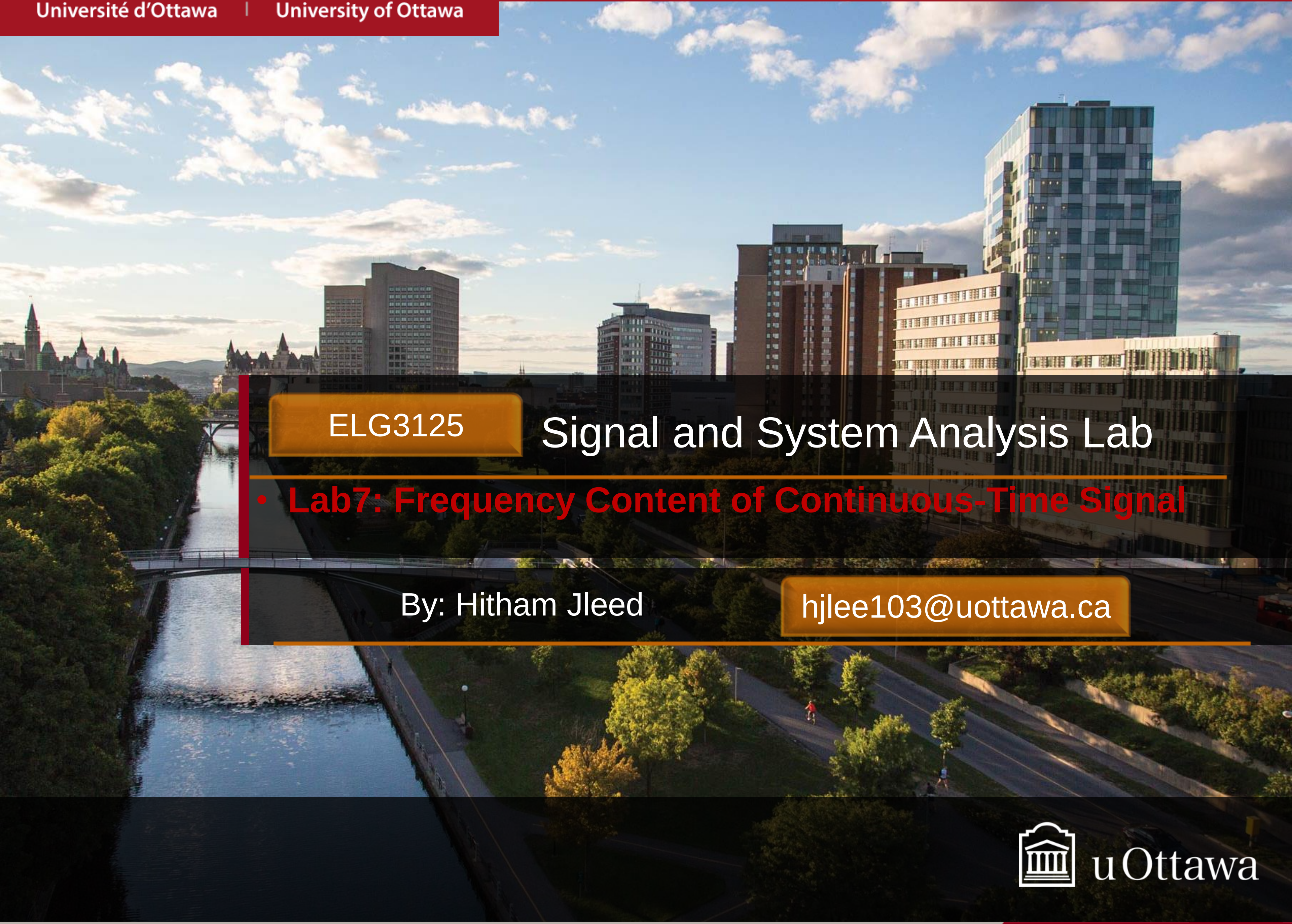

ELG3125 Signal and System Analysis Lab

Lab7: Frequency Content of Continuous-Time Signal

