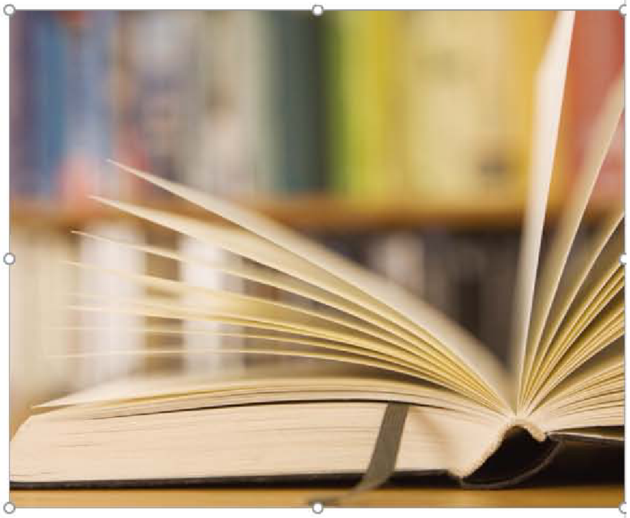
TUTORIAL
ELG3125:SIGNAL AND SYSTEM ANALYSIS



Chapter (3): Fourier Series Representation of Period Signals

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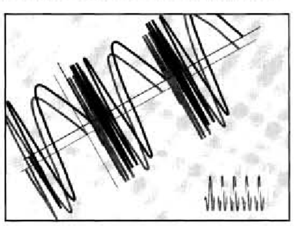
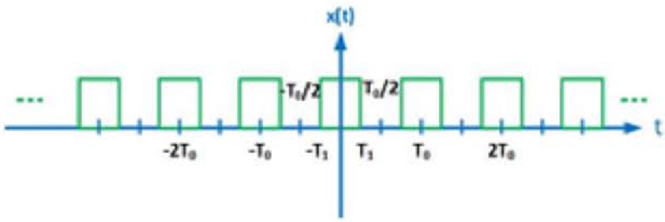
FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS



EXERCISE'S CONTINENTS

- Fourier Series
- Properties of Continuous-Time Fourier Series
- Properties of Discrete-Time Fourier Series.

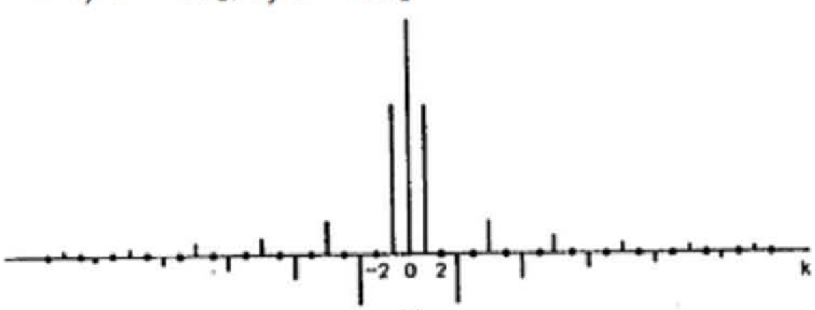
Example: Fourier Series of periodic square wave



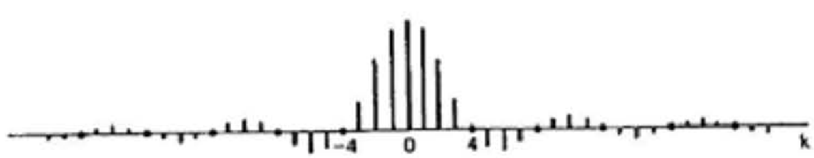
- $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 \leq |t| \leq T/2 \end{cases}$
- dc gain: $a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt = \frac{2T_1}{T}$
- $k \neq 0$: $a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}dt =$
$\frac{-1}{jk\omega_0 T}e^{-jk\omega_0 t}|_{-T_1}^{T_1} = \frac{sink\omega_0 T_1}{k\pi}$

Example Cont'd

- a) $T = 4T_1$, b) $T = 8T_1$



(a)



(b)

3.24. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

(a) We have

$$a_0 = \frac{1}{2}\int_0^1 t dt + \frac{1}{2}\int_1^2 (2-t) dt = 1/2.$$

(b) The signal $g(t) = dx(t)/dt$ is as shown in Figure S3.24.

