TUTORIAL
ELG3125:SIGNAL AND SYSTEM ANALYSIS

Chapter (4) The Continuous-Time Fourier Transform

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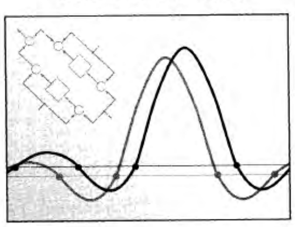
THE CONTINUOUS-TIME FOURIER TRANSFORM



EXERCISE'S CONTINENTS

- The Fourier Transform for periodic signals.

- Properties of the continuous-Time Fourier Transform.

- The convolution properties.

Chapter 4 Problems

The first section of problems belongs to the basic category and the answers are provided in the back of the book. The remaining three sections contain problems belonging to the basic, advanced, and extension categories, respectively.

BASIC PROBLEMS WITH ANSWERS

4.1. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $e^{-2(t-1)}u(t-1)$ **(b)** $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1) + \delta(t-1)$ **(b)** $\frac{d}{dt}\{u(-2-t) + u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

4.3. Determine the Fourier transform of each of the following periodic signals:

(a) $\sin(2\pi t + \frac{\pi}{4})$ **(b)** $1 + \cos(6\pi t + \frac{\pi}{8})$

4.4. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transforms of:

(a) $X_1(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)$

4.1 (1)

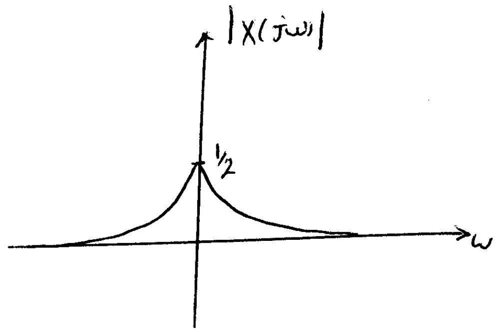
a) $x(t) = e^{-2(t-1)} u(t-1)$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-2(t-1)} u(t-1) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-2(t-1)} e^{-j\omega t} dt$$

$$= e^{-j\omega} / (2 + j\omega)$$

$$|X(j\omega)| = \frac{|e^{-j\omega}|}{|2 + j\omega|} = \frac{1}{\sqrt{4 + \omega^2}}$$



b) $x(t) = e^{-2|t-1|}$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-2|t-1|} e^{-j\omega t} dt$$

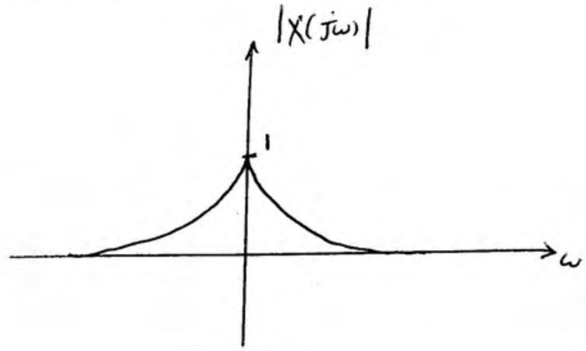
$$= \int_{1}^{+\infty} e^{-2(t-1)} e^{-j\omega t} dt + \int_{-\infty}^{1} e^{2(t-1)} e^{-j\omega t} dt$$

