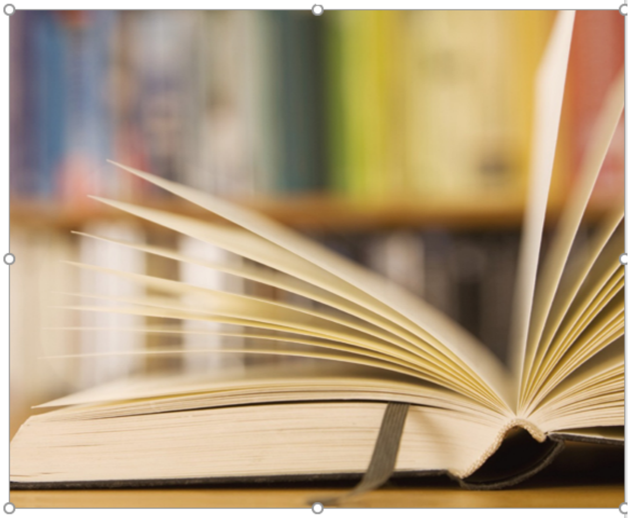
TUTORIAL

ELG3125:SIGNAL AND SYSTEM ANALYSIS

Chapter (5) The Discrete-Time Fourier Transform

By: Hitham Jleed



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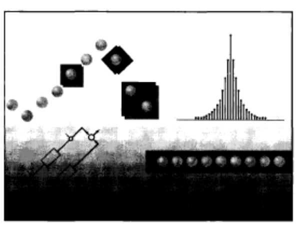
5
THE DISCRETE-TIME FOURIER TRANSFORM



4. (Oppenheim&Willsky) Calculate the discrete time Fourier transform (DTFT) of the following signals

   (a) $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$.

   (b) $x[n] = \left(\frac{1}{2}\right)^{|n-1|}$.

   (c) $x[n] = \delta[n-1] + \delta[n+1]$.

Q. Soln:-

(a) $x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-1)$

The discrete-time fourier Transform (DTFT) is given as:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

$$\therefore X(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} e^{-j\omega(n+1)}$$

$$= \sum_{n=0}^{\infty} e^{-j\omega} \left[\frac{1}{2} e^{-j\omega}\right]^{n}$$

$$= e^{-j\omega}\left[1 + \frac{1}{2} e^{-j\omega} + \left(\frac{1}{2}\right)^2 e^{-j2\omega} + \cdots + \infty\right]$$

↳ Geometric progression

$$\therefore \boxed{X(e^{j\omega}) = e^{-j\omega}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]} \quad \text{——— (1)}$$

Ans.

Since In Geometric Progression

$$\boxed{S_\infty = \frac{a_1}{1-r}}$$

(b) $x(n) = \left(\frac{1}{2}\right)^{|n-1|}$

since DTFT is

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

$$= \underbrace{\sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-(n-1)} e^{-j\omega n}}_{\text{first term}} + \underbrace{\sum_{n=1}^{+\infty} \left(\frac{1}{2}\right)^{(n-1)} e^{-j\omega n}}_{\text{Second term.}}$$

we have already calculated second term in part (a) of this problem; so, we will calculate first term only

$$\therefore \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-(n-1)} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{(n+1)} e^{j\omega n} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{j\omega}\right)^{n}$$

$$= \left(\frac{1}{2} \times \frac{1}{1-\frac{1}{2}e^{j\omega}}\right)$$

[using Geometric progression sum for infinite term as we did in previous problem].

$$\therefore \boxed{X(e^{j\omega}) = \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^{j\omega}}\right] + e^{-j\omega}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]}$$

(using Eqn ①)

(c) $x(n) = \delta(n-1) + \delta(n+1)$

$$\therefore X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} \left[\delta(n-1) + \delta(n+1)\right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} \delta(n-1)\, e^{-j\omega n} + \sum_{n=-\infty}^{+\infty} \delta(n+1)\, e^{-j\omega n}$$

$$= e^{-j\omega} + e^{j\omega}$$

Since $\boxed{\delta(n-n_0) \cdot x(n) = x(n_0)}$

$$X(e^{j\omega}) = \frac{1}{2}\left[e^{j\omega} + e^{-j\omega}\right] \times 2$$

$$\therefore \boxed{X(e^{j\omega}) = 2\cos\omega}$$ Ans.

