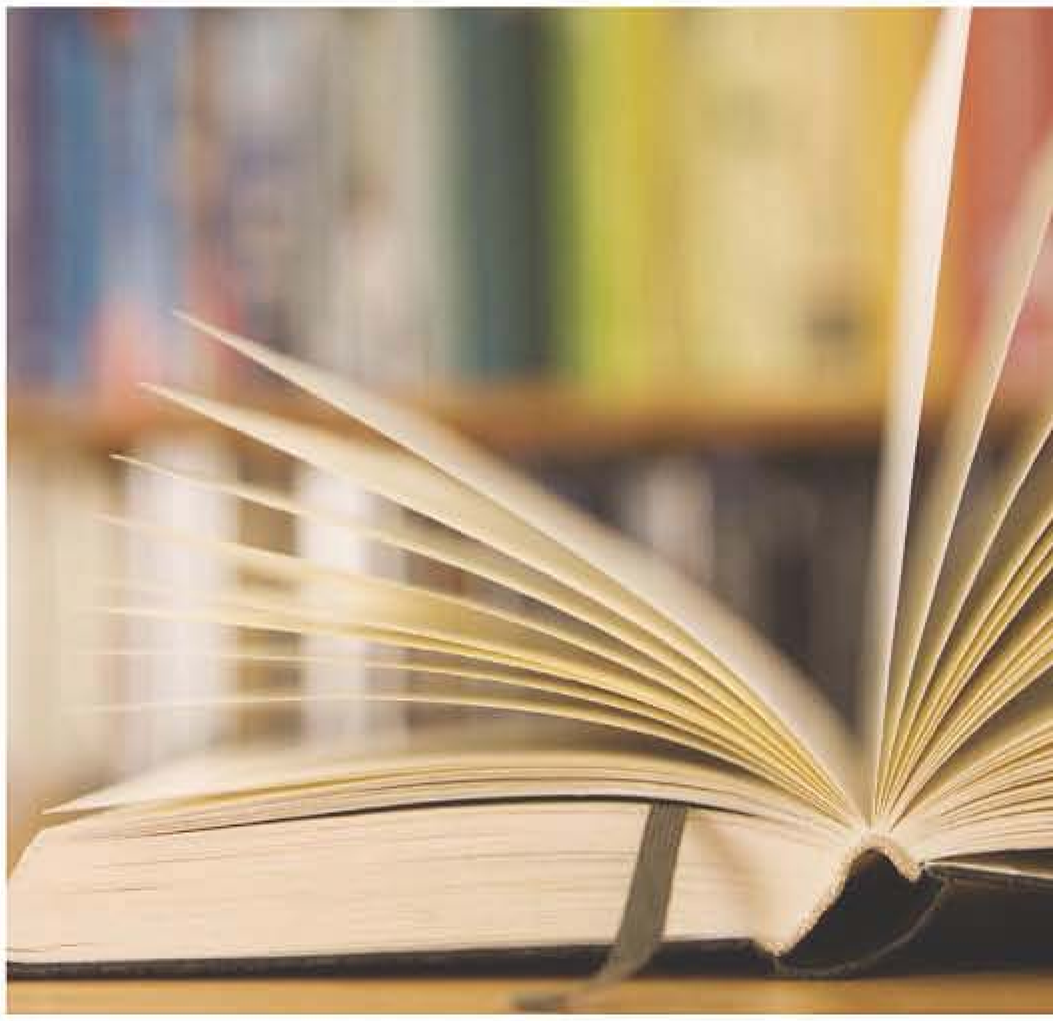
TUTORIAL
ELG3125B:SIGNAL AND SYSTEM ANALYSIS

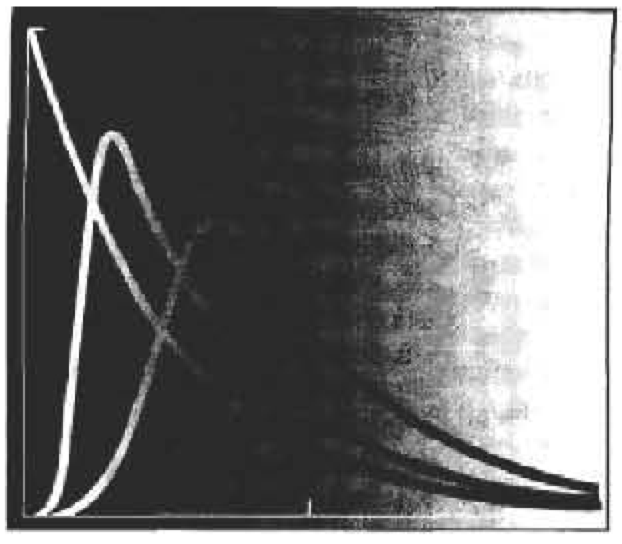
Chapter (2)

By: Hitham Jleed



http://www.site.uottawa.ca/~hjlee103/

2
LINEAR TIME-INVARIANT SYSTEMS



EXERCISE'S CONTINENTS

- ***PROPERTIES OF LINEAR TIME-INVARIANT SYSTEMS***
- **CONVOLUTION.**
    - **Graphical Evaluation.**
    - ***Closed-form Convulsion***
- **Causal LTI Systems Described by Differential and Difference Equations**

LTI - Systems

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h_k[n].$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

We denote convolution as $y[n] = x[n] * h[n]$.

- Equivalent form: Letting $m = n - k$, we can show that

$$\sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{m=-\infty}^{\infty} x[n-m]h[m] = \sum_{k=-\infty}^{\infty} x[n-k]h[k].$$

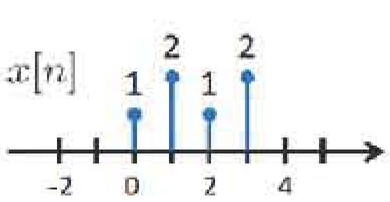
How to Evaluate Convolution?

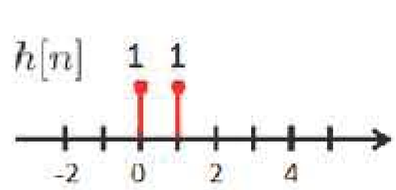
To evaluate convolution, there are three basic steps:

1. Flip
2. Shift
3. Multiply and Add

Example:

Consider the signal x[n] and the impulse response h[n] shown below.





Let's compute the output $y[n]$ one by one. First, consider $y[0]$:

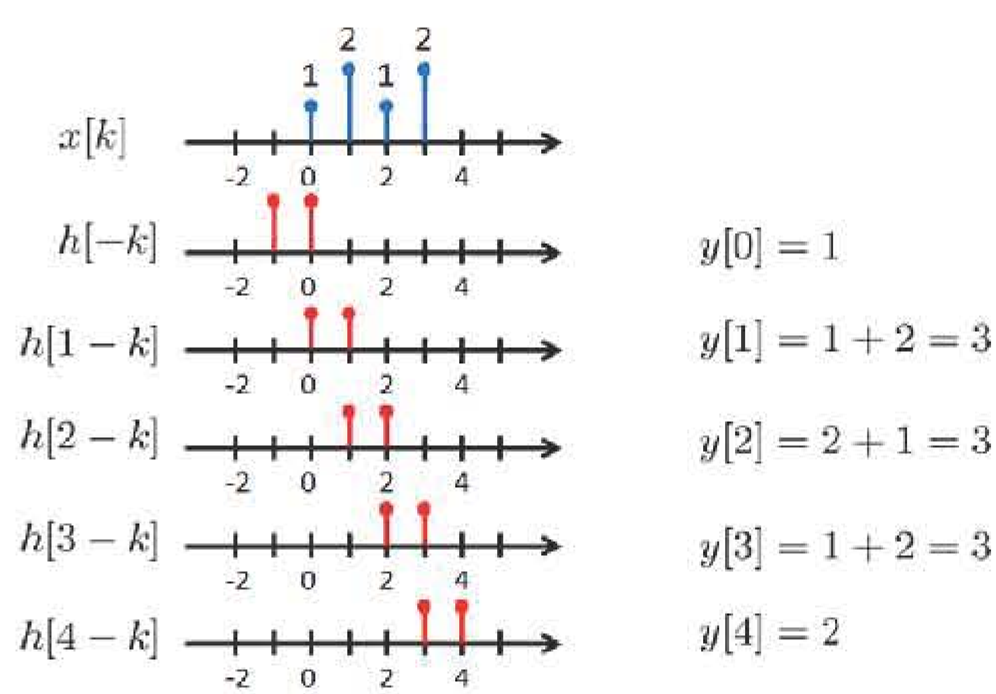
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = \sum_{k=-\infty}^{\infty} x[k]h[-k] = 1.$$

