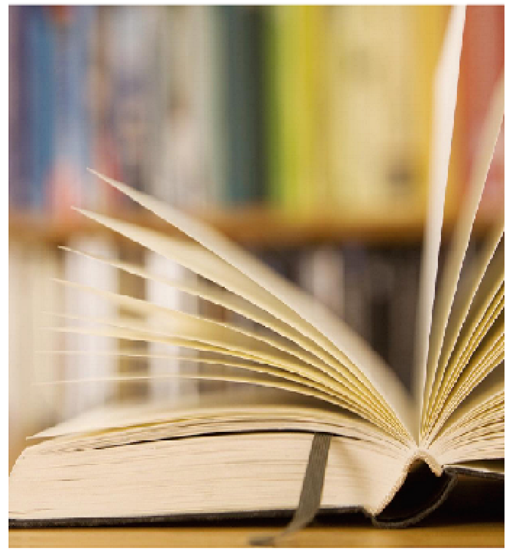
TUTORIAL
ELG3125B:SIGNAL AND SYSTEM ANALYSIS

Chapter (1)

By: Hitham Jleed

(Part 2)



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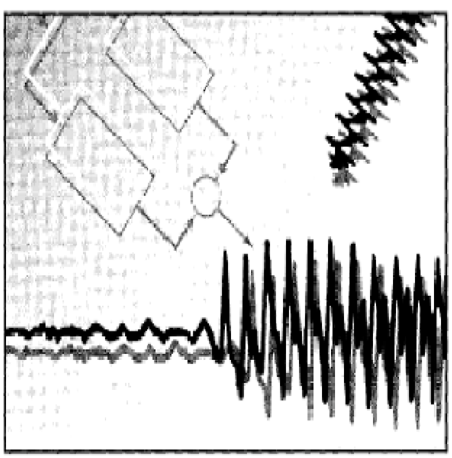
1
SIGNALS AND SYSTEMS



EXERCISE'S CONTINENTS

- Transformations of the independent variable.
- *Periodic Signals Vs Aperiodic Signals.*
- Fundamentals of Systems.
- System Properties.
- *LTI - Systems*

1.10. Determine the fundamental period of the signal $x(t) = 2\cos(10t + 1) - \sin(4t - 1)$.

1.11. Determine the fundamental period of the signal $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

1.12. Consider the discrete-time signal

$$x[n] = 1 - \sum_{k=3}^{\infty} \delta[n - 1 - k].$$

Determine the values of the integers M and n_0 so that $x[n]$ may be expressed as

$$x[n] = u[Mn - n_0].$$

1.13. Consider the continuous-time signal

$$x(t) = \delta(t + 2) - \delta(t - 2).$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

1.14. Consider a periodic signal

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ -2, & 1 < t < 2 \end{cases}$$

with period $T = 2$. The derivative of this signal is related to the "impulse train"

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

with period $T = 2$. It can be shown that

$$\frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2).$$

Determine the values of A_1, t_1, A_2, and t_2.

1.15. Consider a system S with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

$$S_1: \quad y_1[n] = 2x_1[n] + 4x_1[n - 1],$$

$$S_2: \quad y_2[n] = x_2[n - 2] + \frac{1}{2}x_2[n - 3],$$

where $x_1[n]$ and $x_2[n]$ denote input signals.

(a) Determine the input-output relationship for system S.

(b) Does the input-output relationship of system S change if the order in which S_1 and S_2 are connected in series is reversed (i.e., if S_2 follows S_1)?

1.16. Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n - 2].$$

1.18. Determine the Fundamental period of the signal

$$x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$$

Period of first term $= 1$

The second term $e^{j4\pi n/7} = e^{j\frac{4\pi}{7}(n+N)}$

$$m\left(\frac{2\pi}{4\pi/7}\right) = 7 \quad \text{when } m = 2$$

The third term $e^{j2\pi n/5} = e^{j\frac{2\pi}{5}(n+N)}$

$$m\left(\frac{2\pi}{2\pi/5}\right) = 5 \quad \text{when } (m=1)$$

The signal is periodic and Fundamental period is the least common multiple of $(1, 7, 5) = 35$

1.19 For each of the following input-output relationships, determine whether the corresponding system is ~~lean~~ linear, time invariant or both.

