ELG4177 - DIGITAL SIGNAL PROCESSING
Lab2

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Assignment 02

FILTERS AND RESONATORS

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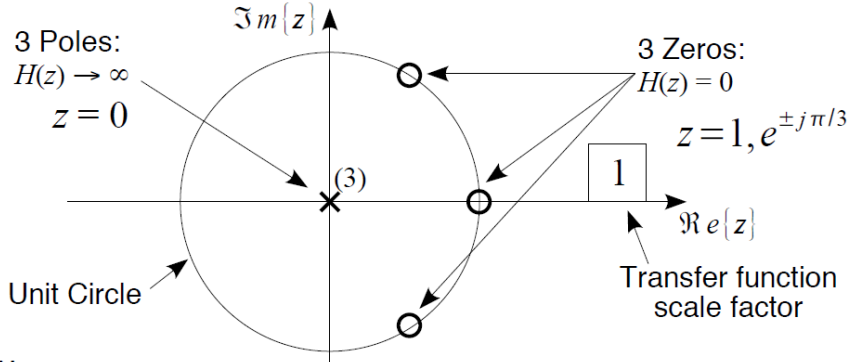
Poles & Zeros

A pole-zero plot is a useful way of expressing a transfer function.
Consider the following for $H(z) = 1\text{-}2z^{-1} + 2z^{-2} - z^{-3}$.



MATLAB: `zplane([1 -2 2 -1],[1])`
or `zplane([1 exp(j*pi/3) exp(-j*pi/3)].',[0 0 0].')`

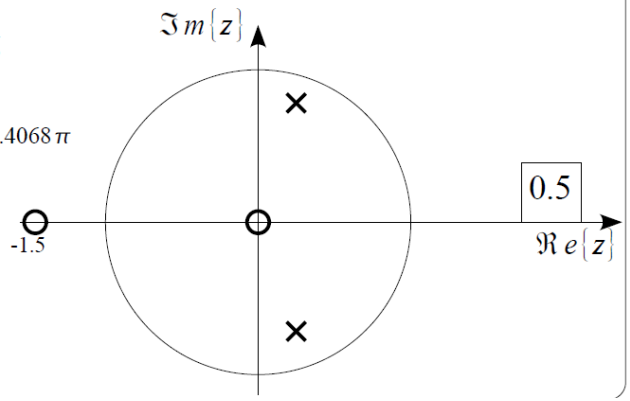
$4y[n] = 2y[n\text{-}1] - 3y[n\text{-}2] + 2x[n] + 3x[n\text{-}1] \Rightarrow$ $H(z) = \dfrac{2 + 3z^{-1}}{4 - 2z^{-1} + 3z^{-2}}$

Zeros:

$z = 0, z = -1.5$

Poles:

$z = 0.886\, e^{\pm j\, 0.4068\pi}$



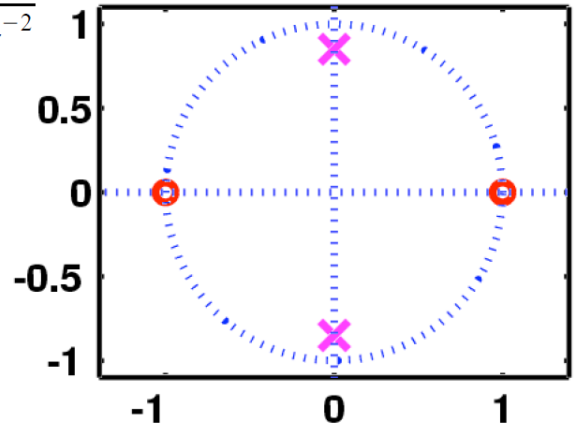
$H(z) = \dfrac{1 - z^{-2}}{1 + 0.7225z^{-2}}$

Zeros:

$z = \pm 1$

Poles:

