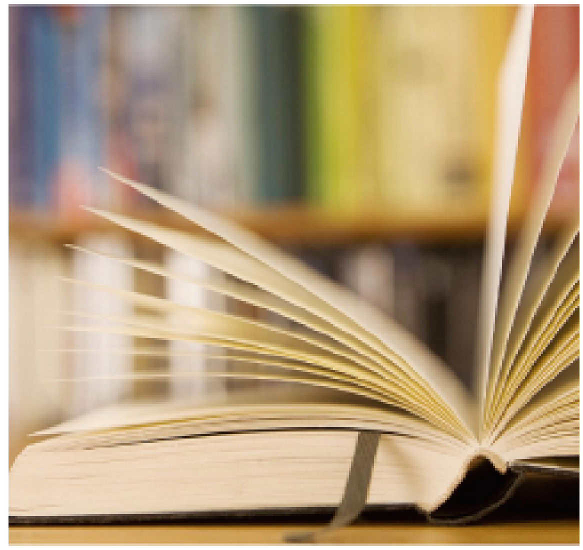
TUTORIAL
ELG3125B:SIGNAL AND SYSTEM ANALYSIS

Chapter (4) -Extra



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Frequency-resposne
Response of a 2nd-order LTI system
Bode plot

Draw the bode plot for the transfer function:

$$H(j\omega)=\frac{1}{(j\omega+5)(j\omega+10)}$$

$$H(j\omega) = \frac{1}{(j\omega+5)\,(j\omega+10)}$$

magnitude $|H(j\omega)| = \dfrac{1}{\sqrt{\omega^2+25}\;\sqrt{\omega^2+100}}$

phase of $|H(j\omega)| = -\tan^{-1}(\omega/5) - \tan^{-1}(\omega/10)$

magnitude in dB (M in dB) $= 20\log\left|\dfrac{1}{\sqrt{\omega^2+25}\;\sqrt{\omega^2+100}}\right|$

$s = j\omega$

Time constant form of $H(j\omega) = \dfrac{1}{(s+5)\,(s+10)} = \dfrac{1}{5(1+s/5)\;10(1+s/10)}$

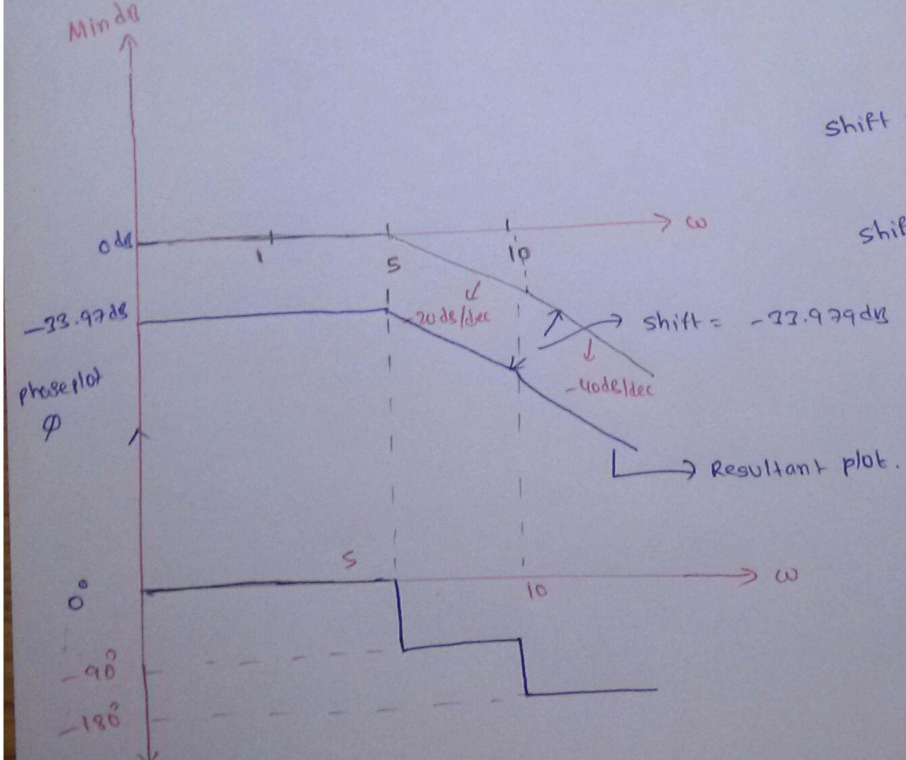
$$H(s) = \frac{0.02}{(1+s/5)\,(1+s/10)}$$