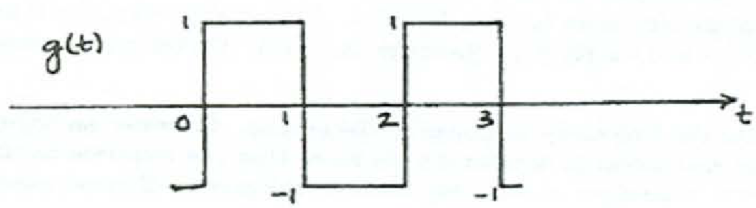


Figure S3.24

The FS coefficients b_k of $g(t)$ may be found as follows:

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

and

$$\begin{aligned} b_k &= \frac{1}{2}\int_0^1 e^{-j\pi kt} dt - \frac{1}{2}\int_1^2 e^{-j\pi kt} dt \\ &= \frac{1}{j\pi k}[1 - e^{-j\pi k}]. \end{aligned}$$

(c) Note that

$$g(t) = \frac{dx(t)}{dt} \overset{FS}{\longleftrightarrow} b_k = jk\pi a_k.$$

Therefore,

$$a_k = \frac{1}{jk\pi} b_k = -\frac{1}{\pi^2 k^2}\{1 - e^{-j\pi k}\}.$$

Let $x(t) = \begin{cases} t & 0 \le t \le 1 \\ -t & -1 \le t \le 0 \end{cases}$ be a periodic signal with fundamental period of $T = 2$ and Fourier series coefficients a_k.

(a) Sketch the waveforms of $x(t)$ and $dx(t)/dt$.

(b) Calculate a_0.

(c) Determine the Fourier series representation of $g(t) = dx(t)/dt$.

(d) Using the results from Part (c) and the property of continuous-time Fourier series to determine the Fourier series coefficients of $x(t)$.

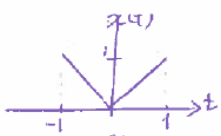
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$$

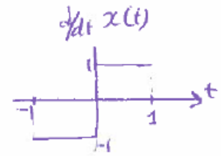
Differentiation $\frac{dx(t)}{dt}$ $\quad jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$

Time Shifting $x(t - t_0)$ $\quad a_k e^{-jk\omega_0 t_0}$

(a)



(1)



(2)

(b) we have for $x(t)$

$$a_0 = \frac{1}{2}\int_0^1 t\,dt + \frac{1}{2}\int_{-1}^0 -t\,dt = 1/2$$

(c) The signal $g(t) = \frac{d}{dt}x(t)$ is as shown in (2). The FS coefficients b_k may be found as.

$$b_0 = \frac{1}{2}\int_{-1}^0 -1 \cdot dt + \frac{1}{2}\int_0^1 dt = 0$$

and
$$b_k = 1/2\int_{-1}^0 -e^{-j\pi k t} dt + \frac{1}{2}\int_0^1 e^{-j\pi k t} dt = -1/2 \left.\frac{e^{-j\pi k t}}{-j\pi k}\right|_{-1}^{0} + 1/2 \left.\frac{e^{-j\pi k t}}{-j\pi k}\right|_{0}^{1}$$

$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

(d) Not that
$$g(t) = \frac{d\,x(t)}{dt} \overset{FS}{\longleftrightarrow} b_k = jk\pi a_k.$$

Therefore
$$a_k = \frac{1}{jk\pi} b_k = -\frac{1}{\pi^2 k^2}\{1 - e^{-j\pi k}\}.$$

