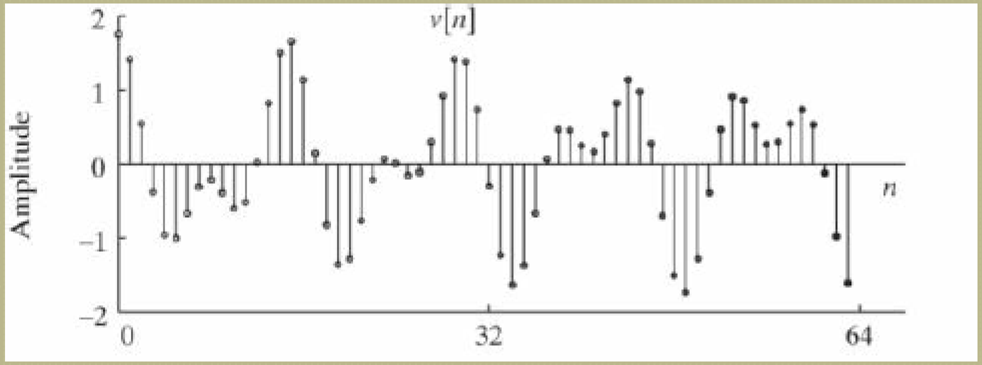
Effect of Spectral Sampling

- DFT obtained by sampling DTFT
  - N samples in 2π period

$$V[k] = V(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k}, \quad 0 \le k \le N-1$$

  - Consider the windowed signal

$$v[n] = \cos\left(\frac{2\pi}{14}n\right) + 0.75\cos\left(\frac{2\pi}{15}n\right), \quad 0 \le n \le 63$$

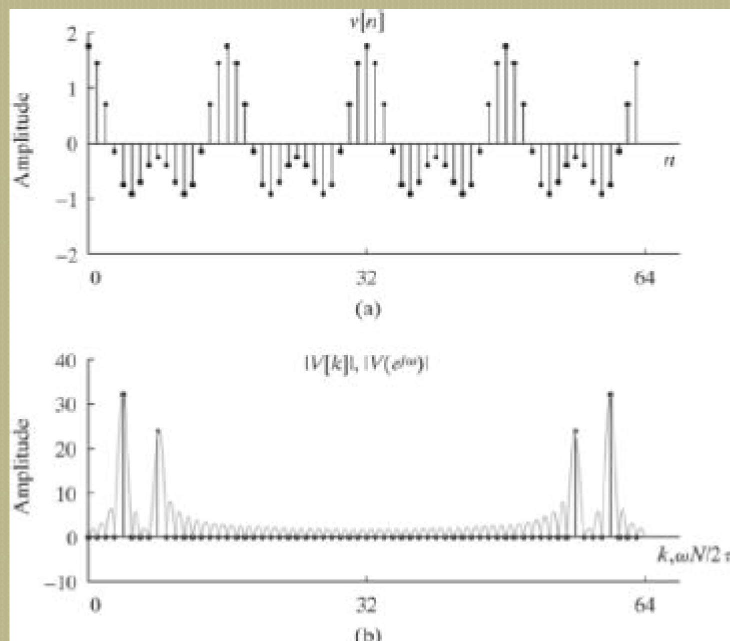


Effect of Spectral Sampling

Case with matching DFT frequencies

- $N = L =$ multiple of signal periods

$$v[n] = \cos\left(\frac{2\pi}{16}n\right) + 0.75\cos\left(\frac{2\pi}{8}n\right), \quad 0 \le n \le 63$$



No apparent leakage

No apparent loss of frequency resolution

Summary

- Spectral leakage due to windowing
  - Choice of type of window is critical
- Two types of loss of frequency resolution:
  - Inability to resolve 2 close components (windowing)
  - Not enough frequency samples (DFT spectral sampling)

- How many samples should we use in a DFT?
  - For no loss of information (no time aliasing): $N \geq L$
  - If we want $V[k]$ to look like $V(e^{j\omega})$: $N >>> L$
    - May require zero-padding

Complete the assignment & Answer all the questions

Thanks