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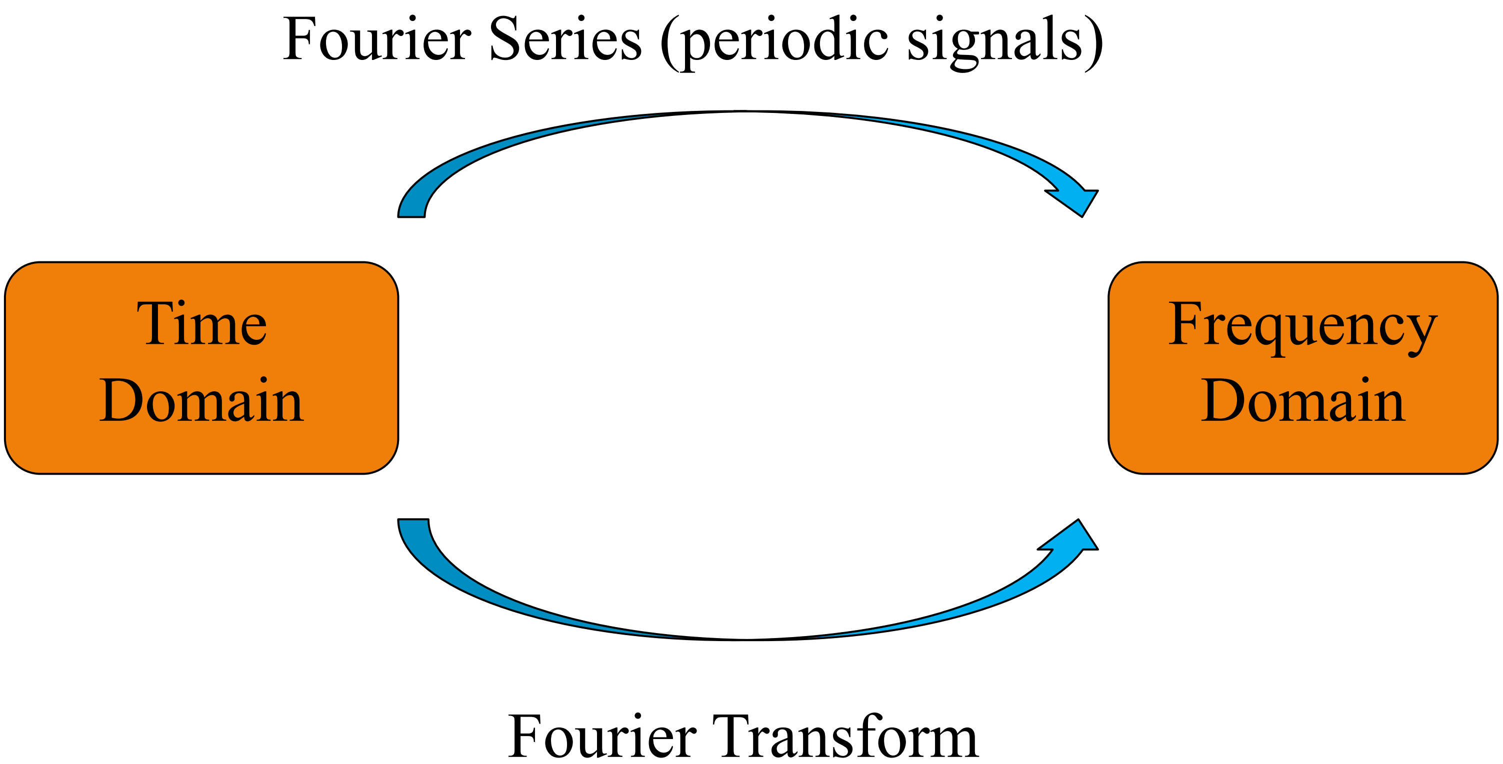
Frequency Content



*Fourier Transform in MATLab can be obtained by fft().