3.31. Let

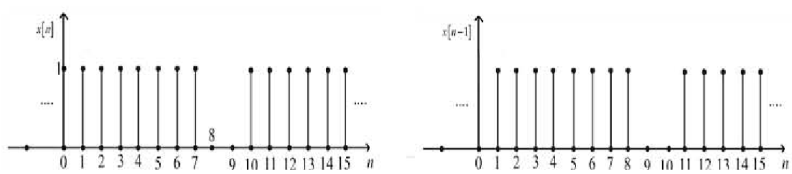
$$x[n] = \begin{cases} 1, & 0 \leq n \leq 7 \\ 0, & 8 \leq n \leq 9 \end{cases}$$

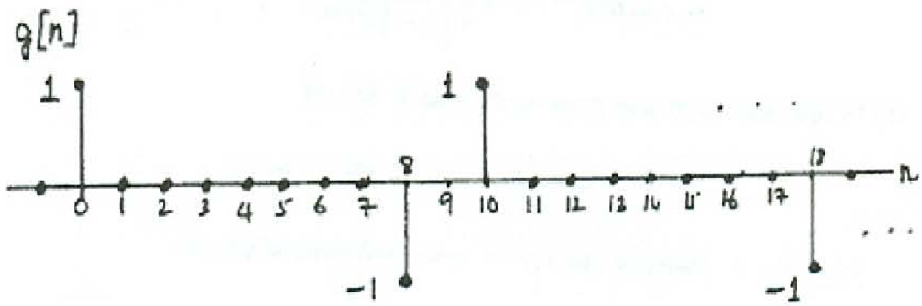
be a periodic signal with fundamental period $N = 10$ and Fourier series coefficients a_k. Also, let

$$g[n] = x[n] - x[n-1].$$

(a) Show that $g[n]$ has a fundamental period of 10.
(b) Determine the Fourier series coefficients of $g[n]$.

(a) $g[n]$ is as shown in Figure S3.31. Clearly, $g[n]$ has a fundamental period of 10.





(b)

$$b_k = \frac{1}{10}\left[e^{-j\frac{2\pi}{10}(0)k} - e^{-jk\frac{2\pi}{10}(8)}\right] \quad = \frac{1}{10}\left[1 - e^{-jk\frac{2\pi}{10}(8)}\right]$$

$$= \frac{1}{10}\left[1 - e^{-j\frac{2\pi}{10}8k}\right]$$

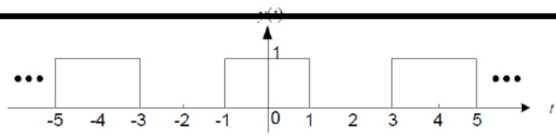
Thus Fourier coefficient of $g[n]$ is $\boxed{\frac{1}{10}\left[1 - e^{-j\frac{2\pi}{10}8k}\right]}$.

The Fourier series coefficiennts of $g[n]$ are $b_k = (1/10)[1 - e^{-j(2\pi/10)8k}]$.

For the periodic signal given below,

(a) Find the Fourier series coefficients of the periodic signal.

(b) Give the expression of the third-order harmonic (that is, $a_k e^{jk\omega_0 t} + a_{-k} e^{-jk\omega_0 t}$ for $k = 3$).



(a) $a_0 = \frac{1}{T}\int_T y(t)\,dt = \frac{1}{4}\int_{-1}^{1} 1\,dt = 1/2$

$a_k = \frac{1}{T}\int_T y(t)\, e^{-jk\omega_0 t}\,dt = \frac{1}{4}\int_{t=-1}^{1} 1\, e^{-jk\omega_0 t}\,dt = \frac{1}{2}\,\frac{\sin \pi k/2}{\pi k/2}$

$\therefore\; a_k = \begin{cases} 1/2 & k = 0 \\ \frac{1}{2}\,\frac{\sin \pi k/2}{\pi k/2} & k \neq 0 \end{cases}$

(b) $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\pi t/2}$

The Third order $k=3$.