Note that $h[-k]$ is the flipped version of $h[k]$, and $\sum_{k=-\infty}^{\infty} x[k]h[-k]$ is the multiply-add between $x[k]$ and $h[-k]$.

To calculate $y[1]$, we flip $h[k]$ to get $h[-k]$, shift $h[-k]$ go get $h[1-k]$, and multiply-add to get $\sum_{k=-\infty}^{\infty} x[k]h[1-k]$. Therefore,

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = 1 \times 1 + 2 \times 1 = 3.$$

Pictorially, the calculation is shown in the figure below.



2.3. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2],$$

$$h[n] = u[n+2].$$

2.3 Let's define the signals $x_1[n] = (1/2)^n u[n]$
and $h_1[n] = u[n]$

we note that

$$x[n] = x_1[n-2] \quad \text{and} \quad h[n] = h_1[n+2]$$

Now

$$y[n] = x[n] * h[n] = x_1[n-2] * h_1[n+2]$$
$$= \sum_{k=-\infty}^{\infty} x_1[k-2]\, h_1[n-k+2]$$

By replacing k with $m+2$ in the above summation, we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m]\, h_1[n-m] = x_1[n] * h_1[n]$$

Using the results of Example 2.3 in the textbook, which states

$$y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha} \quad \text{for } n \geq 0$$

For all n, $\quad y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u[n]$

Set $\alpha = 1/2 \Rightarrow y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$

2.8. Determine and sketch the convolution of the following two signals:

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & \text{elsewhere} \end{cases},$$

$$h(t) = \delta(t+2) + 2\delta(t+1).$$

Using the convolution integral,

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral reduces to

$$x(t) * y(t) = x(t+2) + 2x(t+1)$$

The signals $x(t+2)$ and $2x(t+1)$ are plotted in Figure S2.8.

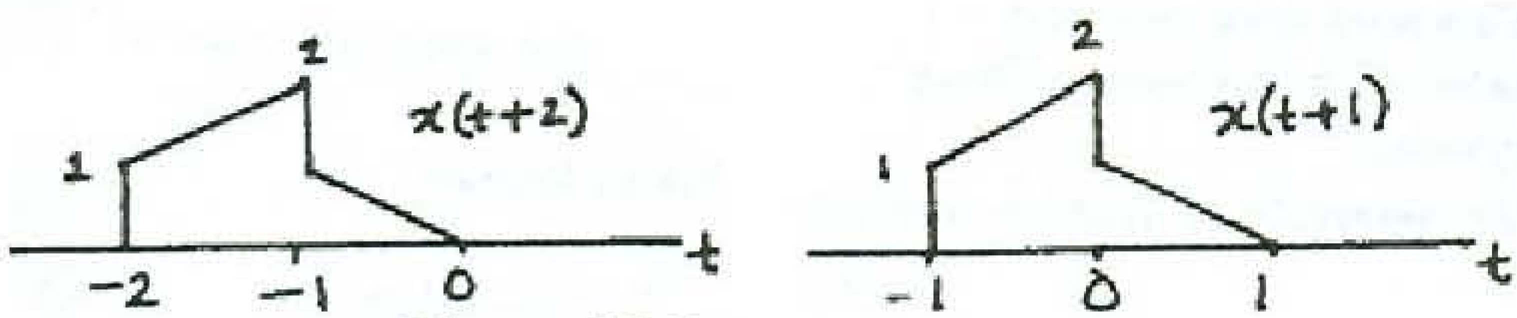
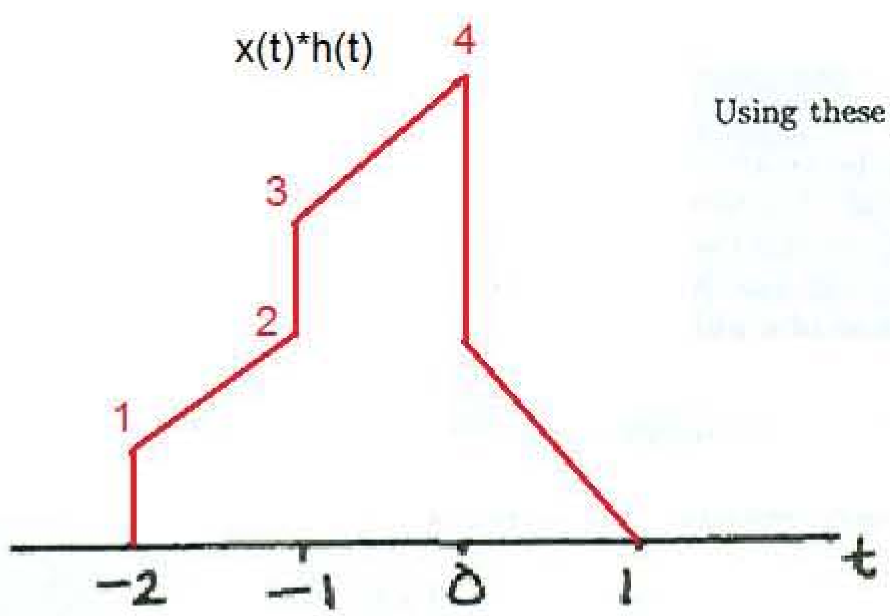


Figure S2.8



Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

2.4. Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases},$$

$$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{otherwise} \end{cases}.$$

We know that

$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure S2.4. From this figure, we see that the above summation reduces to

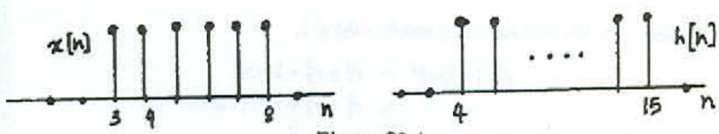


Figure S2.4

x[n] can be written as,

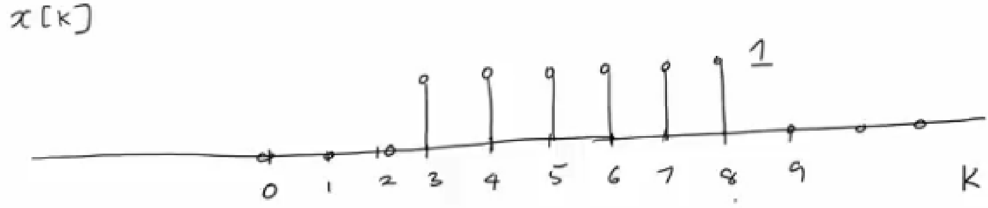
x[n] = δ[n-3]+ δ[n-4]+ δ[n-5]+ δ[n-6]+δ[n-7]+ δ[n-8]