**There is a relationship between Fourier Series and Fourier Transform.

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Notations and abbreviations

Mathematical tools for frequency analysis depends on,

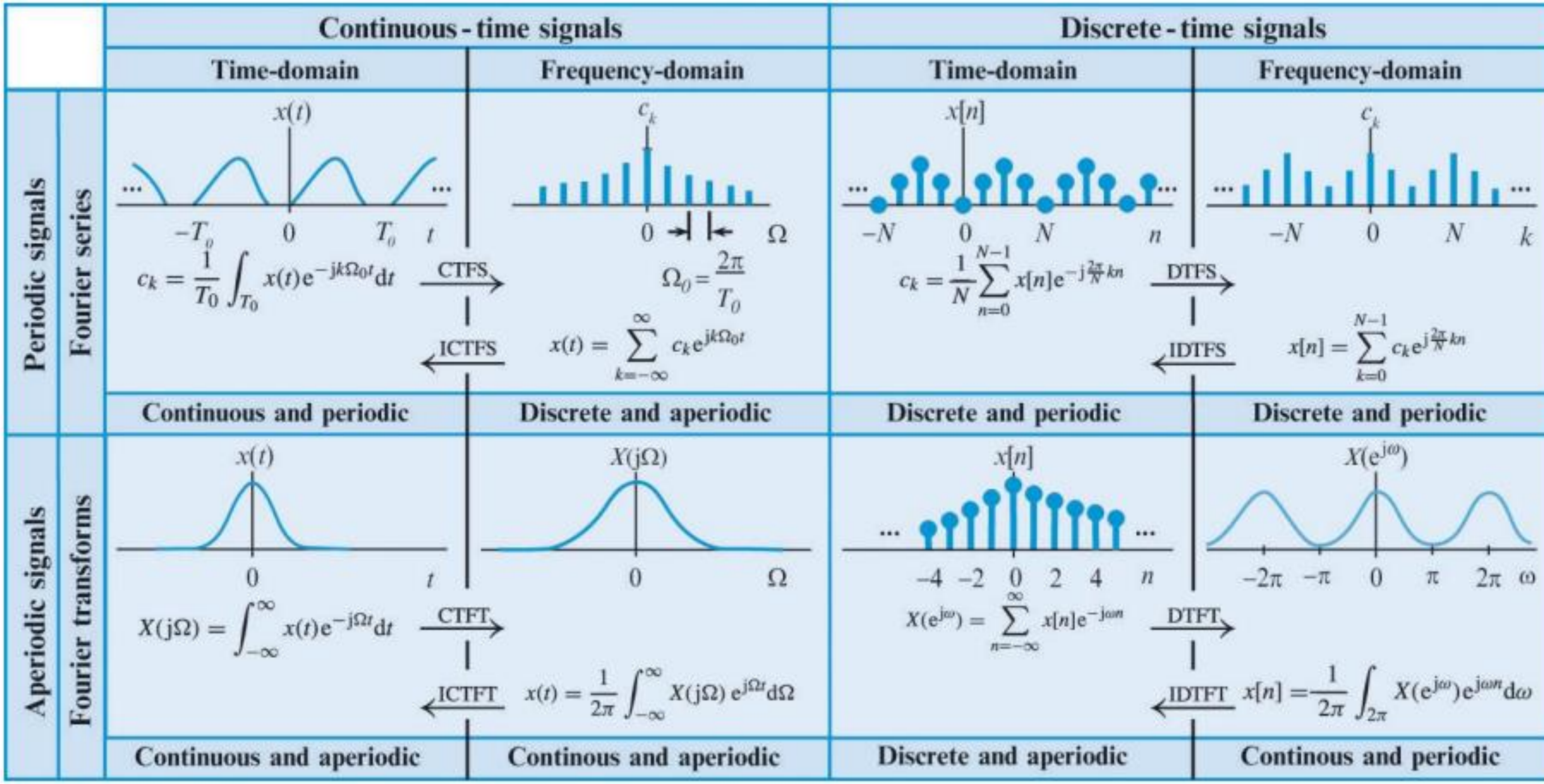
- **Nature of time:** continuous or discrete
- **Existence of harmonic:** periodic or aperiodic