The denominator of s – terms are called corner frequencies. Irrespective of corner frequencies always represent 0.1, 1

shift $= 20\log K$

$= 20\log(0.02)$

shift $= -33.979$ dB



Sketch the magnitude characteristic of the Bode plot for the transfer function:
H(jw)=(10jw+1)/(jw+1)(0.1jw+1)

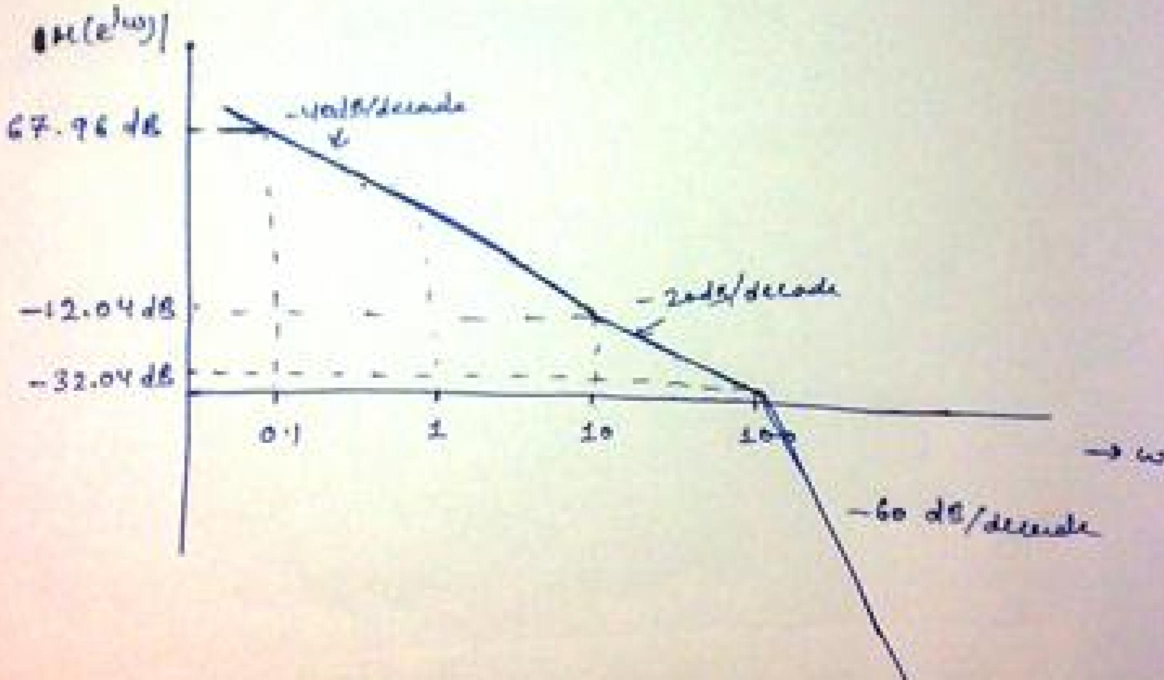
$$H(j\omega) = \frac{250\,(j\omega + 10)}{(j\omega)^2\,(j\omega + 100)^2}$$

$$= \frac{250 \times 10\,(1 + j\omega/10)}{-\omega^2 \times 10000\,\left(1 + \frac{j\omega}{100}\right)^2} = \frac{-1}{4}\,\frac{(1 + j\omega/10)}{\omega^2\left(1 + \frac{j\omega}{100}\right)^2}$$

$$\Rightarrow \quad |H(j\omega)| = \frac{1}{4\omega^2}\,\frac{\sqrt{1 + \omega^2/100}}{\left(1 + \frac{\omega^2}{10000}\right)}$$

at $\omega = 0.1$

$|H(j\omega)| = 67.96$ dB



slope of the lines are shown. ω axis doesn't correspond to 0 dB.

Ans ⇒ $$\boxed{H(j\omega) = \frac{(j\omega+10)}{j\omega(j\omega+100)}}$$

$$s = j\omega, \quad H(s) = \frac{(s+10)}{s(s+100)} = \frac{10\left(\frac{s}{10}+1\right)}{100\, s\left(\frac{s}{100}+1\right)}$$

$$\therefore H(s) = \frac{0.1\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{100}+1\right)} = \frac{K\left(\frac{s}{\omega_1}+1\right)}{s\left(\frac{s}{\omega_2}+1\right)}$$

Gain $K = 0.1$, Corner frequencies ⇒ $\omega_1 = 10$ rad/sec
$\omega_2 = 100$ rad/sec

$\omega_1 = 10 \rightarrow$ Zero Corner frequency

$\omega_2 = 100 \rightarrow$ Pole corner frequency

For a zero corner frequency the magnitude plot slope changes by $+20$ dB/dec & phase plot by $+90°$.