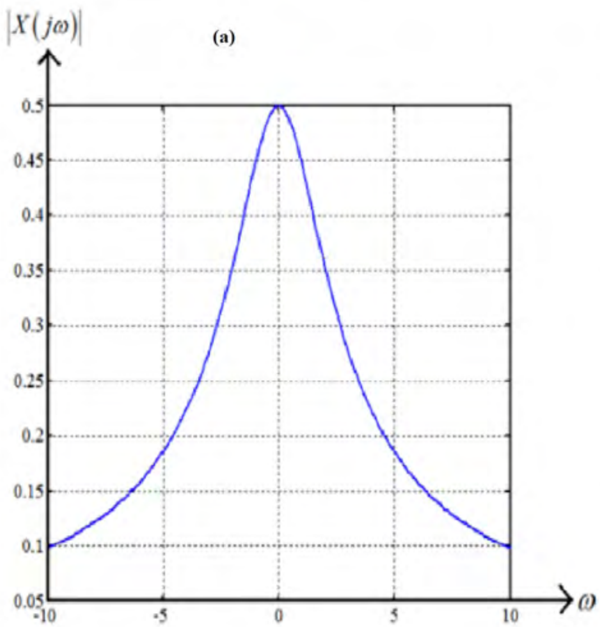
$$= \frac{e^{-j\omega}}{2 + j\omega} + \frac{e^{-j\omega}}{2 - j\omega}$$

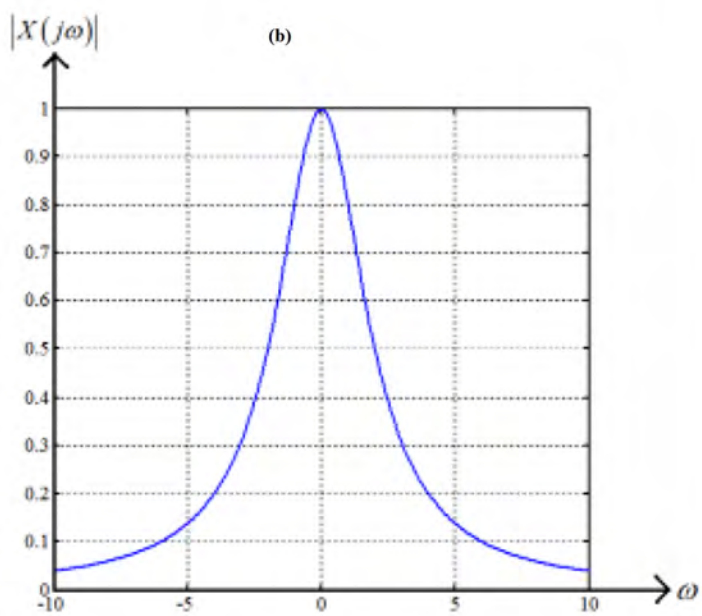
$$= \frac{4 e^{-j\omega}}{4 + \omega^2}$$

②

$$|X(j\omega)| = \frac{|4e^{-j\omega}|}{|4+\omega^2|} = \frac{4}{4+\omega^2}$$







plot using https://www.desmos.com/calculator/

4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1)+\delta(t-1)$ **(b)** $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

(a)

Consider the following signals.

$$x(t)=\delta(t+1)+\delta(t-1)$$

Determine the Fourier transform of $x(t)$.

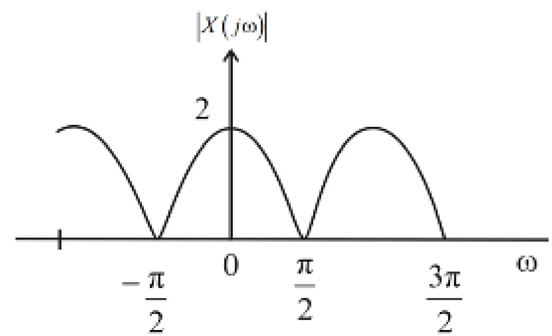
$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}\,dt$$

$$=\int_{-\infty}^{\infty}\left[\delta(t+1)+\delta(t-1)\right]e^{-j\omega t}\,dt$$

$$=\int_{-\infty}^{\infty}\delta(t+1)e^{-j\omega t}\,dt+\int_{-\infty}^{\infty}\delta(t-1)e^{-j\omega t}\,dt$$

The magnitude of the Fourier spectrum is $|X(j\omega)|=2|\cos\omega|$

Draw the magnitude plot of $|X(j\omega)|$



Recall the time shifting property of the impulse function.