$z = 0.85\, e^{\pm j\pi/2}$



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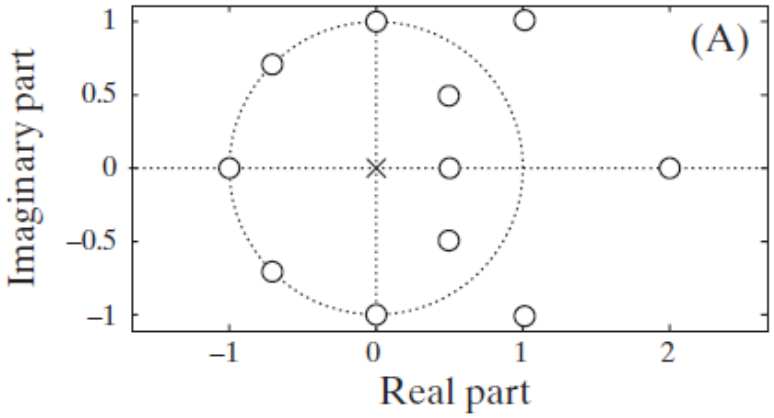
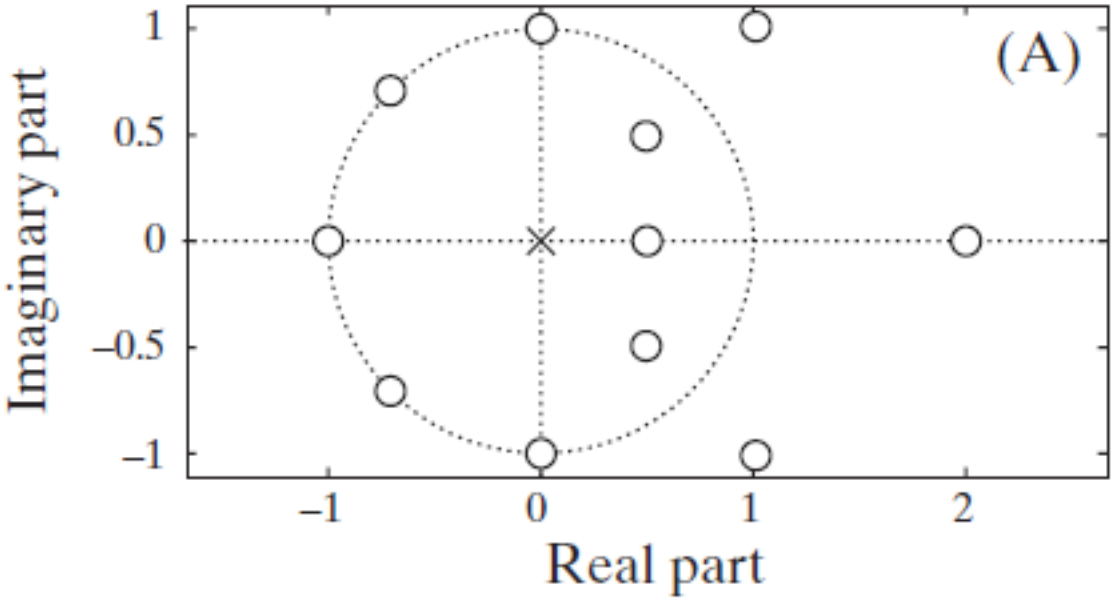
Determine what the filters are by zero-pole