For a pole corner frequency, the magnitude plot slope changes by -20 dB/dec & phase plot by $-90°$.

At origin a pole is present so initial slope $= -20$ dB/dec & initial phase is $-90°$.

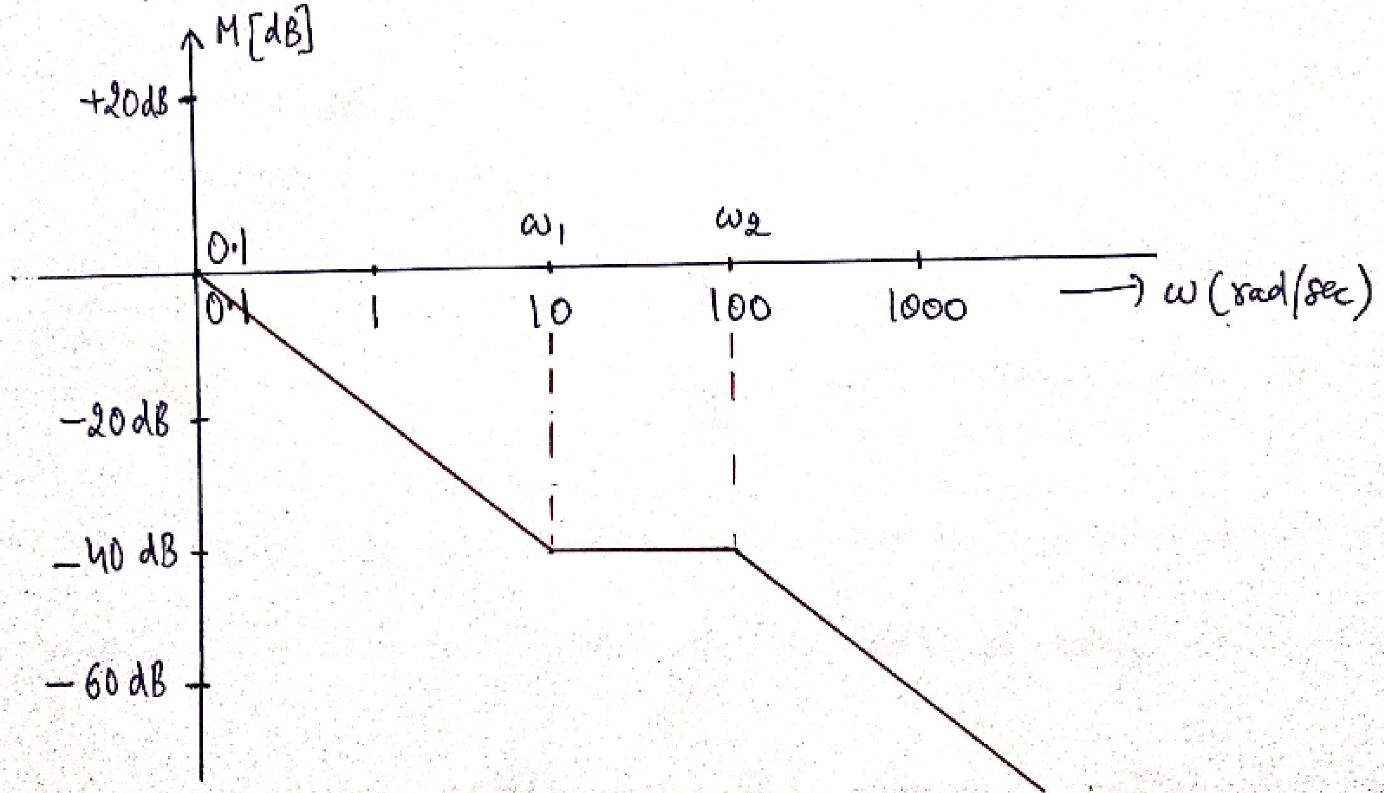
Gain in dB , $M_{dB}\Big|_{at\ \omega=0.1} = 20\log K + 20\log\left(\frac{1}{\omega}\right)$