So, y[n] = h[n-3] + h[n-4] + h[n-5] + h[n-6] + h[n-7] + h[n-8]

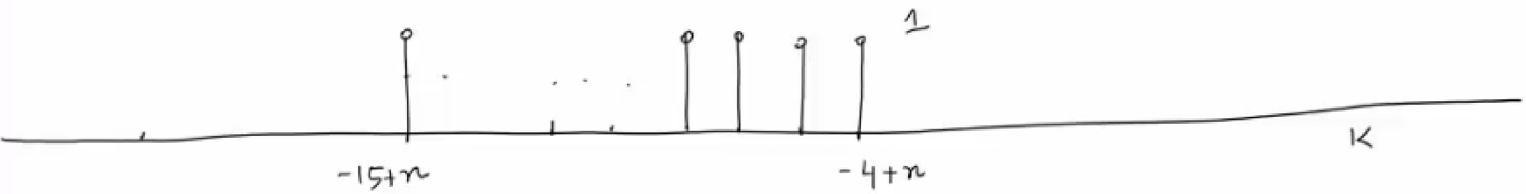
Another solution will be using four cases of summation boundaries, which gives us:

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

To illustrate this step by step, let us fist draw x[k] and h[n-k] and then see the cases for this convolution.



$h[n-k]$ we shift $h[-k]$ to the right by n



Interval 1 $\quad -4+n < 3 \Rightarrow n < 7 \qquad y[n] = 0$

Interval 2 $\quad 3 \leq -4+n \leq 8 \Rightarrow 7 \leq n \leq 12$

$$y[n] = \sum_{k=3}^{-4+n} 1 = n-7+1 = n-6$$

Interval 3

$8 < n-4 \Rightarrow n > 12 \qquad y[n] = \sum_{k=3}^{8} 1 = 6$

$n-15 \leq 3 \Rightarrow n \leq 18$

Interval 4 $\quad 3 < n-15 \leq 8 \Rightarrow 18 < n \leq 23 \qquad y[n] = \sum_{k=n-15}^{8} 1 = 23-n+1 = 24-n$

Interval 5 $\quad n-15 > 8 \qquad n > 23 \qquad y[n] = 0$

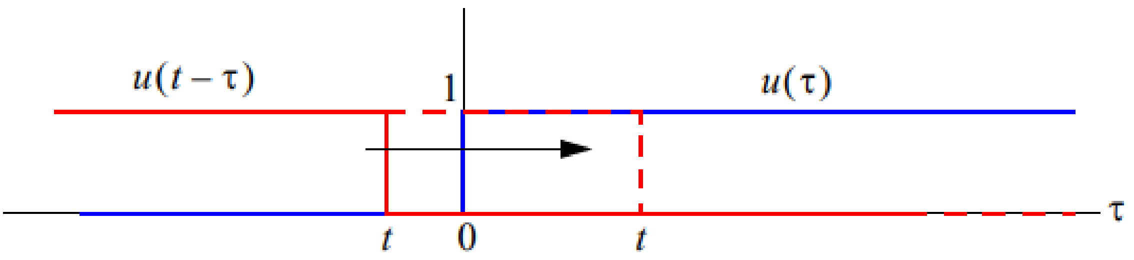
Example of continuous-time convolution

x(t) =u(t) & h(t)=u(t) What is y(t)?

$$y(t) = x(t)*h(t) = u(t)*u(t)$$

Setting up the convolution integral we have

$$y(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau$$



$$y(t) = \begin{cases} 0, & t < 0 \\ \int_0^t d\tau, & t \geq 0 \end{cases}$$

$$= \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

or simply

$$y(t) = tu(t) \equiv r(t),$$

which is known as the *unit ramp*

2.6. Compute and plot the convolution $y[n] = x[n] * h[n]$, where

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1] \quad \text{and} \quad h[n] = u[n-1].$$

From the given information, we have:

$$\begin{aligned} y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \sum_{k=-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1]u[n-k-1] \\ &= \sum_{k=-\infty}^{-1} (\frac{1}{3})^{-k} u[n-k-1] \\ &= \sum_{k=1}^{\infty} (\frac{1}{3})^{k} u[n+k-1] \end{aligned}$$

Replacing k by $p+1$,

$$y[n] = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} u[n+p]$$

For $n \geq 0$ the above equation reduces to,

$$y[n] = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} = \frac{1}{3}\frac{1}{1-\frac{1}{3}} = \frac{1}{2}.$$

For $n < 0$ eq. reduces to,

$$\begin{aligned} y[n] &= \sum_{p=-n}^{\infty} (\frac{1}{3})^{p+1} = (\frac{1}{3})^{-n+1} \sum_{p=0}^{\infty} (\frac{1}{3})^{p} \\ &= (\frac{1}{3})^{-n+1} \frac{1}{1-\frac{1}{3}} = (\frac{1}{3})^{-n} \frac{1}{2} = \frac{3^n}{2} \end{aligned}$$

Therefore,

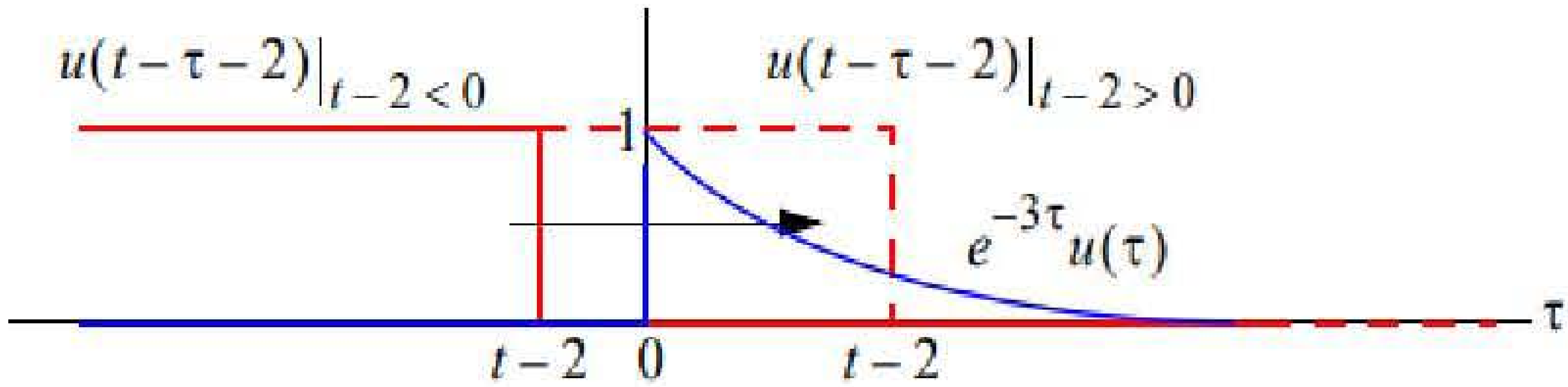
$$y[n] = \begin{cases} (3^n/2), & n < 0 \\ (1/2), & n \geq 0 \end{cases}$$

Consider $x(t) = u(t-2)$ and $h(t) = e^{-3t}u(t)$

We wish to find $y(t) = x(t)*h(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)u(t-\tau-2)d\tau$$

- To evaluate this integral we first need to consider how the step functions in the integrand control the limits of integration