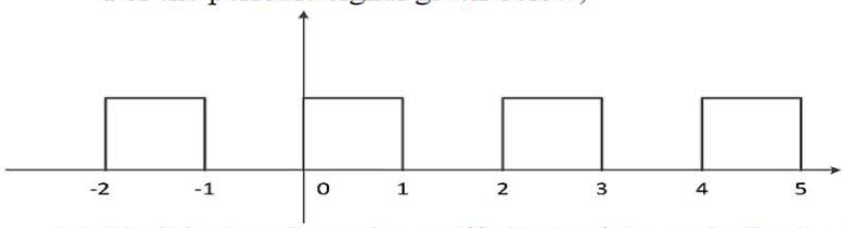
$x(t) = \sum_{k=-3}^{3} a_k e^{jk\pi t/2} =$

$\cdots + -\frac{1}{3\pi} e^{-j\frac{3\pi t}{2}} + 0 + \frac{1}{\pi} e^{-j\frac{\pi}{2}t} + 1/2 + \frac{1}{\pi} e^{j\frac{\pi}{2}t} + 0 - \frac{1}{3\pi} e^{j3\pi t/2} + \cdots$

($k=-3$, $k=-2$, $k=-1$, $k=0$, $k=1$, $k=2$, $k=3$)

$\left(a_k e^{jk\omega_0 t} + a_{-k} e^{-jk\omega_0 t} \text{ for } k=3\right) = \frac{1}{3\pi} e^{-j\frac{3\pi t}{2}} - \frac{1}{3\pi} e^{j\frac{3\pi t}{2}}$ #

For the periodic signal given below,



(a) Find the Fourier series coefficients of the periodic signal.

(b) Plot the frequency spectrum (absolute values of a_k) the first three harmonics.

Here we need to generalize from period one to period T. When T is given, the Fourier coefficients are given by.

$$a_k = \frac{1}{T}\int_T f(t)e^{-j\frac{2\pi k}{T}t}dt$$

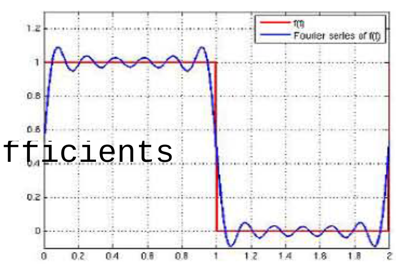
f(t) is periodic with period T = 2. From the Fourier analysis formula, the 0th coefficient can be calculated by,

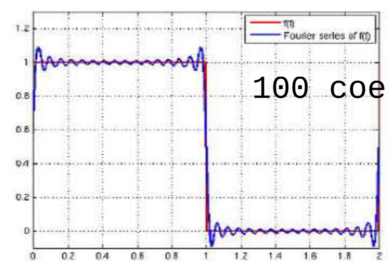
$$a_0 = \frac{1}{T}\int_T f(t)dt = \frac{1}{2}\int_0^2 f(t)dt = \frac{1}{2}\int_0^1 dt = \frac{1}{2}$$

Other coefficients are calculated as,

$$\begin{aligned} a_k &= \frac{1}{T}\int_T f(t)dt = \frac{1}{2}\int_0^2 f(t)e^{-j\frac{2\pi k}{T}t}dt = \frac{1}{2}\int_0^1 e^{-j\pi k t}dt \\ &= \frac{1}{-j2\pi k}\int_0^1 e^{-j\pi k t}d\left(-j\pi k t\right) \\ &= \frac{1}{-j2\pi k}\left(e^{-j\pi k t}\right)\Big|_0^1 = \frac{1}{-j2\pi k}\left(e^{-j\pi k}-1\right) \end{aligned}$$