$$= 20\log(0.1) - 20\log(0.1)$$

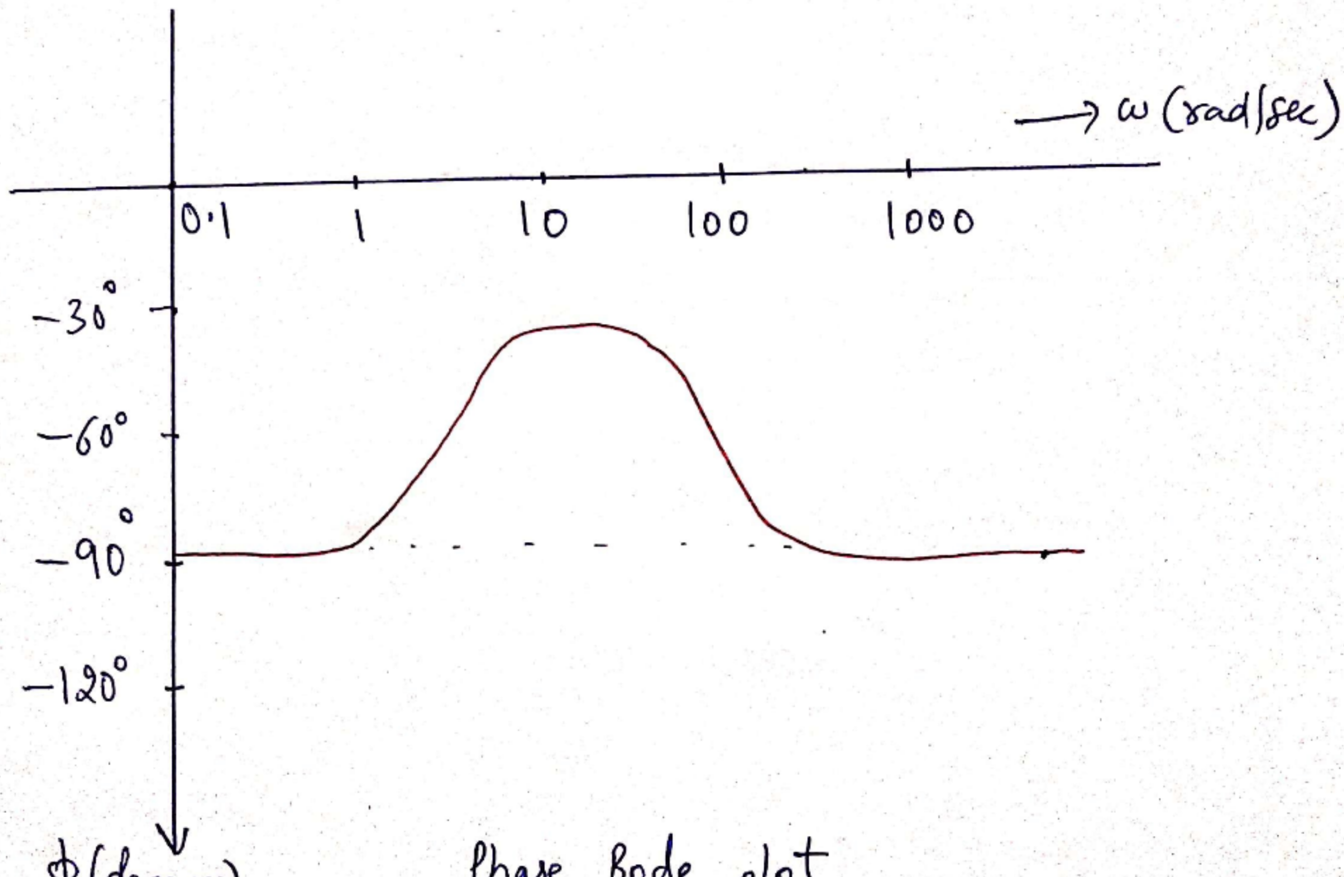
$$= 0\ dB$$

$\therefore$ Initial Magnitude , $M_{dB} = 0\ dB$.
(at $\omega = 10^{-1} = 0.1$ rad/sec)