The signal could be,

Continuous-time and periodic (freq. dom. CTFS)
Continuous-time and aperiodic (freq. dom. CTFT)

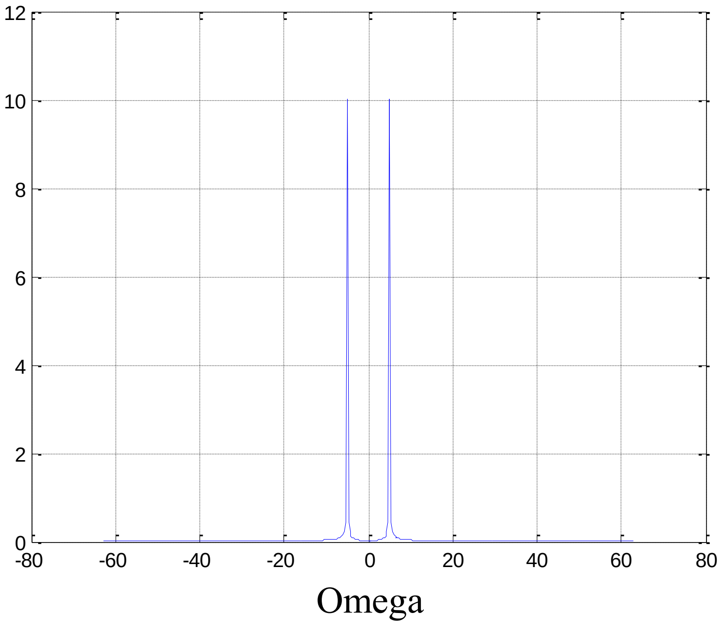
Discrete-time and periodic (freq. dom. DTFS)
Discrete-time and aperiodic (freq. dom. DTFT)

Notice: when the signal is periodic, we talk about Fourier series (FS).


Continuous-time signals
Discrete-time signals
Time-domain
Frequency-domain
Time-domain
Frequency-domain
Periodic signals
Fourier series
$x(t)$
$-T_0$ 0 T_0 t
c_k
0 Ω
$x[n]$
$-N$ 0 N n
c_k
$-N$ 0 N k
$c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\Omega_0 t}dt$
CTFS
ICTFS
$\Omega_0 = \frac{2\pi}{T_0}$
$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}$
$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$
DTFS
IDTFS
$x[n] = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}$
Continuous and periodic
Discrete and aperiodic
Discrete and periodic
Discrete and periodic
Aperiodic signals
Fourier transforms
$x(t)$
0 t
$X(j\Omega)$
0 Ω
$x[n]$
−4 −2 0 2 4 n
$X(e^{j\omega})$
-2π $-\pi$ 0 π 2π ω
$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$
CTFT
ICTFT
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}d\Omega$
$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
DTFT
IDTFT
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$
Continuous and aperiodic
Continous and aperiodic
Discrete and aperiodic
Continous and periodic

Fourier Transform of sin(5t)

```
dt = .05;
t = -10:dt:10;
x = sin(5*t);
X = fft(x)*dt;
X = fftshift(X);
Nw = length(X);
k = -(Nw-1)/2:1:(Nw-1)/2;
w = k*2*pi/Nw/dt; %rad./sample
plot(w,abs(X)); grid on;
```



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Frequency response of the continuous-time LTI system described by the differential equation:

$$a_3\frac{d^3y(t)}{dt^3} + a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = b_0x(t)$$

First, we define coefficient matrices:

A=[a3 a2 a1 a0]

B=[b3 b2 b1 b0]

Second, we use the following function:

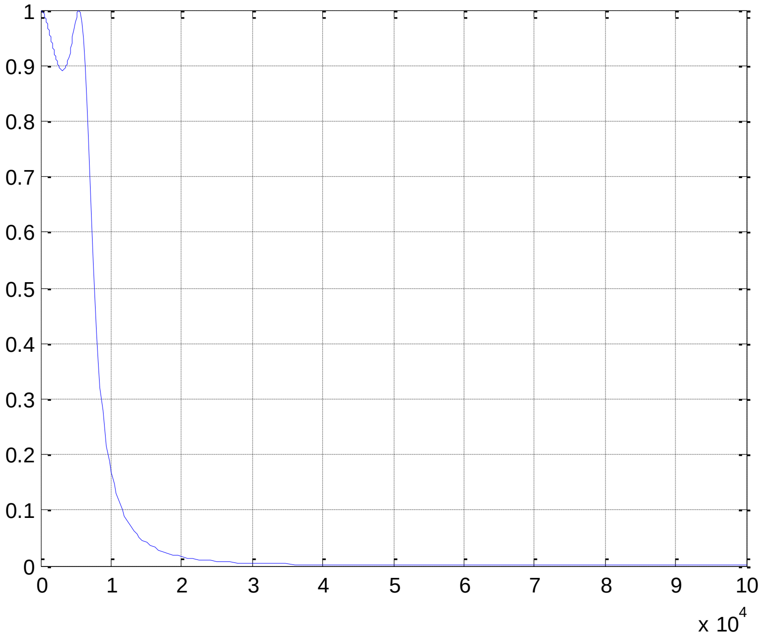
```
[h,w] = freqs(B,A);
plot(w, abs(h))
```

Question 3 – assignment 7:

$$a_3 \frac{d^3y(t)}{dt^3} + a_2 \frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

a0 = 121868727358.1180, a1 = 48890434.5196
a2 = 6209.9310, a3 = 1, b0 = 121868727358.1180;

```
a0 = 121868727358.1180;
a1 = 48890434.5196;
a2 = 6209.9310;
a3 = 1;
a = [a3 a2 a1 a0];
b0 = 121868727358.1180;
b = [0 0 0 b0];
[h,w] = freqs(b,a);
plot(w,abs(h)); grid on;
```



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Example

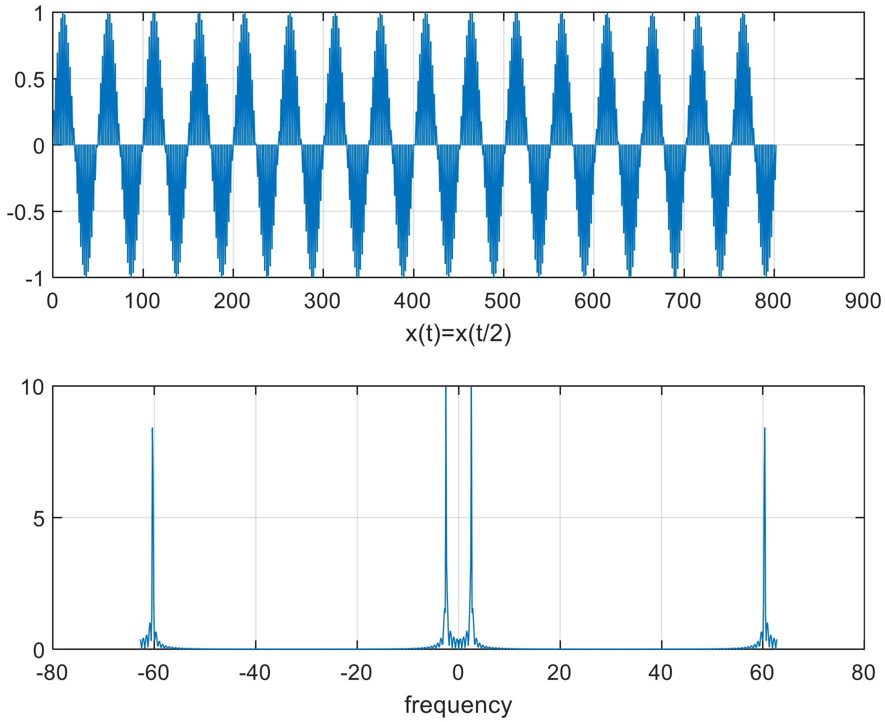
Let $x1(1) = \sin(5t)$ and $x2(t) = \sin(7t)$, $-10 \le t \le 10$, Use matlab to find the Fourier transform, $X(j\omega)$, of $x(t)$ for