$$\int_{a}^{b}\delta(t-T).f(t)\,dt=\begin{cases}f(T), & a<T<b\\ 0, & \text{otherwise}\end{cases}$$

$$X(j\omega)=e^{-j\omega(-1)}+e^{-j\omega(1)}$$

$$=2\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)$$

$$=2\cos\omega$$

Thus, the Fourier transform of $\delta(t+1)+\delta(t-1)$ is $\boxed{2\cos\omega}$

(b)

Consider the following signal.

$$x(t)=\frac{d}{dt}\{u(-2-t)+u(t-2)\}$$

Simplify further by using the relation $\frac{d}{dt}u(t)=\delta(t)$.

$$\begin{aligned}x(t)&=\frac{d}{dt}\left(u(-2-t)\right)+\frac{d}{dt}\left(u(t-2)\right)\\&=-\delta(-t-2)+\delta(t-2)\\&=-\delta(-(2+t))+\delta(t-2)\end{aligned}$$

Since, the impulse function, $\delta(t)$ is even function.

$$\begin{aligned}x(t)&=-\delta(2+t)+\delta(t-2) \qquad (\because \delta(t)\text{ is even function})\\&=\delta(t-2)-\delta(t+2)\end{aligned}$$

Determine the Fourier transform of $x(t)$.

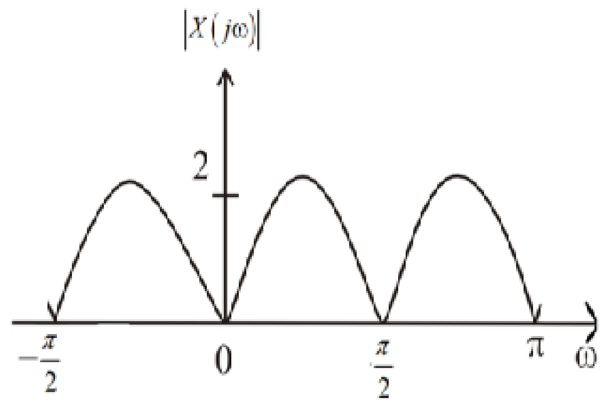
$$\begin{aligned}X(j\omega)&=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt\\&=\int_{-\infty}^{\infty}\left[\delta(t-2)-\delta(t+2)\right]e^{-j\omega t}dt\\&=\int_{-\infty}^{\infty}\delta(t-2)e^{-j\omega t}dt-\int_{-\infty}^{\infty}\delta(t+2)e^{-j\omega t}dt\\&=e^{-j\omega(2)}-e^{-j\omega(-2)} \qquad \left[\int_a^b \delta(t-T).f(t)\,dt=\begin{cases}f(T), & a<T<b\\0, & \text{otherwise}\end{cases}\right]\end{aligned}$$

$$\begin{aligned}X(j\omega)&=-2j\left(\frac{-e^{-j2\omega}+e^{j2\omega}}{2j}\right)\\&=-2j\sin 2\omega\end{aligned}$$

Thus, the Fourier transform of $x(t)=\frac{d}{dt}\{u(-2-t)+u(t-2)\}$ is $\boxed{-2j\sin 2\omega}$.

The magnitude of the Fourier spectrum is $|X(j\omega)|=2|\sin\omega|$

Draw the magnitude plot of $X(j\omega)$.



4.6. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in Table 4.1.