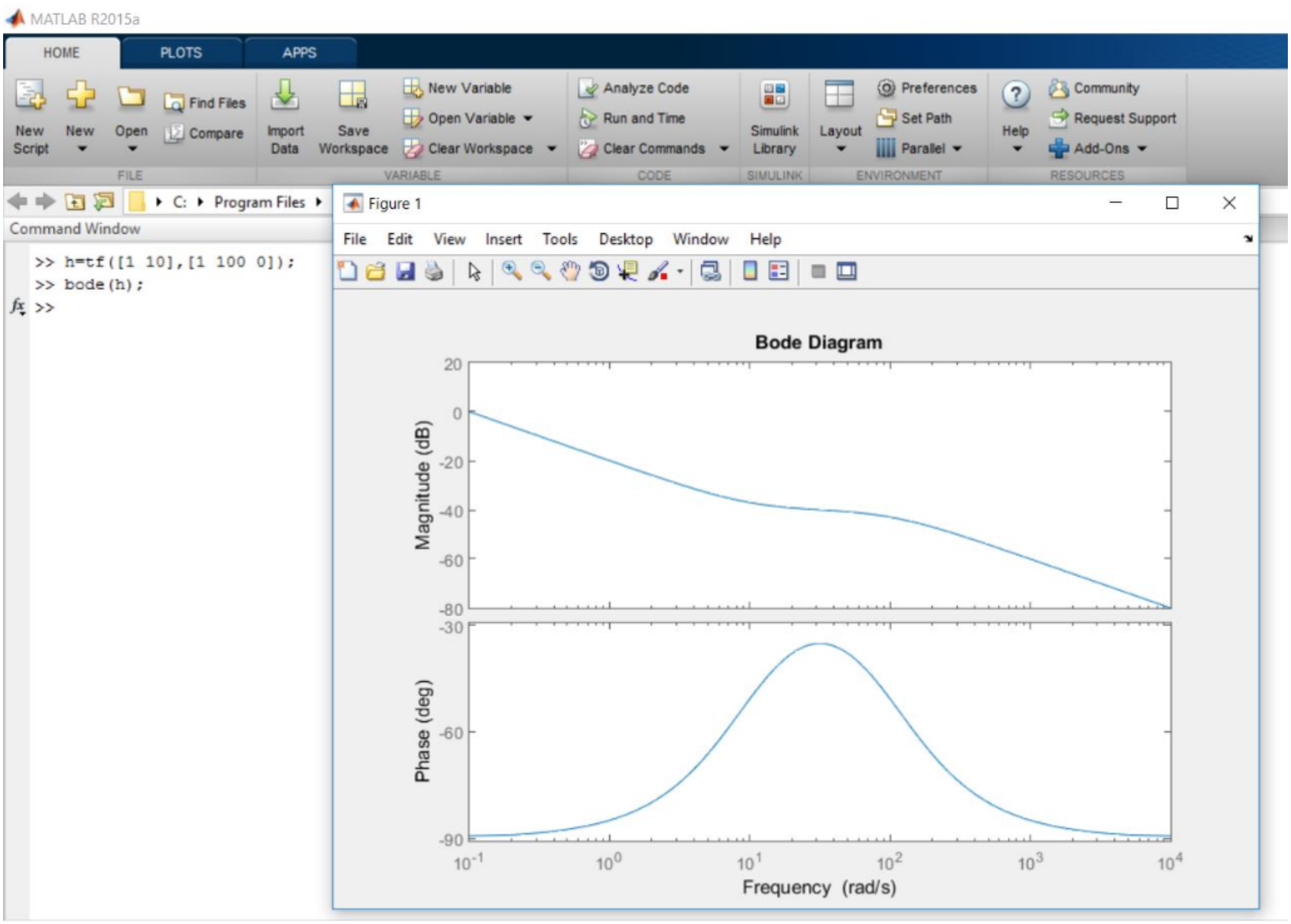
Hence the Bode plot we have :-



Magnitude Bode plot



Phase Bode plot



(c) Both bode plots drawn by hand as well as using MATLAB is almost same.The plot obtained by MATLAB is more accurate as it obtained via software and curves obtained are more smoother.The hand technique is approximate and almost exact not exact

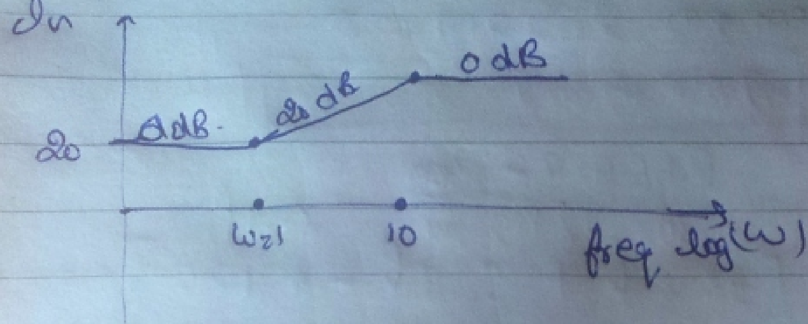
Sketch the amplitude and phase Bode plots of the transfer function:

$$H(w) = \frac{10(1+jw)}{(1+jw/10)}$$

$$H(\omega) = \frac{10(1+j\omega)}{\left(1+j\frac{\omega}{10}\right)}$$

$$= \frac{10(1+s)}{\left(1+\frac{s}{10}\right)}$$

Here $K = 10$ $\quad \omega_{z_1} = 1$ $\quad \omega_{p_1} = 10$

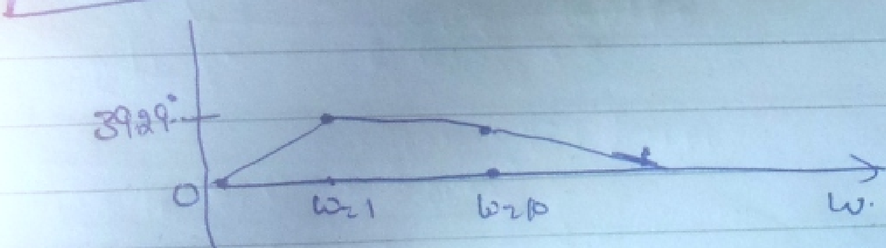


At $\omega = 1$ Magnitude $= 20 \log_{10} K$

$= 20 \log_{10} 10$

$= \underline{20\,dB}$

phase



At $\omega = 1$

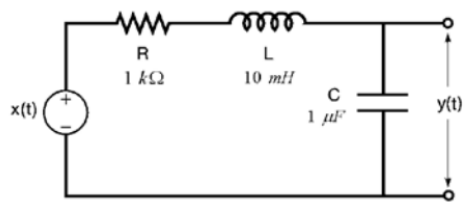
$\angle H(\omega) = \tan^{-1}\omega - \tan^{-1}$

$= \angle 39.29^\circ$

At $\omega = 10$

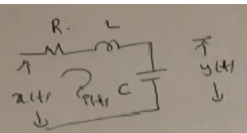
$\angle H(\omega) = \angle 39.28^\circ$

Consider an RLC circuit shown in the figure below. The voltage source $x(t)$ is considered the input to the system, and the voltage $y(t)$ across the capacitor is considered the system output.



a) Find the differential equation governing the input $x(t)$ and output $y(t)$ of this system. **(20)**
b) Calculate the natural frequency of the system. **(20)**
c) Determine if the system is under-, critically, or under damped. **(20)**
d) If the resistance R can be adjusted, determine the value of R required making the system critically damped. **(20)**
e) Draw the Bode plot for the magnitude response. **(20)**

①



$i(t) = 10^{-6} \frac{d}{dt} y(t)$

Apply KVL to the loop i(t)

$-x(t) + 1000\, i(t) + 10^{-2} \frac{d}{dt} i(t) + y(t) = 0$

$1000 \times 10^{-6} \frac{d}{dt} y(t) + 10^{-8} \frac{d^2}{dt^2} y(t) + y(t) = x(t)$

$10^{-8} \left[\frac{d^2}{dt^2} y(t) + 10^{5} \frac{d}{dt} y(t) + 10^{8} y(t) \right] = x(t)$

$$\boxed{\frac{d^2}{dt^2} y(t) + 10^5 \frac{d}{dt} y(t) + 10^8 y(t) = 10^8 x(t)}$$

→ $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}} = 10{,}000$

c) $\alpha = \frac{R}{2L} = \frac{1000}{2 \times 10^{-2}} = 50{,}000$

$\alpha > \omega$, over damped.

d) $\alpha = \omega$ (critical damped)

$\frac{R}{2 \times 10^{-2}} = 10{,}000$ | $\boxed{R = 200\Omega}$

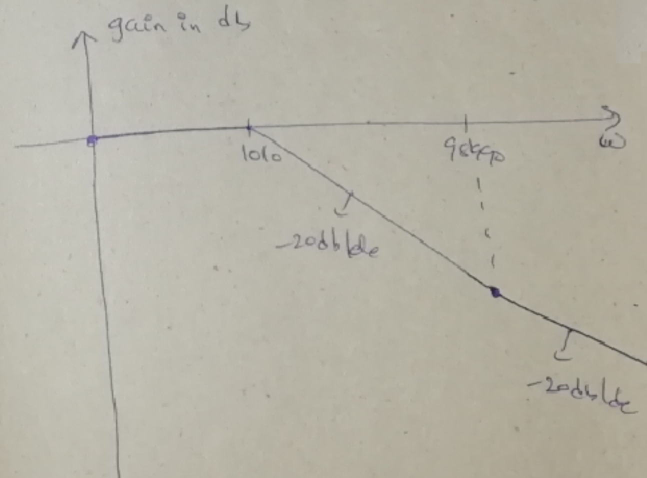
Roots of y(t) is

$D = -98990, -1010.$

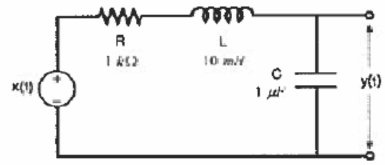
$\frac{Y(s)}{X(s)} = \frac{10^8}{(s+98990)(s+1010)}$

$= \frac{10^8}{10^8 \left[1 + \frac{s}{98990}\right]\left[1 + s/1010\right]}$

$H(j\omega) = \frac{1}{\left(1 + \frac{j\omega}{98990}\right)\left(1 + \frac{j\omega}{1010}\right)}$



Q1: Consider an RLC circuit shown in the figure below. The voltage source $x(t)$ is considered the input to the system, and the voltage $y(t)$ across the capacitor is considered the system output.