- $x(t) = x_1(t/2)$.
- $x(t) = x_1(t) + x_2(t)$.
- $x(t) = x_1(t) \cdot x_2(t)$.

Compare your results with what you expect.

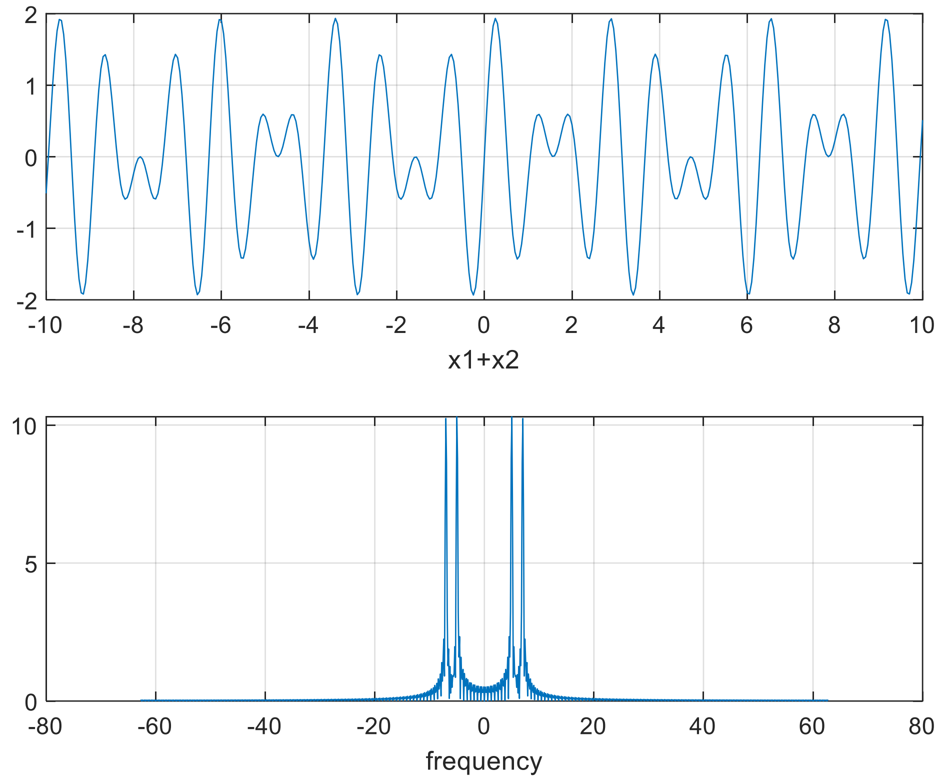
```
clear all;
close all;
%***************************
dt = .05;
t = -10:dt:10;
Nt = length(t);
x1 = sin(5*t);
x = upsample(x1,2);
X = fft(x,max(1001,Nt))*dt;
X = fftshift(X);
Nw = length(X);
k = -(Nw-1)/2:1:(Nw-1)/2;
w = k*2*pi/Nw/dt;
figure(2)
subplot(211);plot(x);grid
xlabel('x(t)=x(t/2)')
subplot(212);plot(w,abs(X));grid
xlabel('frequency')
```

$x(t) = x_1(t/2)$.