FIR

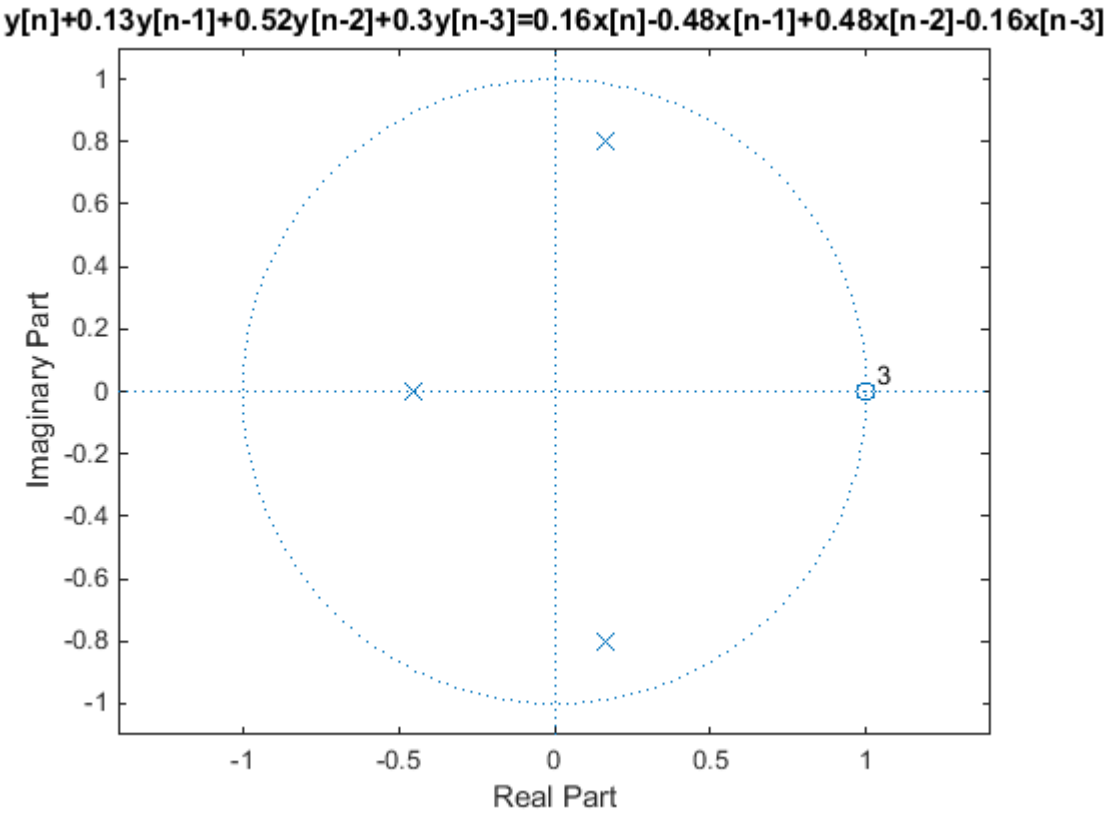


All poles at origin

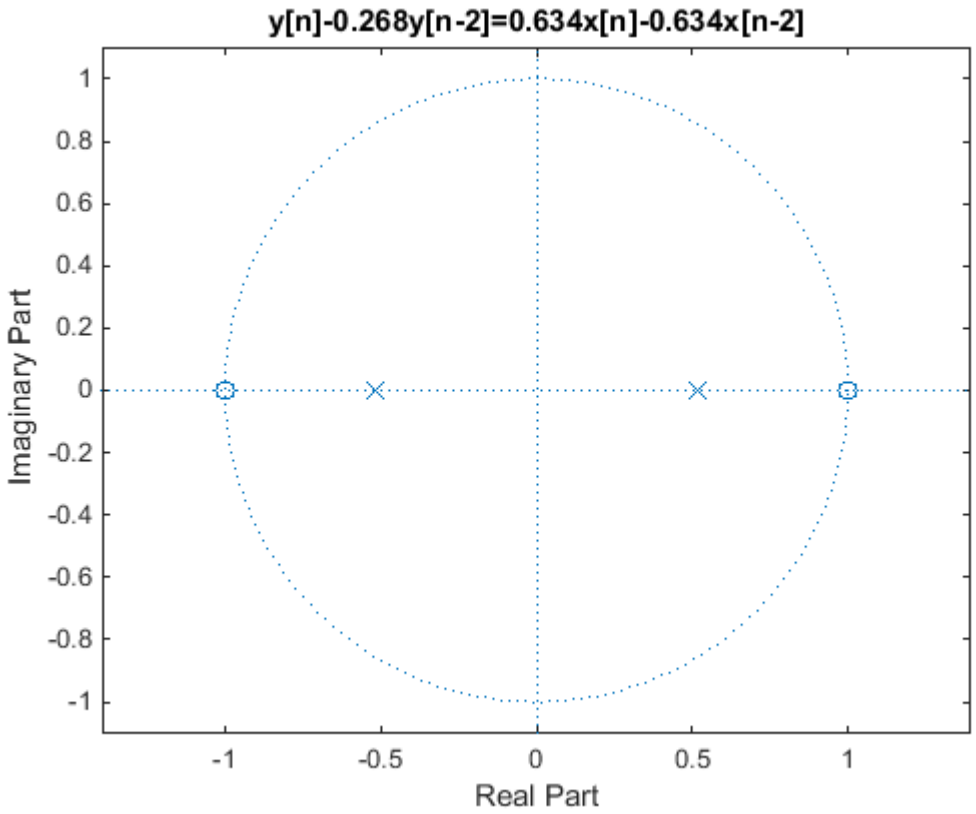
Low-pass filter



High-pass filter

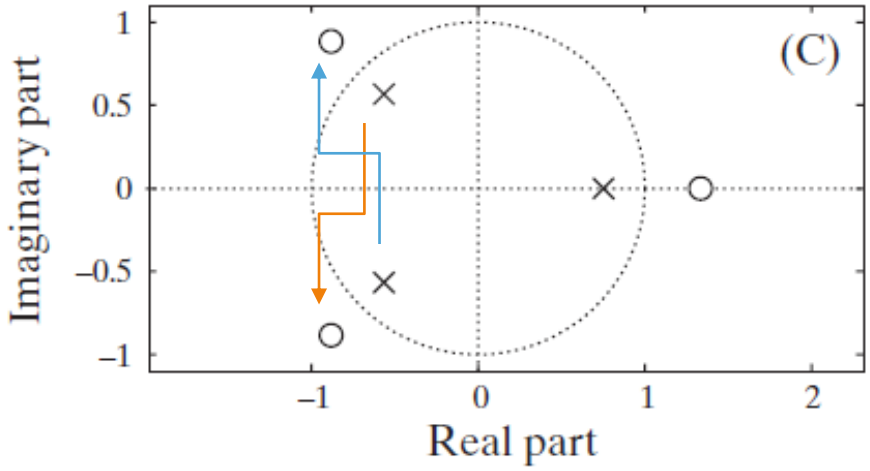