ⓐ $y(t) = t^2 x(t-1)$

linear ?

$$x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$$
$$x_2(t) \rightarrow y_2(t) = t^2 x_2(t-1)$$

Assume $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$

$$x_3(t) = a x_1(t) + b x_2(t)$$

$$y_3(t) = t^2 x_3(t-1) = t^2\big(a x_1(t-1) + b x_2(n-1)\big)$$

$= a\,y_1(t) + b\,y_2(t)$

Therefore, the system is linear.

Time invariant ??

let $y_1(t) = t^2 x_1(t-1)$

consider $x_2(t)$ obtained by shifting $x_1(t)$ in time
$x_2(t) = x_1(t-t_0)$

The output corresponding to this input is.
$y_2(t) = t^2 x_2(t-1) = t^2 x_1(t-1-t_0)$

shifting the output $y_1(t-t_0) = (t-t_0)^2 x_1(t-1-t_0)$
$\neq y_2(t)$

The system is NOT time-invariant.

(b) $y[n] = x^2[n-2]$

Linearity ? $x_1[n] \longrightarrow y_1[n] = x_1^2[n-2]$
$x_2[n] \longrightarrow y_2[n] = x_2^2[n-2]$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$
$x_3[n] = a\,x_1[n] + b\,x_2[n]$

$y_3[n] = x_3^2[n-2]$
$= (a\,x_1[n-2] + b\,x_2[n-2])^2$
$= a^2 x_1^2[n-2] + b^2 x_2^2[n-2] + 2ab\,x_1[n-2]x_2[n-2]$
$\neq a\,y_1[n] + b\,y_2[n]$

Therefore, the system is NOT linear.

Time-invariant ?

let $y_1[n] = x_1^2[n-2]$

consider a shifted signal $x_2[n] = x_1[n-n_0]$

The output corresponding to this input is.

$$y_2[n] = x_2^2[n-2] = x_1^2[n-2-n_0]$$

note that $\quad y_1[n-n_0] = x_1^2[n-2-n_0]$

therefore $\quad y_2[n] = y_1[n-n_0]$

The system is time invariant.

(c) $y[n] = x[n+1] - x[n-1]$

$x_1[n] \longrightarrow y_1[n] = x_1[n+1] - x_1[n-1]$
$x_2[n] \longrightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$

$$x_3[n] = a\,x_1[n] + b\,x_2[n]$$

$$\begin{aligned}
y_3[n] &= x_3[n+1] - x_3[n-1] \\
&= a x_1[n+1] + b\,x_1[n+1] - a\,x_1[n-1] - b\,x_2[n-1] \\
&= a\left(x_1[n+1] - x_1[n-1]\right) + b\left(x_2[n+1] - x_2[n-1]\right) \\
&= a\,y_1[n] + b\,y_2[n]
\end{aligned}$$

the system is linear

Consider an arbitary input $x_1[n]$.

Let

$$y_1[n] = x_1[n+1] - x_1[n-1]$$

be corresponding output. consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time

$$x_2[n] = x_1[n-n_0]$$

The output corresponding to this input.

$$y_2[n] = x_2[n+1] - x_2[n-1] = x_1[n+1-n_0]$$
$$- x_1[n-1+n_0]$$

Also note that.

$$y_1[n-n_0] = x_1[n+1-n_0] - x_1[n-1-n_0]$$

Therefore,

$$y_2[n] = y_1[n-n_0]$$

The system is time invariant.

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$ **(b)** $x[n] = \cos(\frac{n}{8} - \pi)$ **(c)** $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$ **(e)** $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

(a) Periodic, period = 7.

(b) Not periodic.

(c) Periodic, period = 8.

(d) $x[n] = (1/2)[\cos(3\pi n/4) + \cos(\pi n/4)]$. Periodic, period = 8.

(e) Periodic, period = 16.

1.27. In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless **(3)** Linear **(5)** Stable

(2) Time invariant **(4)** Causal

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-2) + x(2-t)$ **(b)** $y(t) = [\cos(3t)]x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$ **(d)** $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$ **(f)** $y(t) = x(t/3)$

(g) $y(t) = \frac{dx(t)}{dt}$

(a) Linear, stable.

(b) Memoryless, linear, causal, stable.

(c) Linear

(d) Linear, causal, stable.

(e) Time invariant, linear, causal, stable.

(f) Linear, stable.

(g) Time invariant, linear, causal.