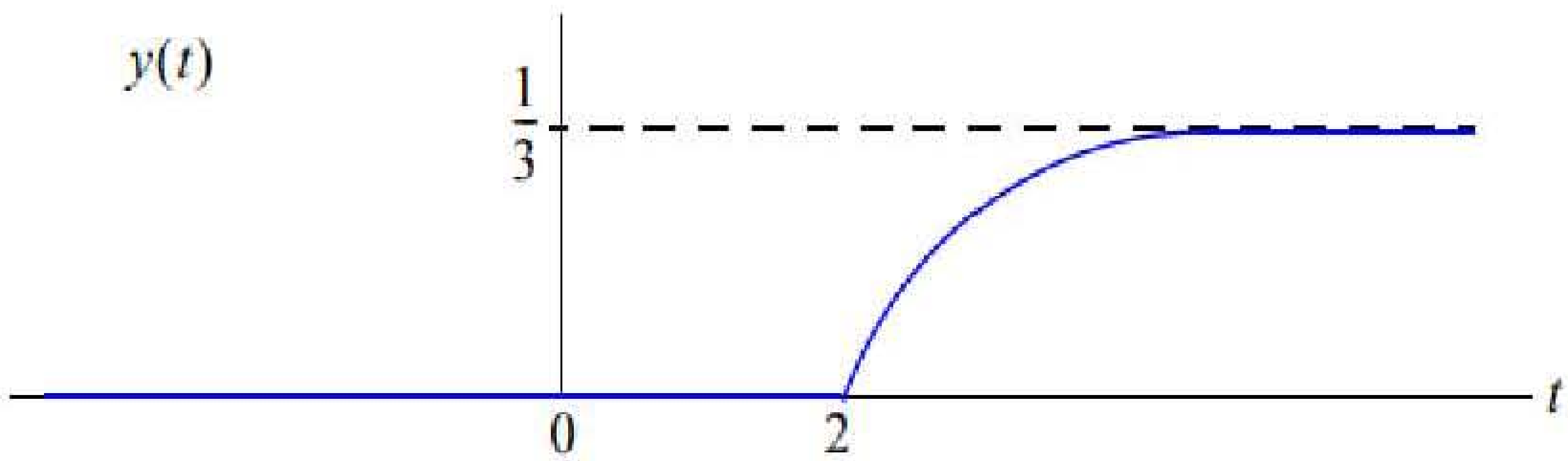


- For $t-2<0$ or $t<2$ there is no overlap in the product that comprises the integrand, so $y(t) = 0$
- For $t-2>0$ or $t>2$ there is overlap for $\tau \in [0, t-2)$, so here

$$y(t) = \int_{0}^{t-2} e^{-3\tau}d\tau$$

$$= \left.\frac{e^{-3\tau}}{-3}\right|_{0}^{t-2}$$

$$= \frac{1}{3}[1-e^{-3(t-2)}]u(t-2)$$



A causal discrete-time LTI system is described by a constant-coefficient difference equation:

$y[n] = \frac{1}{3}y[n-1] + x[n]$.

a) Find the impulse response $h[n]$.

b) Is it a stable system? Justify.

c) Is it an invertible system? Justify.

d) For an input signal given by $x[n] = \left(\frac{1}{2}\right)^n u[n]$, find the output signal $y[n]$.

---

a $h[n] = ?$

when $x[n] = \delta[n] \Rightarrow y[n] = h[n]$

$h[n] = \frac{1}{3} h[n-1] + x[n]$

$n=0 \Rightarrow h[0] = 0 + 1$

$n=1 \Rightarrow h[1] = \frac{1}{3}(1) + 0$

$n=2 \Rightarrow h[2] = \frac{1}{3}(\frac{1}{3}) + 0$

$\Rightarrow h[n] = (\frac{1}{3})^n u[n]$

b stable? Yes, it is

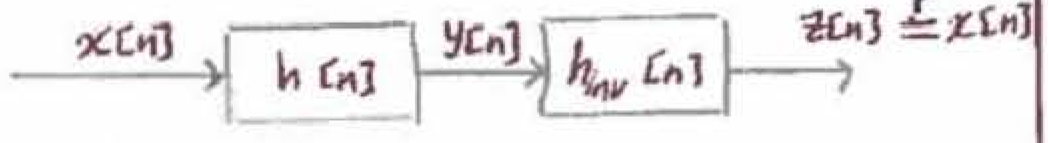
$\sum_{n=-\infty}^{\infty} |h[n]| \overset{?}{<} \infty$

$\sum_{n=0}^{\infty} (\frac{1}{3})^n = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

$\frac{3}{2} < \infty$ It is stable!

c Invertible? Yes, it is

To be invertible system, $h[n] * h_{inv}[n] = \delta[n]$



$h_{inv}[n] = ?$ $\Rightarrow x[n] = \frac{1}{3} x[n-1] + y[n]$

Solve it

so $h_{inv}[n] = \delta[n] - \frac{1}{3}\delta[n-1]$

$h[n] * h_{inv}[n] = \left((\frac{1}{3})^n u(n)\right) * \left(\delta[n] - \frac{1}{3}\delta[n-1]\right)$

$= (\frac{1}{3})^n u(n) - \frac{1}{3}(\frac{1}{3})^{n-1} u[n-1]$

$= (\frac{1}{3})^n \left(u[n] - u[n-1]\right)$

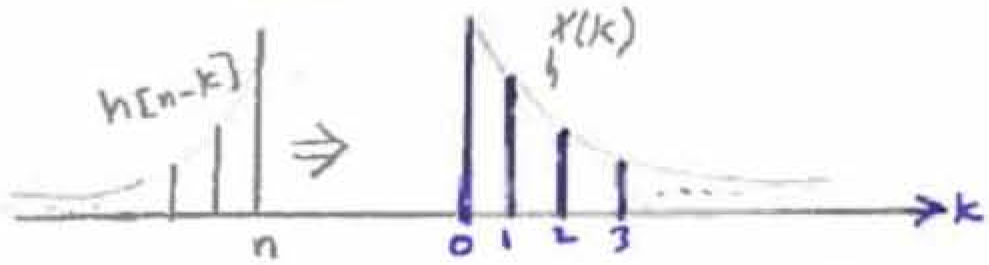
$u[n] - u[n-1] \Rightarrow \delta[n]$

It is invertible system

d $y[n] = ?$ $h[n] = (\frac{1}{3})^n u(n)$

$x[n] = (\frac{1}{2})^n u[n]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$ or $= \sum_{k=-\infty}^{\infty} x[n-k] h[k]$

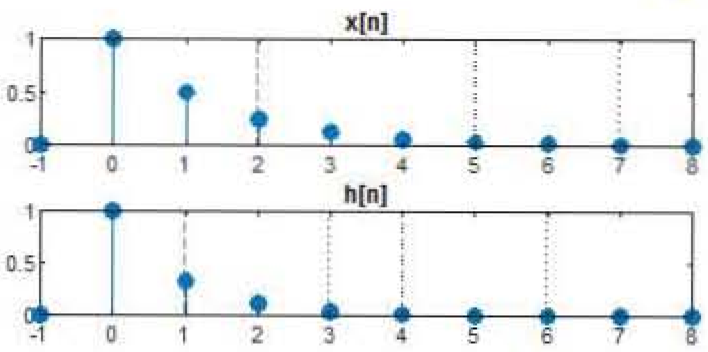


$y[n] = \begin{cases} y[n] = 0 & n < 0 \\ y[n] = \sum_{k=0}^{n} (\frac{1}{2})^k (\frac{1}{3})^{n-k} & n \geq 0 \end{cases}$