1) Find the differential equation governing the input $x(t)$ and output $y(t)$ of this system.
2) Calculate the natural frequency of the system.
3) Determine if the system is under-, critically, or under damped.

use K.V.L : $x(t) = R\,i(t) + L\frac{d}{dt}i(t) + y(t)$ ①

The current in the capacitor : $i(t) = C\frac{d}{dt}y(t)$ ②

substituting ② in ①

$$\boxed{x(t) = RC\frac{d}{dt}y(t) + LC\frac{d^2}{dt^2}y(t) + y(t)}$$ ※ 30

Now use numbers $L = 10\cdot10^{-3}$, $R = 1000$, and $C = 10^{-6}$

$$\frac{d^2y(t)}{dt^2} + 10^5\frac{dy(t)}{dt} + 10^8 y(t) = 10^8 x(t)$$

$\omega_n^2 = 10^8 \Rightarrow f_n = \frac{\omega_n}{2\pi} = \frac{10^4}{2\pi} = \boxed{\frac{10}{2\pi}\text{ KHz}}$ ※ 20

$\xi = \frac{10^5}{2\omega_n} = \frac{10^5}{2\times10^4} = 5$

The system is $\boxed{\text{over damped}}$ ※ 10

The differential equation for a 2nd-order LTI system:

$$\frac{d^2y(t)}{dt^2} + 2\varsigma\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 x(t)$$

where ω_n is the natural frequency and ς is the damping factor. For $\varsigma < 1$, under-damped, $\varsigma = 1$ critically damped, $\varsigma > 1$ over damped.

$2\xi\omega_n = 10^5$

$\omega_n^2 = 10^8$

(a) Sketch the Bode plot of the transfer function (magnitude only). Label the plot.
(b) Based on the plot, determine the filter is a lowpass, bandpass, highpass filter, or none of them.

$$H(j\omega) = \frac{10(10+j\omega)(100+j\omega)}{(1+j\omega)(1000+j\omega)} = \frac{10\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}{(1+j\omega)\left(1+\frac{j\omega}{1000}\right)} \quad \frac{10^3 \cdot 10^{6}}{1000} \Rightarrow \boxed{H(j\omega) = \frac{10\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)}{(j\omega+1)\left(\frac{j\omega}{1000}+1\right)}}$$

$DC = 20\log_{10}(10) = 20$

It is bandstop filter ※ 10

$\omega_b = 1\,(-),\ 10\,(+),\ 100\,(+),\ 1000\,(-)$

※ 30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num=10*conv([1/10 1],[1/100 1]);
den=conv([1 1],[1/1000 1]);
H = tf(num,den);bode(H)
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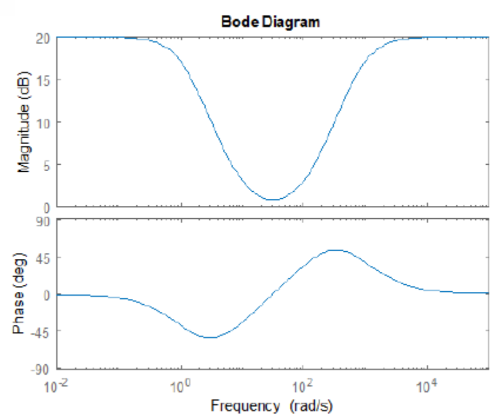


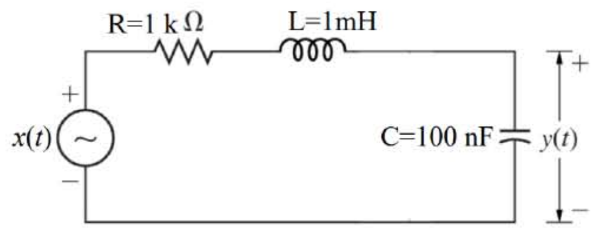
The following system can be modeled as a second-order LTI system with the standard form:

i) Write the differential equation that characterizes the system (using R, L, C, with no numerical values)

ii) Write the frequency response of the system (using R, L, C, with no numerical values)

iii) Determine if the system is under or over damped.

iv) Determine the impulse response $h(t)$ of the system contains oscillations or not (note: it is not necessary to calculate $h(t)$)



Note: $x(t)$ is the input voltage (in Volts), and $y(t)$ is the output voltage over the capacitor (in Volts).