Band-pass Filter



All pass filter

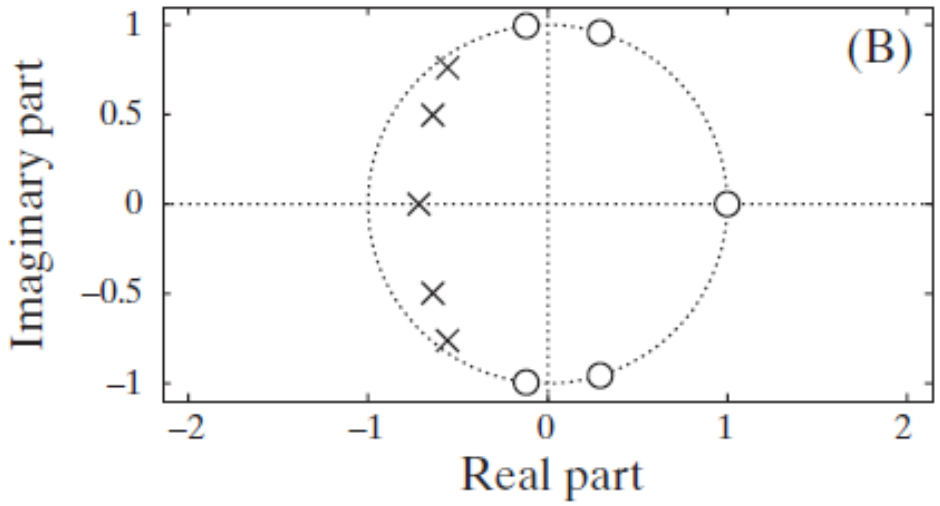
It is the only system for which poles and zeros occur in conjugate reciprocal pairs.