20 coefficients





100 coefficients

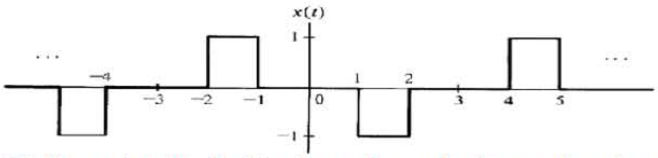
The magnitude of each coefficient can be calculated as

$$|a_k| = \left|\frac{1}{-j2\pi k}\left(e^{-j\pi k}-1\right)\right| = \frac{1}{\pi k} \quad \text{for k odd,}$$

$$= 0 \quad \text{for k even}$$

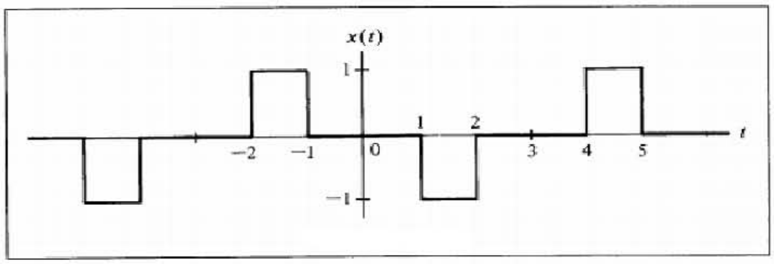
$$a_0 = \frac{1}{2}$$

2. For the periodic signal given below,



(a) By evaluating the Fourier series analysis equation, determine the Fourier series for x(t).

(b) Find $|a_k|$ when k=1.



Note that the period is $T_0 = 6$. Fourier coefficients are given by

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

We take $\omega_0 = 2\pi/T_0 = \pi/3$. Choosing the period of integration as -3 to 3, we have

$$\begin{aligned} a_k &= \frac{1}{6}\int_{-2}^{-1} e^{-jk(\pi/3)t}\,dt - \frac{1}{6}\int_{1}^{2} e^{-jk(\pi/3)t}\,dt \\ &= \frac{1}{6}\frac{1}{-jk(\pi/3)}e^{-jk(\pi/3)t}\Big|_{-2}^{-1} - \frac{1}{6}\frac{1}{-jk(\pi/3)}e^{-jk(\pi/3)t}\Big|_{1}^{2} \\ &= \frac{1}{-j2\pi k}[e^{+j(\pi/3)k} - e^{+j(2\pi/3)k} - e^{-j(2\pi/3)k} + e^{-j(\pi/3)k}] \\ &= \frac{\cos(2\pi/3)k}{j\pi k} - \frac{\cos(\pi/3)k}{j\pi k} \end{aligned}$$

Therefore,

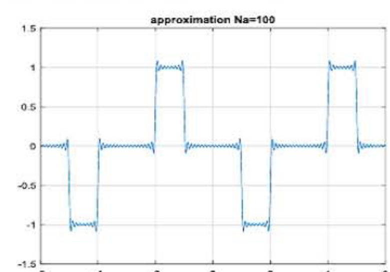
$$x(t) = \sum_k a_k e^{jk\omega_0 t}, \qquad \omega_0 = \frac{\pi}{3}$$

and

$$a_k = \frac{\cos(2\pi/3)k - \cos(\pi/3)k}{j\pi k}$$

a_0 =0, as can be determined by applying

$$a_0 = (1/T_0)\int_{T_0} x(t)dt.$$



(b) $|a_k| = 0.3183$ when k=1

3.30. Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right), \qquad y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right), \qquad z[n] = x[n]y[n].$$

(a) Determine the Fourier series coefficients of $x[n]$.

(b) Determine the Fourier series coefficients of $y[n]$.