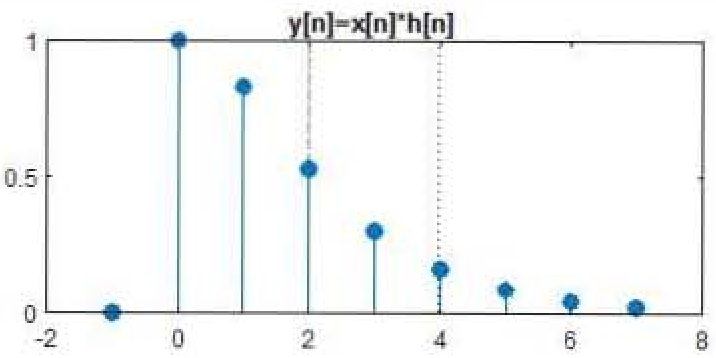
$y[n] = (\frac{1}{3})^n \sum_{k=0}^{n} (\frac{3}{2})^k = (\frac{1}{3})^n \frac{1-(\frac{3}{2})^{n+1}}{1-\frac{3}{2}}$

$y[n] = 3(\frac{1}{3})^n \left(1-(\frac{3}{2})^{n+1}\right) u[n]$ or

$= \left[3(\frac{1}{3})^n - \frac{9}{2}(\frac{1}{2})^n\right] u[n]$




```
x = @(n) ((1/2).^n).*(n>=0);
h = @(n) ((1/3).^n).*(n>=0);
y=conv(x(n),h(n));
```



A continuous-time LTI system has the following impulse response:

$h(t) = 2e^{-3t}u(t)$

a) Is it a causal system? Why? (/2+3)

b) Is it a stable system? Why? (/2+3)

c) What is the output when an input signal $x(t) = u(t-1) - u(t-4)$ is applied to the system? (/10)

---

a causal? Yes, it is

Because $\nexists\, u(t)$, the system does not have any value before $t=0$

In other words, it relies on present and past input values

b stable? Yes, it is

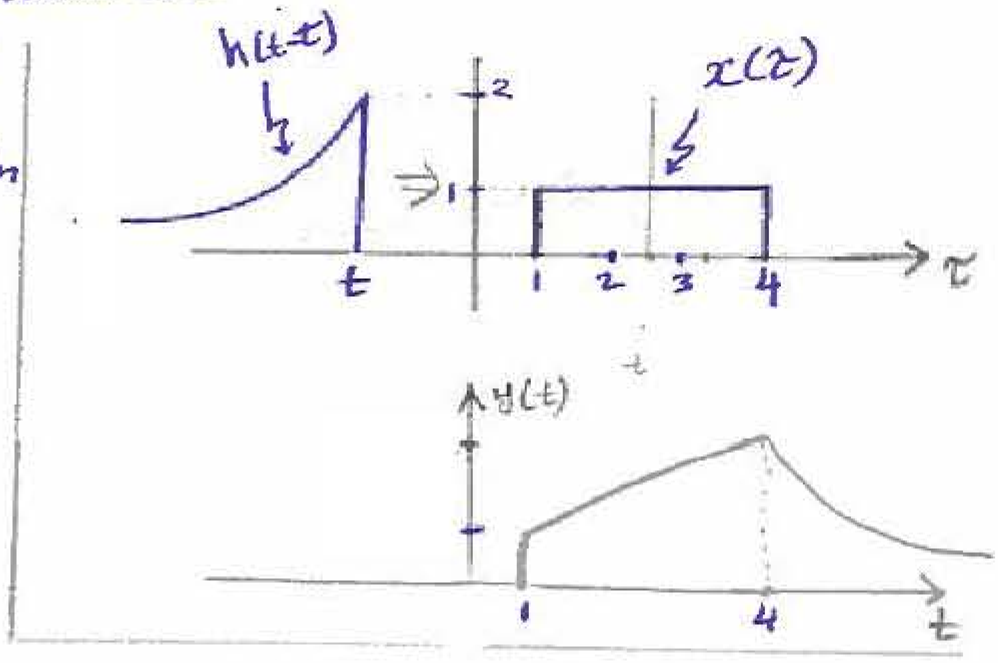
$\int_{-\infty}^{\infty} |h(t)|\,dt \overset{?}{<} \infty$

$= 2\int |e^{-3t}|\,dt = -\frac{2}{3} e^{-3t} = \frac{2}{3}$

$\frac{2}{3} < \infty$, the system is stable

c $y(t) = ?$ $\quad h(t) = 2e^{-3t}u(t)$

$x(t) = u(t-1) - u(t-4)$ $\Rightarrow y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$



we can see there are three cases for our solution

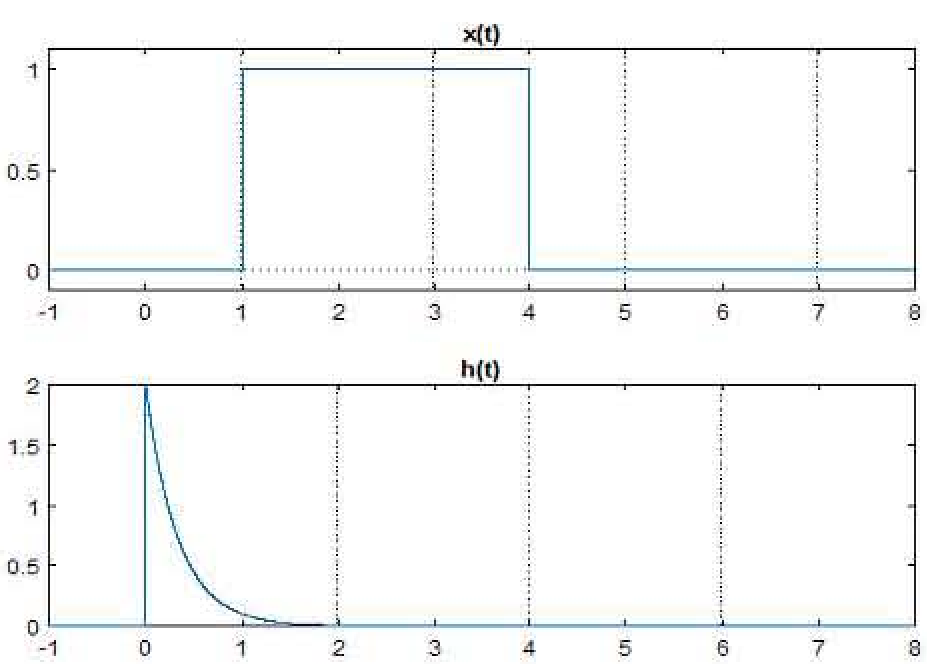
$$y(t) = \begin{cases} 0 & t<1 \quad \text{(case 1)} \\ \int_1^t 2e^{-3(t-\tau)}\,d\tau & 1<t<4 \quad \text{(case 2)} \\ \int_1^4 2e^{-3(t-\tau)} & t>4 \quad \text{(case 3)} \end{cases}$$