Stable system

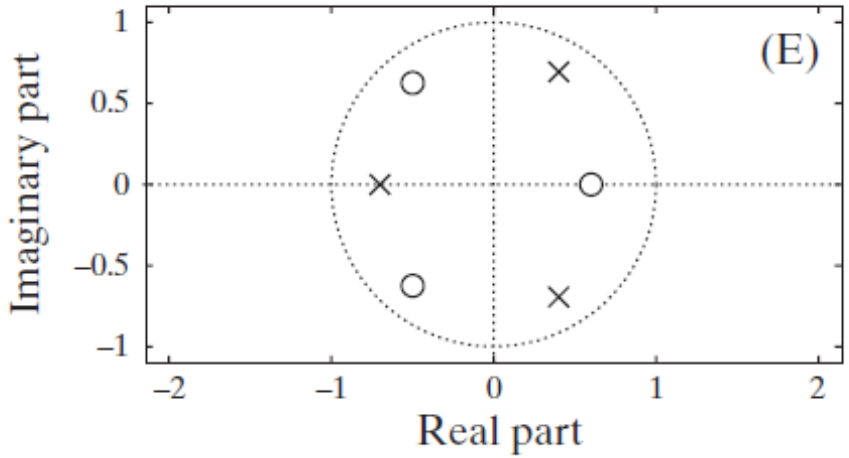
Stable systems include the unit circle in their ROC.