(a)

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right)$$

$$= 1 + \frac{e^{j\frac{2\pi}{6}n} + e^{-j\frac{2\pi}{6}n}}{2}$$

$$x[n] = \frac{1}{2}e^{j(-1)\frac{2\pi}{6}n} + 1e^{j(0)\frac{2\pi}{6}n} + \frac{1}{2}e^{j(1)\frac{2\pi}{6}n} \quad \text{...... (1)}$$

The Fourier series expansion of $x[n]$ is

$$x[n] = \sum_{n=\langle N\rangle} a_k e^{jk\omega_0 n} \quad \text{...... (2)}$$

Comparing equations (1) and (2) we get

$a_0 = 1$,

$a_1 = \frac{1}{2}$, and

$a_{-1} = \frac{1}{2}$

Therefore, the Fourier series coefficients are

$$\boxed{a_0 = 1,\ a_1 = a_{-1} = \frac{1}{2}}$$

(b)

$$y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)$$

$$= \frac{e^{j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)} - e^{-j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)}}{2j}$$

$$= \frac{1}{2j}e^{j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)} - \frac{1}{2j}e^{-j\left(\frac{2\pi}{6}n+\frac{\pi}{4}\right)}$$

$$y[n] = \left(\frac{e^{j\frac{\pi}{4}}}{2j}\right)e^{j(1)\frac{2\pi}{6}n} - \left(\frac{e^{-j\frac{\pi}{4}}}{2j}\right)e^{j(-1)\frac{2\pi}{6}n} \quad \text{...... (3)}$$

The Fourier series expansion of $y[n]$ is

$$y[n] = \sum_{n=\langle N\rangle} b_k e^{jk\omega_0 n} \quad \text{...... (4)}$$

Comparing equations (3) and (4) we get

$b_1 = \frac{e^{\frac{j\pi}{4}}}{2j}$ and

$b_{-1} = -\frac{e^{-\frac{j\pi}{4}}}{2j}$

Therefore, the Fourier series coefficients are

$$\boxed{b_1 = \frac{e^{\frac{j\pi}{4}}}{2j} \text{ and } b_{-1} = -\frac{e^{-\frac{j\pi}{4}}}{2j}}$$

(c) Use the results of parts (a) and (b), along with the multiplication property of the discrete-time Fourier series, to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.

(c) Given

$$z[n] = x[n]y[n]$$

From the multiplication property of the discrete-time Fourier series

$$x[n]y[n] \xleftrightarrow{F\cdot S} \sum_{l=\langle N\rangle} a_l b_{k-l}$$

$$z[n] \xleftrightarrow{F\cdot S} \sum_{l=\langle N\rangle} a_l b_{k-l}$$

Therefore,

$$\begin{aligned} c_k &= \sum_{l=\langle N\rangle} a_l b_{k-l} \\ &= a_{-1}b_{k-(-1)} + a_0 b_k + a_1 b_{k-1} \\ &= a_{-1}b_{k+1} + a_0 b_k + a_1 b_{k-1} \end{aligned}$$

$$c_k = a_{-1}b_{k+1} + a_0 b_k + a_1 b_{k-1} \quad \text{...... (5)}$$

Put $k=0$ in equation (5):

$$\begin{aligned} c_0 &= a_{-1}b_1 + a_0 b_0 + a_1 b_{-1} \\ &= \frac{1}{2}\left(\frac{e^{\frac{j\pi}{4}}}{2j}\right) + (1)(0) + \frac{1}{2}\left(-\frac{e^{-\frac{j\pi}{4}}}{2j}\right) \\ &= \frac{1}{2}\left(\frac{e^{\frac{j\pi}{4}} - e^{-\frac{j\pi}{4}}}{2j}\right) \end{aligned}$$

$$c_0 = \frac{1}{2}\sin\left(\frac{\pi}{4}\right)$$

Put $k=1$ in equation (5):

$$\begin{aligned} c_1 &= a_{-1}b_2 + a_0 b_1 + a_1 b_0 \\ &= \frac{1}{2}(0) + (1)\left(\frac{e^{\frac{j\pi}{4}}}{2j}\right) + \frac{1}{2}(0) \end{aligned}$$

$$c_1 = \frac{e^{\frac{j\pi}{4}}}{2j}$$

Put $k=2$ in equation (5):