5.6. Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties listed in Table 5.1.

(a) $x_1[n] = x[1-n] + x[-1-n]$

(b) $x_2[n] = \frac{x^*[-n]+x[n]}{2}$

(c) $x_3[n] = (n-1)^2 x[n]$

$x[n]$ has Fourier transform $X(e^{j\omega})$.

$$x[n] \xleftrightarrow{\text{F.T}} X(e^{j\omega})$$

(a)

Consider the signal $x_1[n]$.

$$x_1[n] = x[1-n] + x[-1-n]$$

Consider the time shifting property of the Fourier transform.

$$x[n-n_0] \xleftrightarrow{\text{F.T}} e^{-j\omega n_0} X(e^{j\omega})$$

Use time shifting property to obtain the Fourier transform of $x[n+1], x[n-1]$.

$$x[n+1] \xleftrightarrow{\text{FT}} e^{j\omega} X(e^{j\omega})$$
$$x[n-1] \xleftrightarrow{\text{FT}} e^{-j\omega} X(e^{j\omega})$$

Consider the time reversal property of the Fourier transform.

$$x[-n] \xleftrightarrow{\text{F.T}} X(e^{-j\omega})$$

Apply time reversal property to $x[n+1], x[n-1]$.

$$x[-n+1] \xleftrightarrow{\text{FT}} e^{-j\omega} X(e^{-j\omega})$$
$$x[-n-1] \xleftrightarrow{\text{FT}} e^{j\omega} X(e^{-j\omega})$$

Use linearity property to obtain the Fourier transform of $x_1[n]$.

$$x_1[n] \stackrel{\text{FT}}{\longleftrightarrow} e^{-j\omega} X\left(e^{-j\omega}\right) + e^{j\omega} X\left(e^{-j\omega}\right)$$

$$x_1[n] \stackrel{\text{FT}}{\longleftrightarrow} 2X\left(e^{-j\omega}\right)\left[\frac{e^{-j\omega}+e^{j\omega}}{2}\right]$$

$$x_1[n] \stackrel{\text{FT}}{\longleftrightarrow} 2X\left(e^{-j\omega}\right)\cos\omega$$

Therefore, the Fourier transform of $x_1[n]$ is

$$\boxed{2\cos(\omega)X\left(e^{-j\omega}\right)}.$$

(b)

Consider the signal $x_2[n]$.

$$x_2[n] = \frac{x^*[-n]+x[n]}{2}$$

$$= \frac{1}{2}x^*[-n]+\frac{1}{2}x[n]$$

Consider the conjugation property of Fourier transform.

$$x^*[n] \stackrel{\text{F.T}}{\longleftrightarrow} X^*\left(e^{-j\omega}\right)$$

Apply time reversal property to obtain the Fourier transform of $x^*[-n]$.

$$x^*[-n] \stackrel{\text{F.T}}{\longleftrightarrow} X^*\left(e^{j\omega}\right)$$

Use linearity property to obtain the Fourier transform of $x_2[n]$.

$$\frac{1}{2}x^*[-n]+\frac{1}{2}x[n] \stackrel{\text{F.T}}{\longleftrightarrow} \frac{1}{2}X^*\left(e^{j\omega}\right)+\frac{1}{2}X\left(e^{j\omega}\right)$$

$$x_2[n] \stackrel{\text{F.T}}{\longleftrightarrow} \frac{1}{2}\left[X^*\left(e^{j\omega}\right)+X\left(e^{j\omega}\right)\right]$$

$$x_2[n] \stackrel{\text{F.T}}{\longleftrightarrow} \mathcal{Re}\left\{X\left(e^{j\omega}\right)\right\}$$

Therefore, the Fourier transform of $x_2[n]$ is,

$$\boxed{\mathcal{Re}\left\{X\left(e^{j\omega}\right)\right\}}.$$

(c)

Consider the signal $x_3[n]$.