Solution:

i) $\dfrac{dy^2(t)}{dt^2}+\dfrac{R}{L}\dfrac{dy(t)}{dt}+\dfrac{1}{LC}y(t)=\dfrac{1}{LC}x(t)$

$\dfrac{dy^2(t)}{dt^2}+10^6\dfrac{dy(t)}{dt}+10^{10}y(t)=10^{10}x(t)$ $\left(\dfrac{R}{L}=\dfrac{10^3}{1\times10^{-3}}=10^6,\ \dfrac{1}{LC}=\dfrac{1}{1\times10^{-3}\times100\times10^{-9}}=10^{10}\right)$,

ii) Apply FT to $\dfrac{dy^2(t)}{dt^2}+\dfrac{R}{L}\dfrac{dy(t)}{dt}+\dfrac{1}{LC}y(t)=\dfrac{1}{LC}x(t)$, we have

$$(j\omega)^2Y(j\omega)\frac{dy^2(t)}{dt^2}+\frac{R}{L}(j\omega)Y(j\omega)+\frac{1}{LC}Y(j\omega)=\frac{1}{LC}X(j\omega)$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{\frac{1}{LC}}{(j\omega)^2+\frac{R}{L}(j\omega)+\frac{1}{LC}}$$

iii) $\xi=\dfrac{R}{2\sqrt{\dfrac{L}{C}}}=\dfrac{10^3}{2\sqrt{\dfrac{10^{-3}}{100\times10^{-9}}}}=\dfrac{10^3}{2\times100}=5$

Since $\xi=5$, which is greater than 1, the system is overdamped.

iv) Since the system is overdamped, no oscillation would be observed from $h(t)$.

The frequency response is given below. What are the nature frequency and the damping factor? Obtain the Bode plot.

$$H(j\omega) = \frac{2\times 10^4}{(j\omega)^2 + 100(j\omega) + 10^4}$$

(Standard form) $H(j\omega) = \dfrac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$)

Solution:

the nature frequency is $\omega_n = 100$, and the damping factor is

$2\xi\omega_n = 100 \Rightarrow \xi = \dfrac{100}{2\omega_n} = \dfrac{100}{2\times 100} = \dfrac{1}{2}$, which $\angle 1$, the system is underdamped

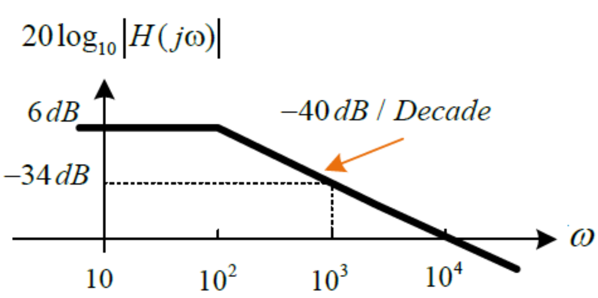
To obtain its Bode plot, we change the expression

$$H(j\omega) = \frac{2\times 10^4}{(j\omega)^2 + 100(j\omega) + 10^4} = \frac{2\times 10^4}{10^4\left[\left(\frac{j\omega}{10^2}\right)^2 + \left(\frac{j\omega}{10^2}\right) + 1\right]} = \frac{2\times 1}{\left(\frac{j\omega}{10^2}\right)^2 + \left(\frac{j\omega}{10^2}\right) + 1}$$

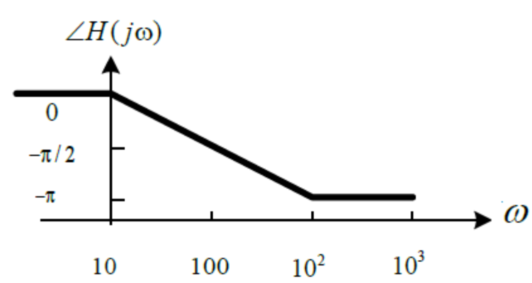
Since $20\log_{10} H(j\omega) = 20\log_{10} 2 + 20\log_{10} \dfrac{1}{\left(\frac{j\omega}{10}\right)^2 + \left(\frac{j\omega}{10}\right) + 1}$

and $20\log_{10} 2 = 6dB$, the magnitude Bode plot would be the one of

$20\log_{10} \dfrac{1}{\left(\frac{j\omega}{10}\right)^2 + \left(\frac{j\omega}{10}\right) + 1}$ shifted up by 6 dB, shown below



(a) magnitude Bode plot



(b) phase Bode plot