$$\begin{aligned} c_2 &= a_{-1}b_3 + a_0 b_2 + a_1 b_1 \\ &= \frac{1}{2}(0) + (1)(0) + \frac{1}{2}\frac{e^{\frac{j\pi}{4}}}{2j} \end{aligned}$$

$$c_2 = \frac{e^{\frac{j\pi}{4}}}{4j}$$

Put $k=-1$ in equation (5):

$$\begin{aligned} c_{-1} &= a_{-1}b_0 + a_0 b_{-1} + a_1 b_{-2} \\ &= \frac{1}{2}(0) + (1)\left(-\frac{e^{-\frac{j\pi}{4}}}{2j}\right) + \frac{1}{2}(0) \end{aligned}$$

$$c_{-1} = -\frac{e^{-\frac{j\pi}{4}}}{2j}$$

Put $k=-2$ in equation (5):

$$\begin{aligned} c_{-2} &= a_{-1}b_{-1} + a_0 b_{-2} + a_1 b_{-3} \\ &= \frac{1}{2}\left(-\frac{e^{-\frac{j\pi}{4}}}{2j}\right) + (1)(0) + \frac{1}{2}(0) \end{aligned}$$

$$c_{-2} = -\frac{e^{-\frac{j\pi}{4}}}{4j}$$

And the remaining all coefficients are zeros.

Therefore, the Fourier series coefficients are

$$\boxed{c_0 = \frac{1}{2}\sin\left(\frac{\pi}{4}\right), c_1 = \frac{e^{\frac{j\pi}{4}}}{2j}, c_2 = \frac{e^{\frac{j\pi}{4}}}{4j}, c_{-1} = -\frac{e^{-\frac{j\pi}{4}}}{2j}, \text{ and } c_{-2} = -\frac{e^{-\frac{j\pi}{4}}}{4j}}$$

3.36. Consider a causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] = x[n]$$

Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sin(\frac{3\pi}{4}n)$

(b) $x[n] = \cos(\frac{\pi}{4}n) + 2\cos(\frac{\pi}{2}n)$

---

(1)

Consider the following difference equation:

$$y[n]-\frac{1}{4}y[n-1]=x[n]$$

Apply the Fourier transform on both the sides.

$$Y(e^{j\omega})-\frac{1}{4}e^{-j\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$\left(1-\frac{1}{4}e^{-j\omega}\right)Y(e^{j\omega})=X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$H(e^{j\omega})=\frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

If $x[n]$ is a periodic signal, then its Fourier series is given by,

$$x[n]=\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

If the input $x[n]$ is applied to an LTI system with impulse response $h[n]$, then the output $y[n]$ would also be periodic with same period.

The expression of the output is,

$$y[n]=\sum_{k=\langle N\rangle} a_k H\left(e^{jk\left(\frac{2\pi}{N}\right)}\right)e^{jk\left(\frac{2\pi}{N}\right)n} \quad \ldots\ldots (1)$$

Substitute 8 for N in equation (1) and obtain the output response of the system $y[n]$.

$$y[n]=\sum_{k=8} a_k H\left(e^{jk\left(\frac{2\pi}{8}\right)}\right)e^{jk\left(\frac{2\pi}{8}\right)n}$$

$$=a_{-3}H\left(e^{j(-3)\left(\frac{2\pi}{8}\right)}\right)e^{j(-3)\left(\frac{2\pi}{8}\right)n}+a_3H\left(e^{j(3)\left(\frac{2\pi}{8}\right)}\right)e^{j(3)\left(\frac{2\pi}{8}\right)n}$$

$$=\left(-\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{-j(-3)\left(\frac{2\pi}{8}\right)}\right)}e^{j(-3)\left(\frac{2\pi}{8}\right)n}+\left(\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{-j(3)\left(\frac{2\pi}{8}\right)}\right)}e^{j(3)\left(\frac{2\pi}{8}\right)n}$$

$$=\left[\left(-\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{j\left(\frac{3\pi}{4}\right)}\right)}\right]e^{-j\left(\frac{3\pi}{4}\right)n}+\left[\left(\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{-j\left(\frac{3\pi}{4}\right)}\right)}\right]e^{j\left(\frac{3\pi}{4}\right)n}$$