If the system is causal, this means all poles have to be inside the unit circle



Stable & Causal inverse system

All zeros of the original system need to be inside the unit circle

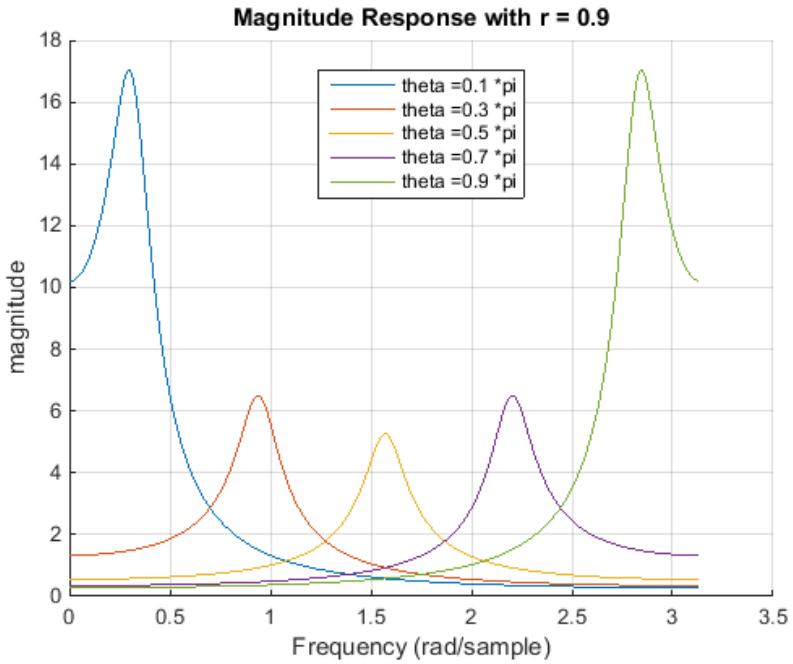


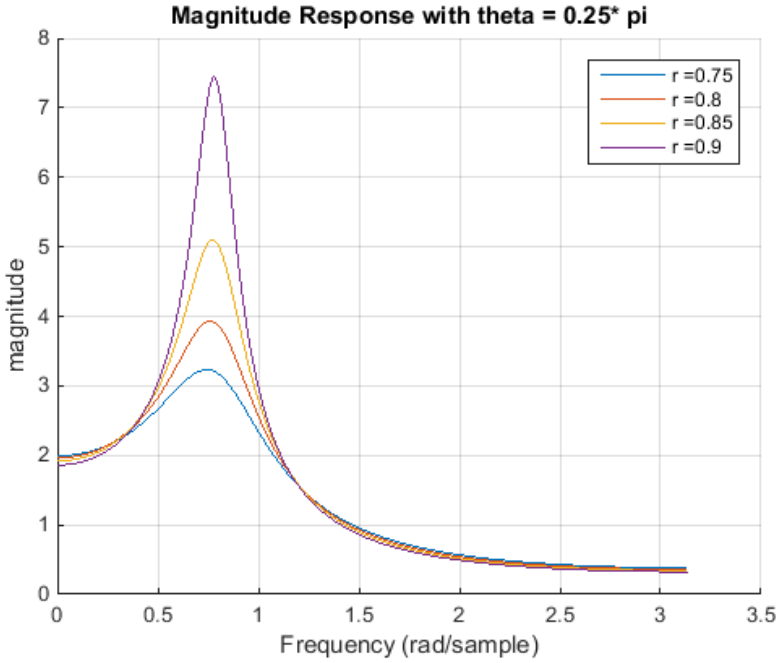
Assignments' Help

- a) Use [h w] = freqz(b,a); Plot h vs. w like in lab 1 for the magnitude and phase response. Describe what time of filter each of the 4 systems are (LPF, HPF, BPF, APF,...)
- Theta&r) You can do this by using mesh to plot in 3D. The function mesh(x, y, z) plots a 3D contour of x, y and z where x is a vector of size 1:N, y is a vector of size 1:M and z is a matrix of size MxN.
- Or let **theta** be constant and change **r**, then let r constant and change theta.

Resonators

$$H(z) = \frac{1}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$



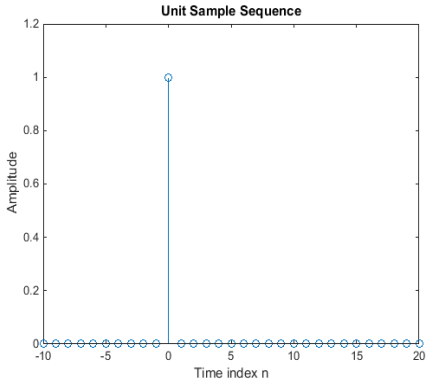


$$\textit{The Gain}\quad G = \left|H(e^{j\omega})\right| = \frac{1}{1 - r\sqrt{1 - 2r\cos(2\theta) + r^2}}$$

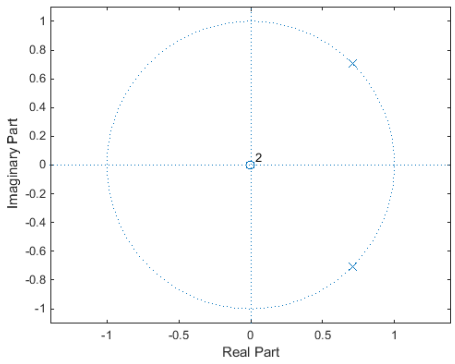
Sinusoid Generator

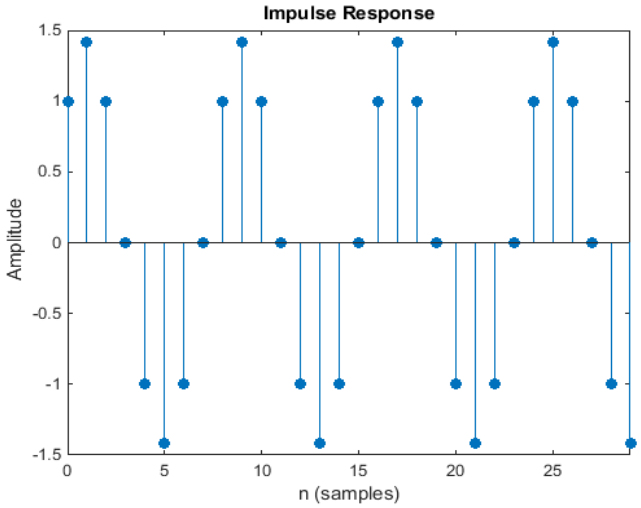
$$H(z) = \frac{1}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