$$x_3[n] = (n-1)^2 x[n]$$
$$= (n^2 - 2n + 1)x[n]$$
$$x_3[n] = n^2 x[n] - 2nx[n] + x[n]$$

Consider the Differentiation in frequency Property.

$$nx[n] \xleftrightarrow{\text{F.T}} j\frac{dX(e^{j\omega})}{d\omega}$$

$$n^2 x[n] \xleftrightarrow{\text{F.T}} (j)^2 \frac{d^2 X(e^{j\omega})}{d\omega^2}$$

$$n^2 x[n] \xleftrightarrow{\text{F.T}} -\frac{d^2 X(e^{j\omega})}{d\omega^2}$$

Use linearity property to obtain the Fourier transform of $x_3[n]$.

$$n^2 x[n] - 2nx[n] + x[n] \xleftrightarrow{\text{F.T}} -\frac{d^2 X(e^{j\omega})}{d\omega^2} - 2\left(j\frac{dX(e^{j\omega})}{d\omega}\right) + X(e^{j\omega})$$

$$n^2 x[n] - 2nx[n] + x[n] \xleftrightarrow{\text{F.T}} -\frac{d^2 X(e^{j\omega})}{d\omega^2} - 2j\frac{dX(e^{j\omega})}{d\omega} + X(e^{j\omega})$$

$$x_3[n] \xleftrightarrow{\text{F.T}} -\frac{d^2 X(e^{j\omega})}{d\omega^2} - 2j\frac{dX(e^{j\omega})}{d\omega} + X(e^{j\omega})$$

Hence, the Fourier transform of $x_3[n]$ is

$$\boxed{-\frac{d^2 X(e^{j\omega})}{d\omega^2} - 2j\frac{dX(e^{j\omega})}{d\omega} + X(e^{j\omega})}.$$

Q1 For a discrete-time LTI system with an impulse response of $h[n] = \left(\frac{1}{3}\right)^n$,

1) If a signal of $x[n] = \left(\frac{1}{2}\right)^n$ is applied to the input, find the output signal. **(25%)**
2) If a signal of $x[n] = \left(\frac{1}{3}\right)^n$ is applied to the input, find the output signal. **(25%)**

$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n]\begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n + 1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$

Q2 Consider a causal LTI system described by the difference equation $y[n] - \frac{1}{2}y[n-1] = 2x[n]$

1) What is the impulse response $h[n]$ of this system? **(25%)**
2) If an input to the system is $x[n] = \left(\frac{1}{4}\right)^n u[n]$, what is the system response $y[n]$? **(25%)**)

5.13. An LTI system with impulse response $h_1[n] = (\frac{1}{3})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$

Determine $h_2[n]$.

Considering two LTI systems that are connected in parallel the impulse response of the overall system is the sum of the impulse responses of the individual system.

$$h[n] = h_1[n] + h_2[n]$$

Form the linearly of the Fourier transform we can write

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

We have the impulse response of $h_1[n]$ is

$$h_1[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$H_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

We can write

$$H_2(e^{j\omega}) = H(e^{j\omega}) - H_1(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}$$

Therefore

$$H_2(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}} - \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

$$H_2(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{(3 - e^{-j\omega})(4 - e^{-j\omega})} - \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

$$H_2\left(e^{j\omega}\right)=\frac{1}{12}\left[\frac{-12+5e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}\right]-\frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$H_2\left(e^{j\omega}\right)=\frac{-1+\frac{5}{12}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}-\frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$H_2\left(e^{j\omega}\right)=\left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)\left(\frac{-1+\frac{5}{12}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}-1\right)$$

$$H_2\left(e^{j\omega}\right)=\left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)\left(\frac{-1+\frac{5}{12}e^{-j\omega}-1+\frac{1}{4}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}\right)$$

$$H_2\left(e^{j\omega}\right)=\left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)\left(\frac{-2+\frac{8}{12}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}\right)$$

$$H_2\left(e^{j\omega}\right)=\left(\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right)\left(\frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}\right)(-2)$$

$$H_2\left(e^{j\omega}\right)=\frac{-2}{1-\frac{1}{4}e^{-j\omega}}$$

Taking the inverse Fourier transform we have

$$\therefore \boxed{h_2[n]=-2\left(\frac{1}{4}\right)^n u[n]}$$

5.29. (a) Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the response to each of the following input signals:

(i) $x[n] = (\frac{3}{4})^n u[n]$

(ii) $x[n] = (n+1)(\frac{1}{4})^n u[n]$

(iii) $x[n] = (-1)^n$

(a)

Consider the following impulse response of a discrete-time LTI (linear time-invariant) system:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Refer to Table 5.2 in the textbook to find the Fourier transform of $h[n]$.

$$H\left(e^{j\omega}\right) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \quad \left(\text{since, } a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - ae^{-j\omega}}\right)$$

The output of the system $y[n]$ and its Fourier transform from convolution property of discrete-time Fourier transform (DTFT) is,

$$Y\left(e^{j\omega}\right) = X\left(e^{j\omega}\right) H\left(e^{j\omega}\right)$$

(i)

Consider the following input of the system:

$$x[n] = \left(\frac{3}{4}\right)^n u[n]$$

The Fourier transform of input $x[n]$ is,

$$X\left(e^{j\omega}\right) = \frac{1}{1 - \frac{3}{4}e^{-j\omega}} \quad \left(\text{since, } a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - ae^{-j\omega}}\right)$$

Substitute the input and impulse response in the equation of output of the system.

$$\begin{aligned} Y\left(e^{j\omega}\right) &= X\left(e^{j\omega}\right) H\left(e^{j\omega}\right) \\ &= \left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1 - \frac{3}{4}e^{-j\omega}}\right] \\ &= \frac{1}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{3}{4}e^{-j\omega}\right)} \end{aligned}$$

Use partial fraction expansion to simplify the equation.

$$Y\left(e^{j\omega}\right)=\frac{A}{1-\frac{1}{2}e^{-j\omega}}+\frac{B}{1-\frac{3}{4}e^{-j\omega}}$$

Here,

$$A=\left.\frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{3}{4}e^{-j\omega}\right)}\left(1-\frac{1}{2}e^{-j\omega}\right)\right|_{e^{-j\omega}=2}$$

$$=\left.\frac{1}{\left(1-\frac{3}{4}e^{-j\omega}\right)}\right|_{e^{-j\omega}=2}$$

$$=\frac{1}{1-\frac{3}{4}(2)}$$

$$=-2$$

$$B=\left.\frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{3}{4}e^{-j\omega}\right)}\left(1-\frac{3}{4}e^{-j\omega}\right)\right|_{e^{-j\omega}=\frac{4}{3}}$$

$$=\left.\frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)}\right|_{e^{-j\omega}=\frac{4}{3}}$$

$$=\frac{1}{1-\frac{1}{2}\left(\frac{4}{3}\right)}$$

$$=3$$

Hence, the expression of the output function is,

$$Y\left(e^{j\omega}\right)=\frac{-2}{1-\frac{1}{2}e^{-j\omega}}+\frac{3}{1-\frac{3}{4}e^{-j\omega}}$$

Take the inverse Fourier transforms for $Y\left(e^{j\omega}\right)$.

$$y[n]=3\left(\frac{3}{4}\right)^n u[n]-2\left(\frac{1}{2}\right)^n u[n] \qquad \left(\text{since, } a^n u[n]\xleftrightarrow{DTFT}\frac{1}{1-ae^{-j\omega}}\right)$$

Thus, the response $y[n]$ of the system is $\boxed{3\left(\frac{3}{4}\right)^n u[n]-2\left(\frac{1}{2}\right)^n u[n]}$.