(a) $x_1(t) = x(1-t) + x(-1-t)$

(b) $x_2(t) = x(3t-6)$

Solution

4.6

a) $x(t) \xleftrightarrow{FT} X(j\omega)$

using the time reversal property ($x(-t) \xleftrightarrow{FT} X(-j\omega)$), we have:

$$x(-t) \xleftrightarrow{FT} X(-j\omega)$$

using the time shifting property ($x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)$), we have

$$x(-t+1) \xleftrightarrow{FT} e^{-j\omega} X(-j\omega)$$

$$x(-t-1) \xleftrightarrow{FT} e^{j\omega} X(-j\omega)$$

therefore,

$$x_1(t) = x(-t+1) + x(-t-1) \xleftrightarrow{FT} e^{-j\omega} X(-j\omega) + e^{j\omega} X(-j\omega) = 2\cos\omega \cdot X(-j\omega)$$

(3)

b) using the time scaling property $\left(x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{j\omega}{a}\right) \right)$, we have:

$$x(3t) \overset{FT}{\longleftrightarrow} \frac{1}{3} X\left(j\frac{\omega}{3}\right)$$

using the time shifting property on this, we have:

$$x_2(t) = x(3(t-2)) \overset{FT}{\longleftrightarrow} e^{-2j\omega} \frac{1}{3} X\left(j\frac{\omega}{3}\right)$$

4.19. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}.$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

4.19)

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Since it is given that $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$, we can compute $Y(j\omega)$ to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

Since, $H(j\omega) = \frac{1}{3+j\omega}$, we have

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

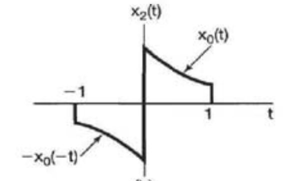
Taking the inverse Fourier transform of $X(j\omega)$, we have

$$x(t) = e^{-4t}u(t).$$

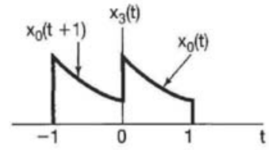
4.23. Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Determine the Fourier transform of each of the signals shown in Figure P4.23. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.



(b)



(c)

4.23 b)

$$x_0(t) = \begin{cases} e^{-t} & 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\Rightarrow \quad X_0(j\omega) = \frac{1 - e^{-(1+j\omega)}}{1 + j\omega}$$

$x_2(t) = x_0(t) - x_0(-t)$

using the linearity and time reversal properties of the Fourier transform we have:

$$X_2(j\omega) = X_0(j\omega) - X_0(j\omega) = j\left[\frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}\right]$$

c)

$x_3(t) = x_0(t) + x_0(t+1)$

using the linearity and time-shifting properties of the Fourier transform we have.

$$X_3(j\omega) = X_0(j\omega) + e^{j\omega} X(j\omega) = \frac{1 + e^{j\omega} - e^{-1}(1 + e^{-j\omega})}{1 + j\omega}$$

4.26. **(a)** Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ by calculating $X(j\omega)$ and $H(j\omega)$, using the convolution property, and inverse transforming.

(i) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$

(ii) $x(t) = e^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$

4.26

i)

$$Y(j\omega) = X(j\omega)\,H(j\omega) = \left[\frac{1}{(2+j\omega)^2}\right]\left[\frac{1}{4+j\omega}\right]$$

$$= \frac{1/4}{4+j\omega} - \frac{1/4}{2+j\omega} + \frac{1/2}{(2+j\omega)^2}$$

Taking the inverse Fourier transform we obtain:

$$y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)$$

ii)

$$Y(j\omega) = X(j\omega)\,H(j\omega)$$

$$= \left[\frac{1}{1+j\omega}\right]\left[\frac{1}{1-j\omega}\right]$$

$$= \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}$$

Taking the inverse Fourier transform, we obtain

$$y(t) = \frac{1}{2}e^{-|t|}$$

4.33. The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.
(b) What is the response of this system if $x(t) = te^{-2t}u(t)$?
(c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

4.33

a) Taking the Fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 2j\omega + 8}$$

using partial fraction expansion, we obtain:

$$H(j\omega) = \frac{1}{j\omega+2} - \frac{1}{j\omega+4}$$

Taking the inverse Fourier transform,

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

---

b)