Let r=1



$$\frac{1}{1 - 2\cos\theta\, z^{-1} + z^{-2}}$$





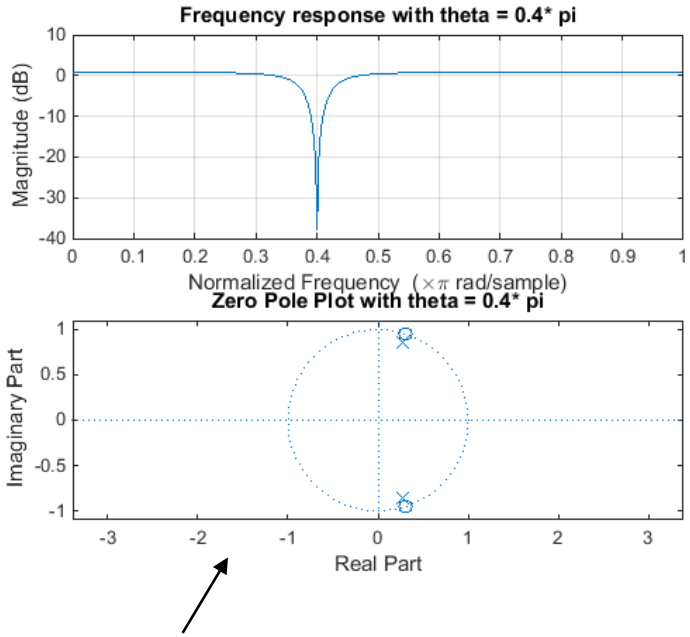
$$\text{Frequency} = f = \frac{\theta}{2\pi}.fs$$

Notch & Comb Filters

Notch filter: If zeros are closer than poles to unit circle

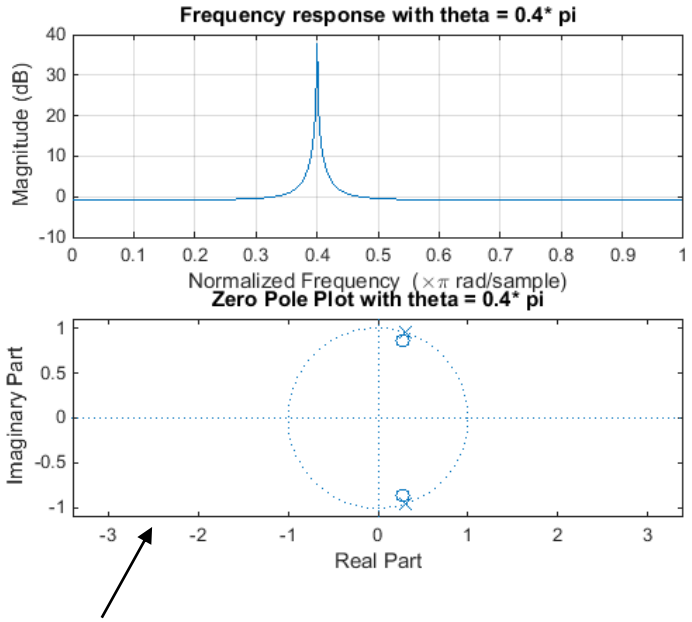
Comb Filter: If poles are closer than zeros to unit circle

$$H(z) = \frac{1 - 2\cos\theta\, z^{-1} + z^{-2}}{1 - 1.8\cos\theta\, z^{-1} + 0.81z^{-2}}$$



Notch

$$H(z) = \frac{1 - 1.8\cos\theta\, z^{-1} + 0.81z^{-2}}{1 - 2\cos\theta\, z^{-1} + z^{-2}}$$