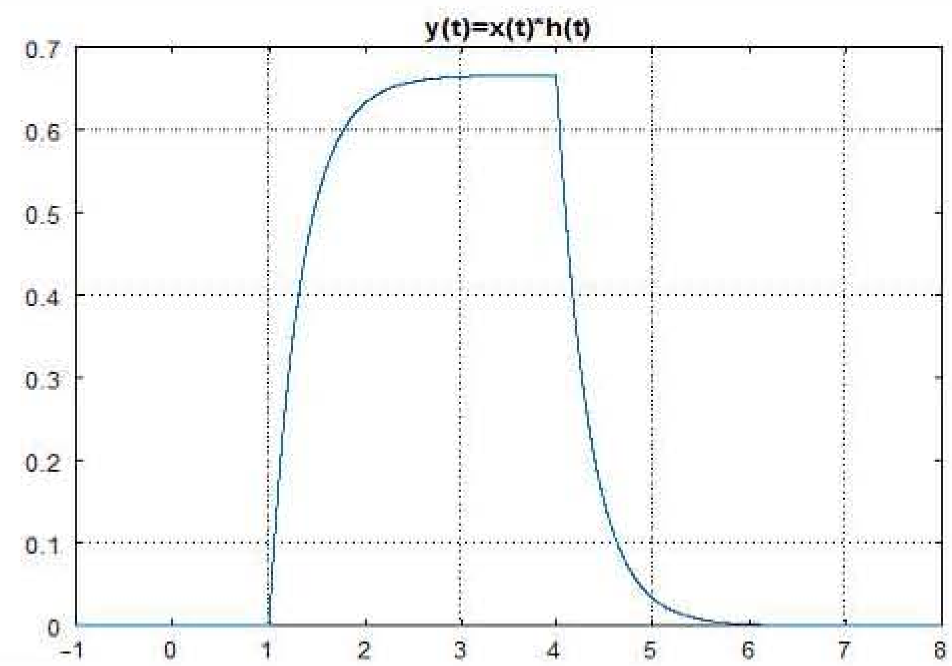
case 2: $2\int_1^t e^{-3(t-\tau)}\,d\tau = \frac{2}{3} e^{-3(t-\tau)}\Big|_1^t = \frac{2}{3} e^{-3t}\left[e^{3t} - e^{3}\right]$

case 3: $2\int_1^4 e^{-3(t-\tau)}\,d\tau = \frac{2}{3} e^{-3(t-\tau)}\Big|_1^4 = \frac{2}{3} e^{-3t}\left[e^{12} - e^{3}\right]$

we can write in one signal

$$y(t) = \frac{2}{3}\left[1 - e^{3-3t}\right]\left[u(t-1) - u(t-4)\right] + \frac{2}{3} e^{-3t}\left[e^{12} - e^{3}\right]u(t-4)$$





A causal continuous-time LTI system has the following differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

a) Find the impulse response (do not use transforms)

b) What is the output when an input signal $x(t) = u(t) - u(t-4)$ is applied to the system?

---

a $h(t) = ?$

$h(t) = y(t)$ when $\delta(t) = x(t)$

$$\frac{dh(t)}{dt} + 2h(t) = \delta(t)$$

Homogeneous solution $\dot{h}(t) + 2h(t) = 0$

Using hypothesis $h(t) = A e^{st}$

substitute :- $A s e^{st} + 2A e^{st} = 0$

$s = -2$

Finding A

$$\int_{0^-}^{0^+} \dot{h}(t) + 2\int_{0^-}^{0^+} h(t) = \int_{0^-}^{0^+} \delta(t)$$

$h(t) = 1$ at $t = 0 \Rightarrow A = 1$

$$\boxed{h(t) = e^{-2t}\, u(t)}$$

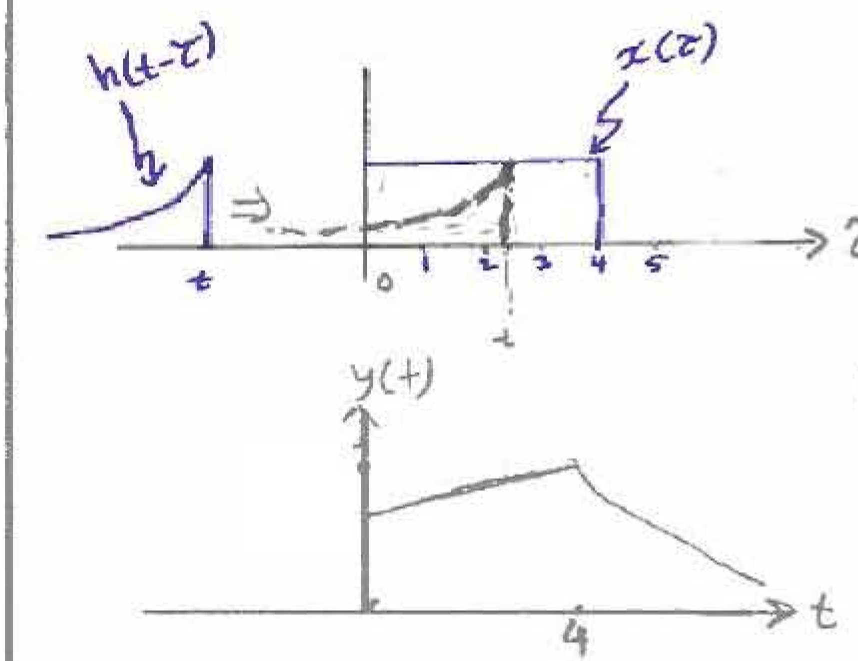
---

b $y(t) = ?$ $\quad h(t) = e^{-2t} u(t)$ & $x(t) = u(t) - u(t-4)$

$$y(t) = \int_{\tau=-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

we can see we have three cases :-

$$y(t) = \begin{cases} 0 & t < 0 \quad \text{(case 1)} \\ \int_0^t e^{-2(t-\tau)}\, d\tau & 0 < t < 4 \quad \text{(case 2)} \\ \int_0^4 e^{-2(t-\tau)}\, d\tau & t > 4 \quad \text{(case 3)} \end{cases}$$