For the given signal $x(t)$, we have

$$X(j\omega) = \frac{1}{(2+j\omega)^2}$$

Therefore,

$$Y(j\omega) = X(j\omega)\,H(j\omega) = \frac{2}{(-\omega^2 + 2j\omega + 8)(2+j\omega)^2}$$

Using partial fraction expansion, we obtain

$$Y(j\omega) = \frac{1/4}{j\omega+2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

Taking the inverse Fourier transform,

$$y(t) = \frac{1}{4} e^{-2t} u(t) - \frac{1}{2} t e^{-2t} u(t) + t^2 e^{-2t} u(t) - \frac{1}{4} e^{-4t} u(t)$$

c) Taking the fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(-\omega^2-1)}{-\omega^2 + \sqrt{2}\, j\omega + 1}$$

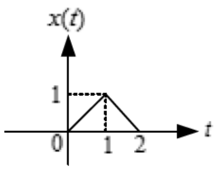
Using partial fraction expansion, we obtain

$$H(j\omega) = 2 + \frac{-\sqrt{2} - 2\sqrt{2}\, j}{j\omega - \frac{-\sqrt{2} + j\sqrt{2}}{2}} + \frac{-\sqrt{2} + 2\sqrt{2}\, j}{j\omega - \frac{-\sqrt{2} - j\sqrt{2}}{2}}$$

Taking the inverse Fourier transform:

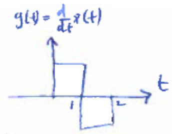
$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)\, e^{-(1+j)t/\sqrt{2}} u(t) - \sqrt{2}\,(1-2j)\, e^{-(1-j)t/\sqrt{2}} u(t)$$

Us the Fourier Transform properties (shifting and differentiation) to find the Fourier transform of $y(t)$ **(50%)**



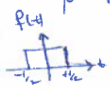
$$x(t-t_0) \xleftrightarrow{F} e^{-j\omega t_0} X(j\omega)$$
$$\frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(j\omega)$$

way # 4



shifting & differentiation properties

Assume



$\Rightarrow F(j\omega) = \frac{\sin \omega/2}{\omega/2}$

$$g(t) = f(t-1/2) - f(t-3/2)$$
$$G(j\omega) = e^{-j\omega/2} \frac{\sin \omega/2}{\omega/2} - e^{-j3\omega/2} \frac{\sin \omega/2}{\omega/2}$$
$$G(j\omega) = \frac{\sin \omega/2}{\omega/2} e^{-j\omega} \left[e^{j\omega/2} - e^{-j\omega/2} \right]$$
$$g(t) = \frac{d\, x(t)}{dt} \Longleftrightarrow G(j\omega) = j\omega X(j\omega)$$
$$j\omega X(j\omega) = 2j \frac{\sin^2 \omega/2}{\omega/2} e^{-j\omega} \Rightarrow$$
$$\underline{X(j\omega) = \left(\frac{\sin(\omega/2)}{\omega/2}\right)^2 e^{-j\omega}} + \pi \cancel{X(0)} \delta(\omega)$$
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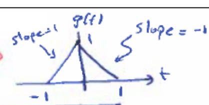
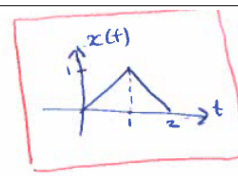
Other Solutions

Now use g(t) as



so x(t) is just a shifted function of g(t) $\boxed{x(t) = g(t-1)}$



Now we can compute Fourier in different ways.