Therefore, the Fourier series representation of $y[n]$ is

$$\boxed{\left[\left(-\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{j\left(\frac{3\pi}{4}\right)}\right)}\right]e^{-j\left(\frac{3\pi}{4}\right)n}+\left[\left(\frac{1}{2j}\right)\frac{1}{\left(1-\frac{1}{4}e^{-j\left(\frac{3\pi}{4}\right)}\right)}\right]e^{j\left(\frac{3\pi}{4}\right)n}}.$$

Substitute 8 for N in equation (1) to obtain the output response.

$$y[n]=\sum_{k=8} a_k H\left(e^{jk\left(\frac{2\pi}{8}\right)}\right)e^{jk\left(\frac{2\pi}{8}\right)n}$$

$$b_1 = a_1H(e^{j\pi/4}) = \frac{1}{2(1-(1/4)e^{-j\pi/4})}, \quad b_{-1} = a_{-1}H(e^{-j\pi/4}) = \frac{1}{2(1-(1/4)e^{j\pi/4})},$$

$$b_2 = a_2H(e^{j\pi/2}) = \frac{1}{(1-(1/4)e^{-j\pi/2})}, \quad b_{-2} = a_{-2}H(e^{-j\pi/2}) = \frac{1}{(1-(1/4)e^{j\pi/2})}.$$

(2)

(a) Consider the expression of the input.

$$x[n]=\sin\left(\frac{3\pi}{4}n\right)$$
$$=\sin\left(\frac{2\pi}{8}3n\right)$$
$$=\frac{e^{j\frac{2\pi}{8}3n}-e^{-j\frac{2\pi}{8}3n}}{2j}$$

Here, the value of N is 8, so,

$$x[n]=\frac{1}{2j}e^{j\frac{2\pi}{8}3n}+\left(-\frac{1}{2j}\right)e^{-j\frac{2\pi}{8}3n} \quad \ldots\ldots (2)$$

Write down the general equation for $x[n]$.

$$x[n]=\sum_{\langle N\rangle} a_k e^{jk\omega_0 n}$$
$$=\sum_{\langle N\rangle} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

Compare equation (2) with the general form of $x[n]$.

The Fourier coefficients of $x[n]$ are,

$$a_3=\frac{1}{2j}$$
$$a_{-3}=-\frac{1}{2j}$$

(b)

Consider the expression of the input.

$$x[n]=\cos\left(\frac{\pi}{4}n\right)+2\cos\left(\frac{\pi}{2}n\right)$$
$$=\cos\left(\frac{2\pi}{8}n\right)+2\cos\left(\frac{2\pi}{8}2n\right)$$
$$=\frac{e^{j\frac{2\pi}{8}n}+e^{-j\frac{2\pi}{8}n}}{2}+2\left(\frac{e^{j\frac{2\pi}{8}(2n)}+e^{-j\frac{2\pi}{8}(2n)}}{2}\right)$$

$$x[n]=e^{-j\frac{2\pi}{8}(2n)}+\frac{1}{2}e^{-j\frac{2\pi}{8}n}+\frac{1}{2}e^{j\frac{2\pi}{8}n}+e^{j\frac{2\pi}{8}(2n)} \quad \ldots\ldots (3)$$

Write down the general equation for $x[n]$.

$$x[n]=\sum_{\langle N\rangle} a_k e^{jk\omega_0 n}$$
$$=\sum_{\langle N\rangle} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

Compare equation (3) with the general form of $x[n]$.

The Fourier coefficients of $x[n]$ are,

$$a_1=a_{-1}=\frac{1}{2}$$
$$a_2=a_{-2}=1$$