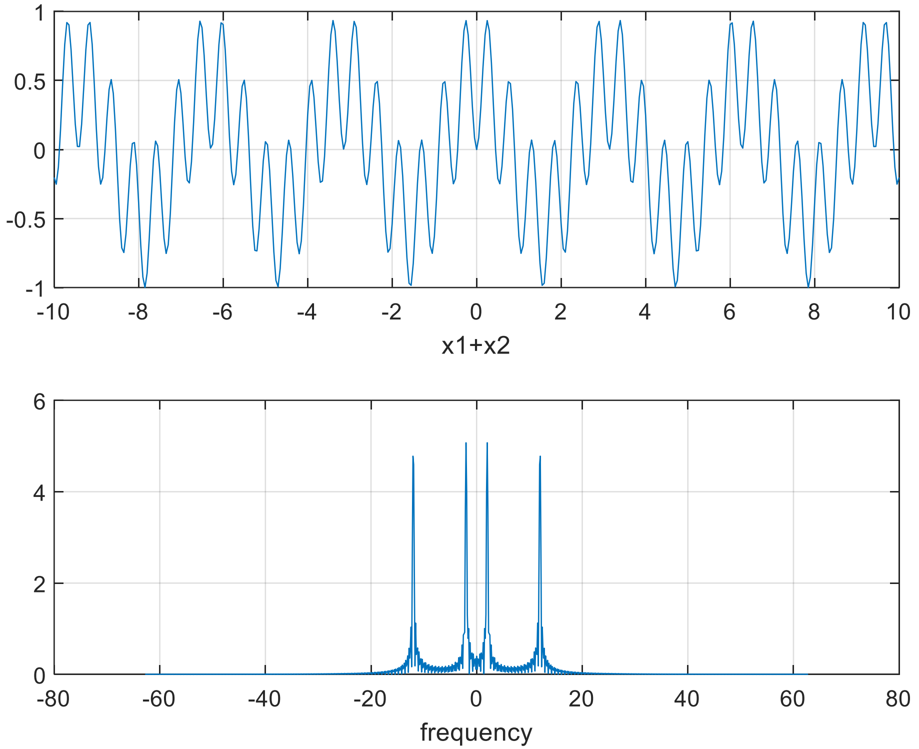
$$x(t) = x_1(t) + x_2(t).$$

```
clear all;
close all;
%*******************************
dt = .05;
t = -10:dt:10;
Nt = length(t);
x1 = sin(5*t);
x2 = sin(7*t);
x = x1+x2;
X = fft(x,max(1001,Nt))*dt;
X = fftshift(X);
Nw = length(X);
k = -(Nw-1)/2:1:(Nw-1)/2;
w = k*2*pi/Nw/dt;
figure(2)
subplot(211);plot(t,x);grid
xlabel('x1+x2')
subplot(212);plot(w,abs(X));grid
xlabel('frequency')
```



$$x(t) = x_1(t) \,.\, x_2(t).$$

```
clear all;
close all;
%*******************************
dt = .05;
t = -10:dt:10;
Nt = length(t);
x1 = sin(5*t);
x2 = sin(7*t);
x = x1.*x2;
X = fft(x,max(1001,Nt))*dt;
X = fftshift(X);
Nw = length(X);
k = -(Nw-1)/2:1:(Nw-1)/2;
w = k*2*pi/Nw/dt;
figure(2)
subplot(211);plot(t,x);grid
xlabel('x1+x2')
subplot(212);plot(w,abs(X));grid
xlabel('frequency')
```



Exercise 1

Find the amplitude spectrum of the two-frequency signal:

$x(t) = \cos(2\pi\, 100t) + \cos(2\pi\, 500t)$

Begin by creating a vector, x, with sampled values of the continuous time function. If we want to sample the signal every 0.0002 seconds and create a sequence of length 250, this will cover a time interval of length 250*0.0002 = 0.05 seconds.

Exercise 2

plot magnitude and phase spectrum of $\boldsymbol{x}(\mathrm{t}) = \mathbf{0.6} \times \mathbf{sinc}\,(\mathbf{0.6}\mathrm{t})$ for $\mathrm{t} = -\mathbf{200}:\mathbf{0.1}:\mathbf{200}$.

The END