$\boxed{\text{way\#1}}$

integration by part

$$g(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ -t+1 & 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases} \Rightarrow G(j\omega) = \int_{-1}^{0} (t+1) e^{-j\omega t} dt + \int_{0}^{1} (-t+1) e^{-j\omega t} dt$$

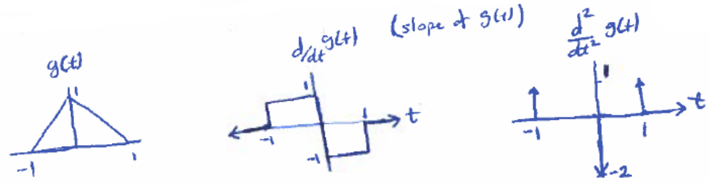
$$G(j\omega) = \left[(t+1) \frac{e^{-j\omega t}}{(-j\omega)} - \int 1 \frac{e^{-j\omega t}}{(-j\omega)} dt \right]_{-1}^{0} + \left[(-t+1) \frac{e^{-j\omega t}}{(-j\omega)} - \int -1 \frac{e^{-j\omega t}}{(-j\omega)} dt \right]_{0}^{1}$$

$$= \left[-\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{e^{j\omega}}{\omega^2} \right] + \left[-\frac{e^{-j\omega}}{\omega^2} + \frac{1}{j\omega} + \frac{1}{\omega^2} \right] = \frac{2}{\omega^2} [1 - \cos \omega]$$

$$G(j\omega) = \frac{2}{\omega^2} \cdot 2 \sin^2 \frac{\omega}{2} = \left(\frac{\sin \omega/2}{\omega/2} \right)^2$$

$$X(j\omega) = G(j\omega) e^{-j\omega} = \left(\frac{\sin \omega/2}{\omega/2} \right)^2 e^{-j\omega}$$

$\boxed{\text{way \#2}}$



$$\frac{d^2}{dt^2} g(t) = \delta(t+1) - 2\delta(t) + \delta(t-1)$$

Take Fourier

$$\underset{(-\omega^2)}{(j\omega)^2} G(j\omega) = e^{j\omega} - 2 + e^{-j\omega} \Rightarrow G(j\omega) = \frac{1}{-\omega^2}\left[2\cos\omega - 2\right] + \pi \cancel{G(0)}^{\,0}\,\delta(\omega)$$

$$\frac{2}{-\omega^2}\left[\cos\omega - 1\right]$$

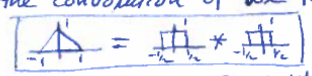
$$G(j\omega) = \frac{4}{\omega^2}\sin^2\frac{\omega}{2} = \left(\frac{\sin\omega/2}{\omega/2}\right)^2$$

$$X(j\omega) = G(j\omega)e^{-j\omega} = \left(\frac{\sin\omega/2}{\omega/2}\right)^2 e^{-j\omega} \quad \#$$

way # 3

Using Convolution property

Triangle function is actually the convolution of ~~box~~ box function with itself.



$$box(t) = \begin{cases} 1 & |t| \leq 0.5 \\ 0 & \text{otherwise} \end{cases} \Rightarrow Box(j\omega) = \frac{2\sin\omega/2}{\omega} = \frac{\sin\omega/2}{\omega/2}$$

$$box(t) * box(t) \Longleftrightarrow \underbrace{Box(j\omega)\cdot Box(j\omega)}_{G(j\omega)} = \left(\frac{\sin\omega/2}{\omega/2}\right)^2$$

$$X(j\omega) = G(j\omega)e^{-j\omega} = \left(\frac{\sin(\omega/2)}{\omega/2}\right)^2 e^{-j\omega}$$

4.31. **(a)** Show that the three LTI systems with impulse responses

$$h_1(t) = u(t),$$

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t),$$

and

$$h_3(t) = 2te^{-t}u(t)$$

all have the same response to $x(t) = \cos t$.

(b) Find the impulse response of another LTI system with the same response to $\cos t$.

This problem illustrates the fact that the response to $\cos t$ cannot be used to specify an LTI system uniquely.