(ii) We have

$$X(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2}\right]\left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{4}{1 - \frac{1}{2}e^{-j\omega}} - \frac{2}{1 - \frac{1}{4}e^{-j\omega}} - \frac{3}{(1 - \frac{1}{4}e^{-j\omega})^2} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n] - 3(n+1)\left(\frac{1}{4}\right)^n u[n].$$

(iii) We have

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - (2k+1)\pi).$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - (2k+1)\pi)\right]\left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{4\pi}{3} \sum_{k=-\infty}^{\infty} \delta(\omega - (2k+1)\pi) \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$x[n] = \frac{2}{3}(-1)^n.$$

Problem 4

The discrete-time waveform $x[n]$ is the input to a discrete-time LTI system with impulse response $h[n]$. Both functions are given below. Find the Fourier transform (DTFT) of the input, the frequency response $H(e^{j\omega})$ of the LTI system, and the DTFT $Y(e^{j\omega})$ of the output of the LTI system. Using partial fraction techniques, find the output time function $y[n]$ from the DTFT of the output.

$$x[n] = 5(3/4)^n \, u[n]$$
$$h[n] = 10(1/2)^n \, u[n]$$

Given $x(n) = 5\left(\frac{3}{4}\right)^n u(n)$

DTFT of $x(n) = X(e^{j\omega}) = 5 \frac{1}{1 - \frac{3}{4}e^{-j\omega}}$

$h(n) = 10\left(\frac{1}{2}\right)^n u(n)$

DTFT of $h(n) = H(e^{j\omega}) = 10 \times \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$

$\therefore \quad Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$

$$= \frac{5}{1 - \frac{3}{4}e^{-j\omega}} \cdot \frac{10}{1 - \frac{1}{2}e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{50\, e^{j\omega}}{e^{j\omega} - \frac{3}{4}} \cdot \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}}$$

$$\frac{Y(e^{j\omega})}{e^{j\omega}} = \frac{50 e^{j\omega}}{\left(e^{j\omega} - \frac{3}{4}\right)\left(e^{j\omega} - \frac{1}{2}\right)}$$

$$\frac{Y(e^{j\omega})}{e^{j\omega}} = \frac{A}{e^{j\omega} - \frac{3}{4}} + \frac{B}{e^{j\omega} - \frac{1}{2}}$$

$$A = \frac{50 \times \frac{3}{4}}{\frac{3}{4} - \frac{1}{2}} = 150 \qquad B = \frac{50 \times \frac{1}{2}}{\frac{1}{2} - \frac{3}{4}} = -100$$

$$Y(e^{j\omega}) = \frac{150\, e^{j\omega}}{e^{j\omega} - \frac{3}{4}} - \frac{100\, e^{j\omega}}{e^{j\omega} - \frac{1}{2}}$$

Apply inverse Tranform

$$y(n) = 150\left(\frac{3}{4}\right)^n u(n) - 100\left(\frac{1}{2}\right)^n u(n)$$

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Problem 1. Determine the Discrete Time Fourier Series (DTFS) coefficients of the periodic discrete time sequence $x[n]$ with one fundamental period defined as

$$x[n] = 0.5^n u[n], \quad 0 \leq k \leq 14.$$

Plot the signal and its Fourier coefficients (both magnitude and phase) using MATLAB.

Problem 2. Determine the DTFS coefficients of the following periodic signal

$$x[n] = Ae^{j\left(\frac{2\pi m}{N}k+\theta\right)},$$

where the greatest common divisor between the fundamental period N and the integer constant m is one. Using MATLAB plot the magnitude and phase spectra for the case when $A = 2, N = 6, m = 5$, and $\theta = \frac{\pi}{4}$.