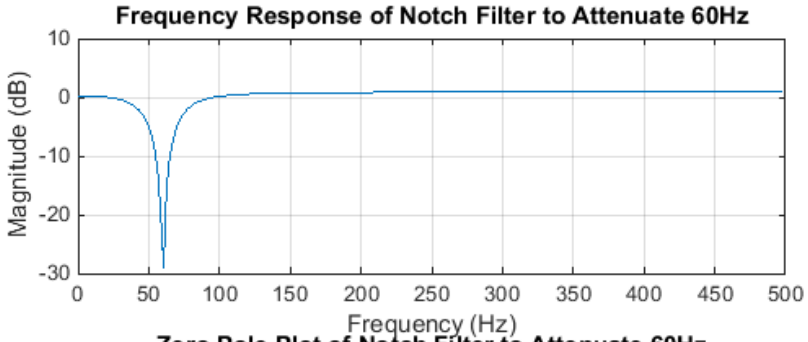


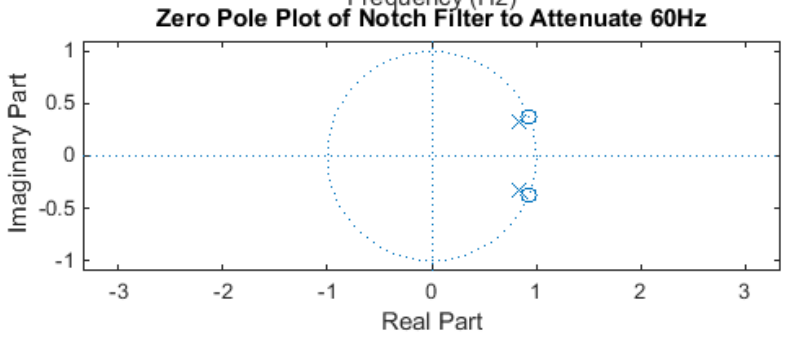
Comb

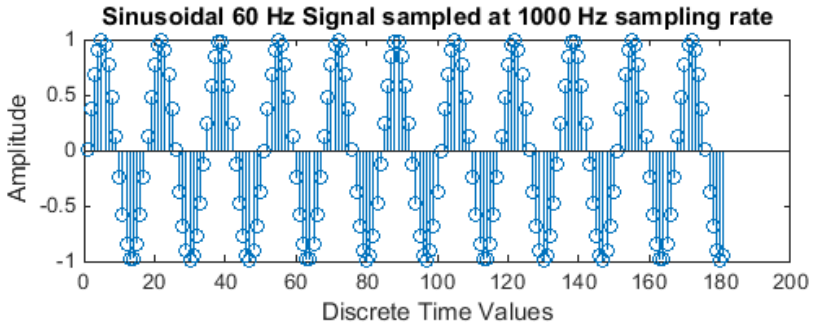
Remove certain frequency

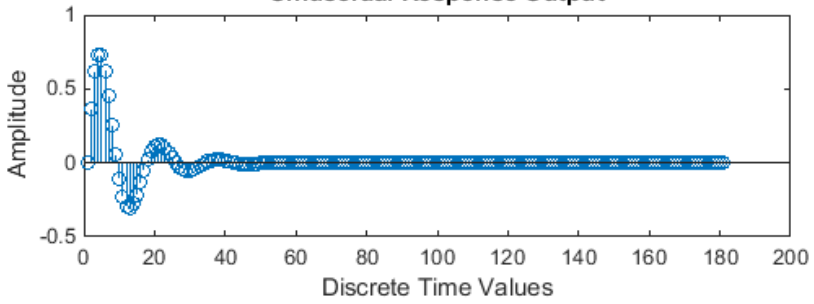
Removing 60 Hz

$$\theta = \frac{f}{fs}.2\pi = 60/1000*2\pi$$









Complete the assignment with mathematical proof if required.

Thanks