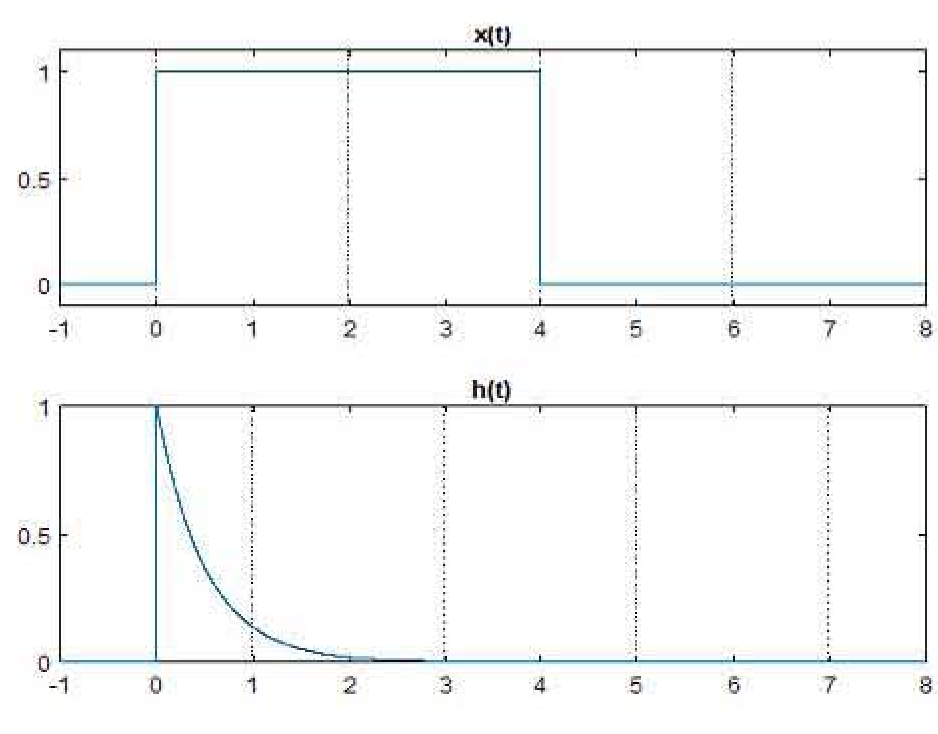


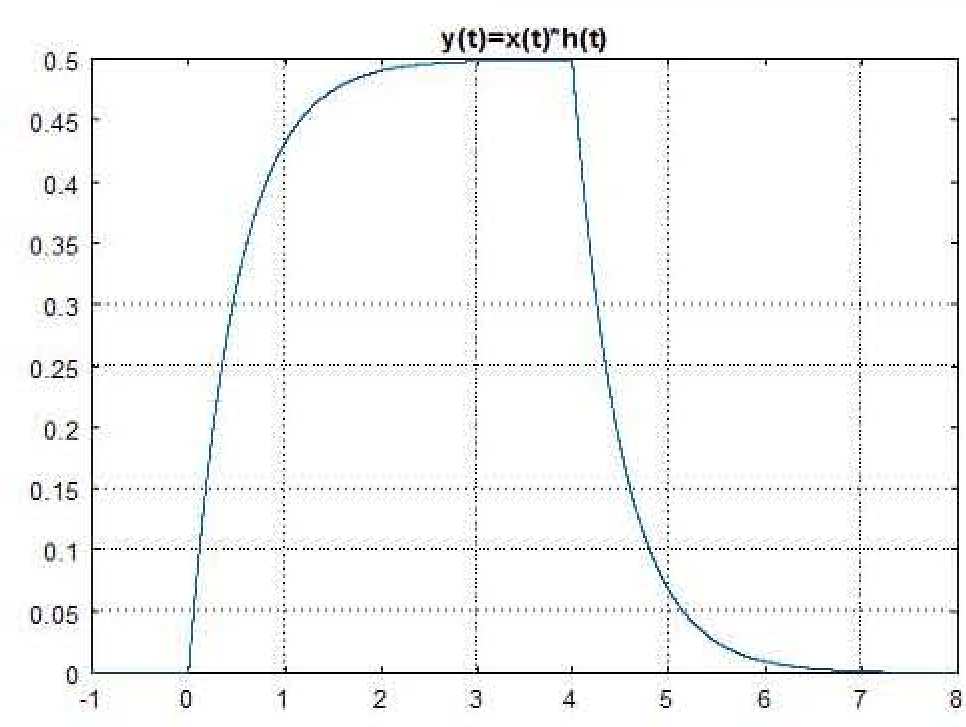
Case 2 :- $\int_0^t e^{-2(t-\tau)}\, d\tau = \frac{1}{2} e^{-2(t-\tau)}\Big|_0^t = \frac{1}{2}\left[1 - e^{-2t}\right]$

Case 3 : $\int_0^4 e^{-2(t-\tau)}\, d\tau = \frac{1}{2} e^{-2(t-\tau)}\Big|_0^4 = \frac{1}{2} e^{-2t}\left[e^{8} - 1\right]$

$$y(t) = \tfrac{1}{2}\left[1 - e^{-2t}\right]\left[u(t) - u(t-4)\right] + \tfrac{1}{2} e^{-2t}\left[e^{8} - 1\right] u(t-4)$$

Note: The other correct answers are accept





2.33. Consider a system whose input $x(t)$ and output $y(t)$ satisfy the first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t). \qquad \text{(P2.33–1)}$$

The system also satisfies the condition of initial rest.

(a) (i) Determine the system output $y_1(t)$ when the input is $x_1(t) = e^{3t}u(t)$.

(ii) Determine the system output $y_2(t)$ when the input is $x_2(t) = e^{2t}u(t)$.

page (118) in the book& Page (20/2) Lecture Notes

(a) (i) From Example 2.14, we know that

$$y_1(t) = \left[\frac{1}{5}e^{3t} - \frac{1}{5}e^{-2t}\right]u(t).$$

(ii) We solve this along the lines of Example 2.14. First assume that $y_p(t)$ is of the form Ke^{2t} for $t > 0$. Then using eq. (P2.33-1), we get for $t > 0$

$$2Ke^{2t} + 2Ke^{2t} = e^{2t} \quad \Rightarrow \quad K = \frac{1}{4}.$$

We now know that $y_p(t) = \frac{1}{4}e^{2t}$ for $t > 0$. We may hypothesize the homogeneous solution to be of the form

$$y_h(t) = Ae^{-2t}.$$

Therefore,

$$y_2(t) = Ae^{-2t} + \frac{1}{4}e^{2t}, \qquad \text{for } t > 0.$$

Assuming initial rest, we can conclude that $y_2(t) = 0$ for $t \leq 0$. Therefore

$$y_2(0) = 0 = A + \frac{1}{4} \quad \Rightarrow \quad A = -\frac{1}{4}.$$

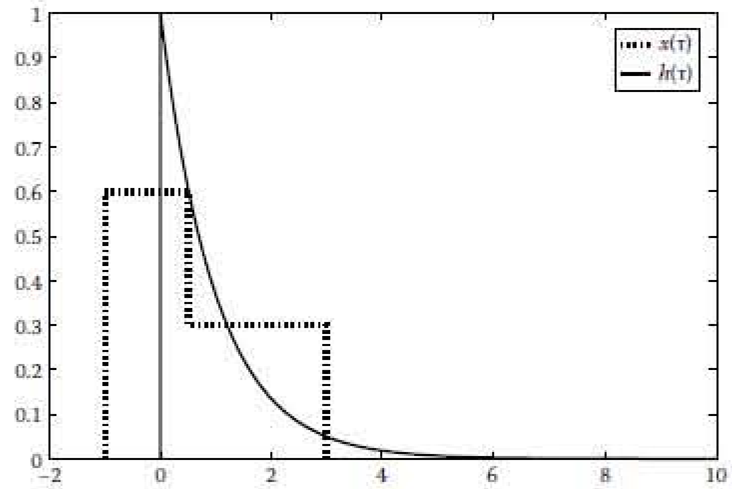
Then,

$$y_2(t) = \left[-\frac{1}{4}e^{2t} + \frac{1}{4}e^{-2t}\right]u(t).$$

Suppose that a linear time-invariant (LTI) system is described by the impulse response $h(t) = e^{-t}u(t)$. Compute the response of the system to the input signal

$$x(t) = \begin{cases} 0.6, & -1 \le t \le 0.5 \\ 0.3, & 0.5 \le t \le 3 \\ 0, & t < -1 \text{ and } t > 3 \end{cases}.$$

Commands	Results	Comments

`th1 = linspace(0,10,1001);`
`h1 = exp(-th1);`
`h = [0 h1];`
`th = [0 th1];`
`tx = [-1 -1 0.5 0.5 3 3];`
`x = [0 0.6 0.6 0.3 0.3 0];`
`plot(tx,x,':',th,h)`
`legend('x(\tau)','h(\tau)')`
		Notice that a zero element is embedded into the impulse response vector and its corresponding time vector. This is done in order to plot the vertical line at $\tau = 0$.

- First stage: Zero overlap.

  For $t < -1$, the input and impulse response signals do not overlap; thus the output of the system is $y(t) = 0$.

- Second stage: Partial overlap of $h(t-\tau)$ with the first part of $x(\tau)$.

  For $-1 < t < 0.5$, the impulse response signal $h(t-\tau)$ overlaps partially with the first part of $x(\tau)$, while there is no overlap with the second part of $x(\tau)$. The convolution integral in this stage is computed as

$$y(t) = \int_{-1}^{t} x(\tau)h(t-\tau)d\tau = \int_{-1}^{t} 0.6e^{-(t-\tau)}d\tau$$
$$= 0.6e^{-t}\int_{-1}^{t} e^{\tau}d\tau = 0.6 - 0.6e^{-t-1}.$$

- Third stage: The impulse response signal $h(t-\tau)$ overlaps completely with the first part of $x(\tau)$ and partially with the second part of $x(\tau)$.