4.32. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

(a) $x_1(t) = \cos(6t + \frac{\pi}{2})$

(b) $x_2(t) = \sum_{k=0}^{\infty}(\frac{1}{2})^k \sin(3kt)$

(c) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)}$

(d) $x_4(t) = (\frac{\sin 2t}{\pi t})^2$

4.33. The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.

(b) What is the response of this system if $x(t) = te^{-2t}u(t)$?

(c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

4.34. A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

(a) Take Fourier Transform for both sides.

$$-\omega^2 Y(j\omega) + 6j\omega Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 6j\omega + 8}$$

using partial Fraction Expansion, we obtain

$$H(j\omega) = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}$$

Taking the inverse of Fourier Transform.

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

(b) For given signal $x(t)$, we have,

$$X(j\omega) = \frac{1}{(2+j\omega)^2}$$

Therefore,

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{2}{(-\omega^2 + 6j\omega + 8)} \frac{1}{(2+j\omega)^2}$$

Using partial fraction expansion, we obtain,

$$Y(j\omega) = \frac{1/4}{j\omega+2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

Taking the inverse Fourier Transform.

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t} + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$

(c) Taking the Fourier Transform of both sides.

$$-\omega^2 Y(j\omega) + \sqrt{2} j\omega\, Y(j\omega) + Y(j\omega) = -2\omega^2 X(j\omega) - 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{-2\omega^2 - 2}{-\omega^2 + \sqrt{2} j\omega + 1}$$

Using partial fraction expansion.

$$H(j\omega) = 2 + \frac{-\sqrt{2} - 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}+j\sqrt{2}}{2}} + \frac{-\sqrt{2} + 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}-j\sqrt{2}}{2}}$$

Taking the inverse of Fourier Transform.

$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)e^{-(1+j)t/\sqrt{2}}u(t) - \sqrt{2}(1-2j)e^{-(1-j)t/\sqrt{2}}u(t)$$

#

ADVANCED PROBLEMS

4.37. Consider the signal $x(t)$ in Figure P4.37.
(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.
(b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

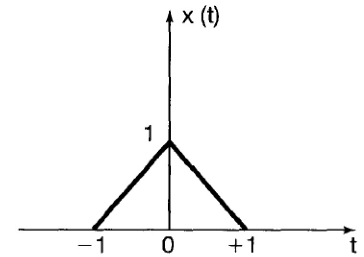


Figure P4.37

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

(a) Note that

$$x(t) = x_1(t) * x_1(t),$$

where

$$x_1(t) = \begin{cases} 1, & |\omega| < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Also, the Fourier transform $X_1(j\omega)$ of $x_1(t)$ is

$$X_1(j\omega) = 2\frac{\sin(\omega/2)}{\omega}.$$

Using the convolution property we have

$$X(j\omega) = X_1(j\omega)X_1(j\omega) = \left[2\frac{\sin(\omega/2)}{\omega}\right]^2.$$

(b) The signal $\tilde{x}(t)$ is as shown in Figure S4.37

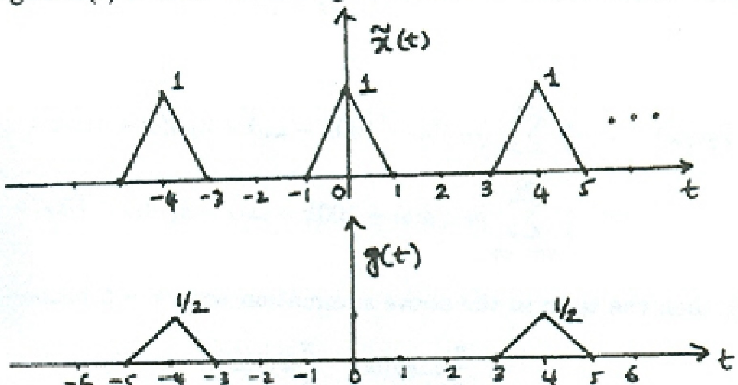


Figure S4.37

(c) One possible choice of $g(t)$ is as shown in Figure S4.37.