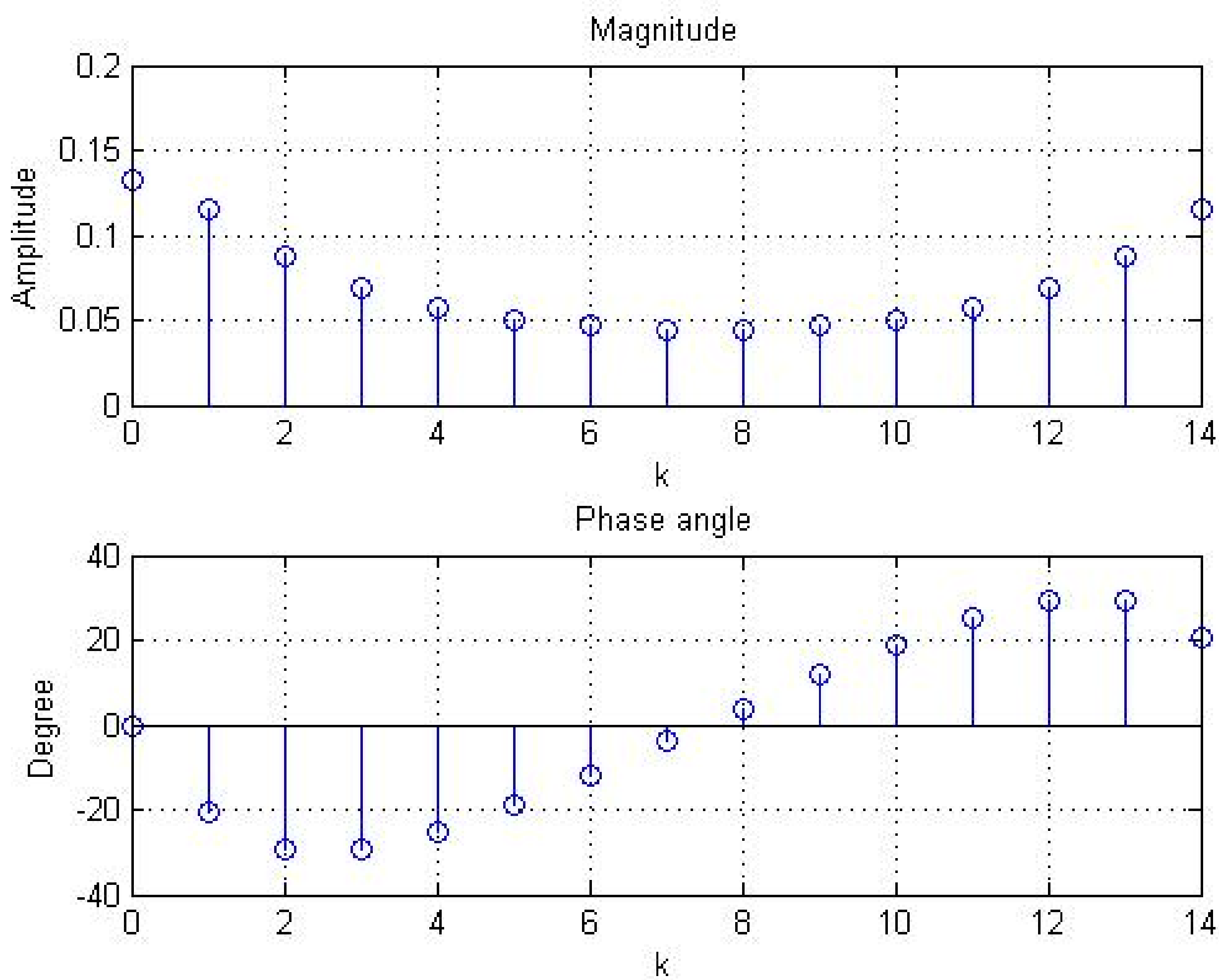
MATLAB code is given below in bold letters.

```
clc;
close all;
clear all;

% Problem (1)
n = 0:14;
k = n;
N = length(n);
x = (0.5).^n;

for l = 1:length(k)
   for m = 1:length(n)
       ak(m) = (1/N) * x(m) * exp(-1j*2*pi*k(l)*n(m)/N);
   end
   Xk(l) = sum(ak);
end

figure;
subplot(211);stem(k,abs(Xk));grid;title('Magnitude');xlabel('k');ylabel('Amplitude');
subplot(212);stem(k,angle(Xk)*57.3);grid;title('Phase angle');xlabel('k');ylabel('Degree');
```



```
% Problem (2)

A = 2;N = 6; m = 5; theta = pi/4;
x2 = A*exp(1j*(2*pi*m*k/N+theta));

for l = 1:length(k)
    for m = 1:length(n)
        ak(m) = (1/N) * x2(m) * exp(-1j*2*pi*k(l)*n(m)/N);
    end
    X2k(l) = sum(ak);
end

figure;
subplot(211);stem(k,abs(X2k));grid;title('Magnitude');xlabel('k');ylabel
('Amplitude');
subplot(212);stem(k,angle(X2k)*57.3);grid;title('Phase angle');xlabel
('k');ylabel('Degree');
```