  This stage takes place for $0.5 < t < 3$. There are two integrals that need to be calculated, corresponding to the different values of $x(\tau)$. Hence, the output signal is

$$y(t) = \int_{-1}^{0.5} 0.6e^{-(t-\tau)}d\tau + \int_{0.5}^{t} 0.3e^{-(t-\tau)}d\tau = 0.6e^{-t}\int_{-1}^{0.5} e^{\tau}d\tau + 0.3e^{-t}\int_{0.5}^{t} e^{\tau}d\tau$$
$$= 0.6e^{-t}(e^{0.5} - e^{-1}) + 0.3e^{-t}(e^{t} - e^{0.5}) = 0.3e^{-t+0.5} - 0.6e^{-t-1} + 0.3.$$

- Fourth stage: Complete overlap of $h(t-\tau)$ with both parts of $x(\tau)$.

  The fourth stage takes place for $t > 3$. The convolution integral is calculated as

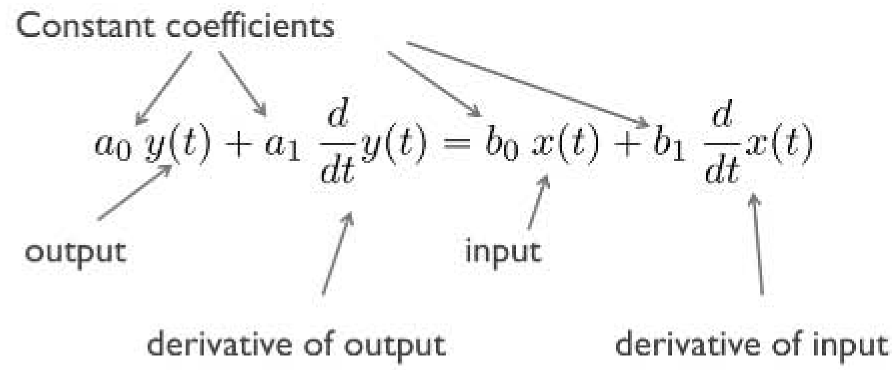
$$y(t) = \int_{-1}^{0.5} 0.6e^{-(t-\tau)}d\tau + \int_{0.5}^{3} 0.3e^{-(t-\tau)}d\tau = 0.6e^{-t}\int_{-1}^{0.5} e^{\tau}d\tau + 0.3e^{-t}\int_{0.5}^{3} e^{\tau}d\tau$$
$$= 0.6e^{-t}(e^{0.5} - e^{-1}) + 0.3e^{-t}(e^{3} - e^{0.5}) = 0.3e^{-t+0.5} - 0.6e^{-t-1} + 0.3e^{-t+3}.$$

$$y(t) = \begin{cases} 0, & t < -1 \\ 0.6 - 0.6e^{-t-1}, & -1 \le t \le 0.5 \\ 0.3e^{-t+0.5} - 0.6e^{-t-1} + 0.3, & 0.5 \le t \le 3 \\ 0.3e^{-t+0.5} - 0.6e^{-t-1} + 0.3e^{-t+3}, & t > 3 \end{cases}.$$

Differential Equations

Solving a continuous-time differential equation

- Determine the homogenous solution given initial conditions
- Determine the impulse response
- Determine the particular solution given an input signal
- Compute the total solution as homogenous + particular solutions



Problems

Solve the following differential equation

$$y(t) + 3\frac{dy(t)}{dt} + 2\frac{d^2y(t)}{dt^2} = 1$$

for $t \geq 0$ assuming the initial conditions $y(0) = 1$ and $\left.\frac{dy(t)}{dt}\right|_{t=0} = 2$. Express the solution in closed form. Enter your closed form expression the the box below.

[Hint: assume the homogeneous solution has the form $Ae^{s_1 t} + Be^{s_2 t}$.]

$y(t) =$ $-4e^{-t} + 4e^{-t/2} + 1$

First solve the homogeneous equation: $y_h(t) + 3\dot{y}_h(t) + 2\ddot{y}_h(t) = 0$. Assume $y_h(t) = Ae^{st}$. Then $\dot{y}_h(t) = sAe^{st}$ and $\ddot{y}_h(t) = s^2Ae^{st}$. Substitute into the homogeneous differential equation to obtain $(1 + 3s + 2s^2)Ae^{st} = 0$. Since e^{st} is never equal to zero, either A must be 0 or $1 + 3s + 2s^2$ must be zero. If A were zero, then the solution would be trivial (i.e., $y_h(t) = 0$), so the latter must be true to get a non-zero solution. From the factored form $(1 + s)(1 + 2s) = 0$, it is clear that s could be -1 or -0.5. Therefore the complete homogeneous solution could be written as

$$y_h(t) = Ae^{-t} + Be^{-t/2}$$

as in the hint. The particular solution has the same form as the inhomogenous part, so that $y_p(t) = 1$. To satisfy the initial conditions, we require that $y(t)$ (the sum of the homogeneous and particular parts) satisfies $y(0) = A+B+1 = 1$ and $\dot{y}(0) = -A-B/2 = 2$ so that $A = -4$ and $B = 4$. The final solution is

$$y(t) = -4e^{-t} + 4e^{-t/2} + 1\,.$$

Solve the following difference equation

$$8y[n] - 6y[n-1] + y[n-2] = 1$$

for $n \geq 0$ assuming the initial conditions $y[0] = 1$ and $y[-1] = 2$. Express the solution in closed form. Enter your closed form expression the the box below.

[Hint: assume the homogeneous solution has the form $Az_1^n + Bz_2^n$.]

$y[n] =$	$\frac{1}{6}\left(\frac{1}{4}\right)^n + \frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{1}{3}$

First solve the homogeneous system: $8y_h[n] - 6y_h[n-1] + y_h[n-2] = 0$. Assume $y_h[n] = Az^n$. Then $y_h[n-1] = Az^{n-1} = z^{-1}Az^n$ and $y_h[n-2] = Az^{n-2} = z^{-2}Az^n$. Substitute into the original difference equation to obtain $(8 - 6z^{-1} + z^{-2})Az^n = 0$. Since z^n is never equal to zero, either A must be 0 or $(8 - 6z^{-1} + z^{-2})$ must be zero. If A were zero, then the solution would be trivial (i.e., $y_h[n] = 0$), so the latter must be true to get a non-zero solution. From the factored form $(4 - z^{-1})(2 - z^{-1}) = 0$, it is clear that z^{-1} could be 4 or 2. Therefore the complete homogeneous solution could be written as

$$y_h[n] = A\left(\frac{1}{4}\right)^n + B\left(\frac{1}{2}\right)^n$$

as in the hint. The non-homogeneous part of the original difference equation is a constant 1. Thus, we expect a particular solution of the form $y_p[n] = C$ where C is a constant. Substituting this $y_p[n]$ into the original difference equation determines C, since $8C - 6C + C = 3C = 1$, so that $C = \frac{1}{3}$, and

$$y[n] = A\left(\frac{1}{4}\right)^n + B\left(\frac{1}{2}\right)^n + \frac{1}{3}$$

will solve the original difference equation. To satisfy the initial conditions, we require $y[n]$ satisfies $y[0] = A + B + \frac{1}{3} = 1$ and $y[-1] = 4A + 2B + \frac{1}{3} = 2$ so that $A = \frac{1}{6}$ and $B = \frac{1}{2}$. The final solution is

$$y[n] = \frac{1}{6}\left(\frac{1}{4}\right)^n + \frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{1}{3}$$