ELG3125B

Signal and System Analysis Lab
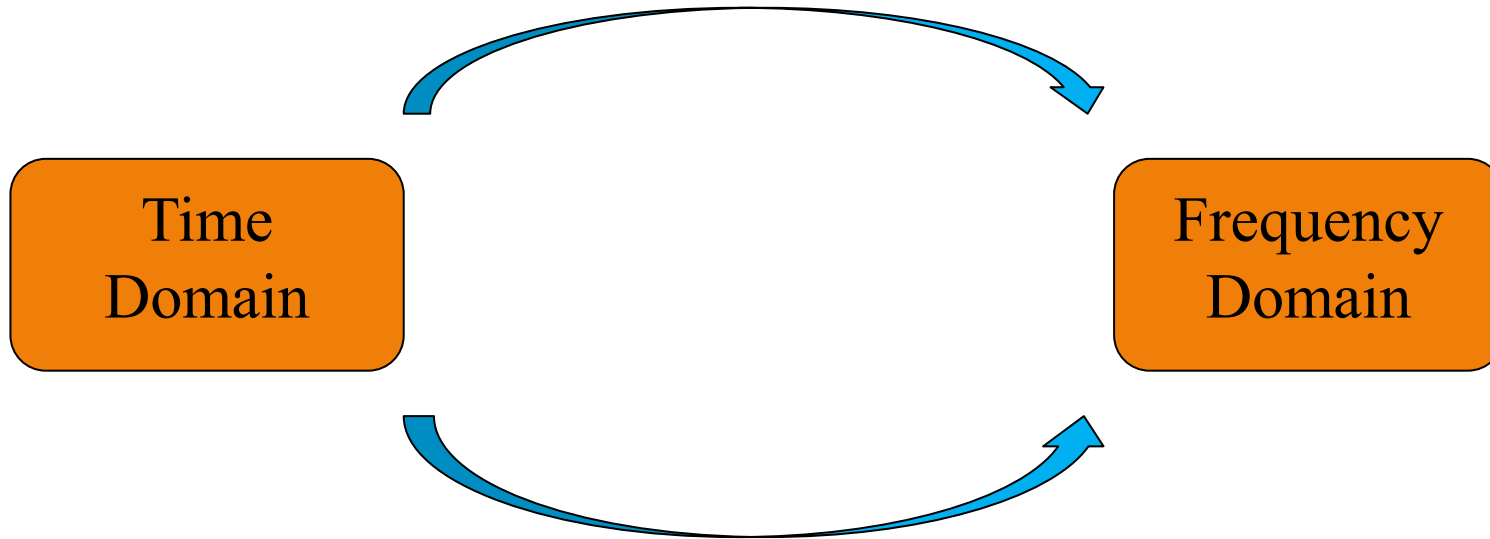
- **Lab6: Frequency Content of Discrete-Time Signal**

By: Hitham Jleed

hjlee103@uottawa.ca

uOttawa

Frequency Content

Fourier Series (periodic signals)

Time
Domain

Frequency
Domain

Fourier Transform (non-periodic signals)

*Fourier Transform in MATLab: fft()
**There is a relationship between Fourier Series and Fourier Transform

uOttawa

The relation between Fourier series and Fourier Transform

- In this lab, we learn to investigate the frequency content of the discrete-time signals. For a periodic signal with period N.

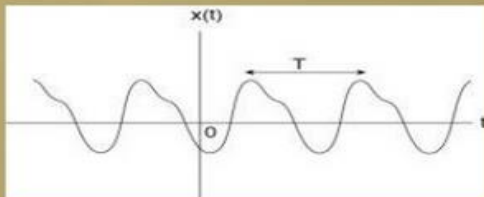Fourier series: $x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N}n} = \frac{1}{N} X(k)$$

uOttawa

Periodic VS Aperiodic Signals

Periodic Signal

- A signal which repeats itself after a specific interval of time is called periodic signal.
- A signal that repeats its pattern over a period is called periodic signal
- They can be represented by a mathematical equation
- Their value can be determined at any point of time
- They are deterministic signals
- Example: sine cosine square sawtooth etc
- Figure:
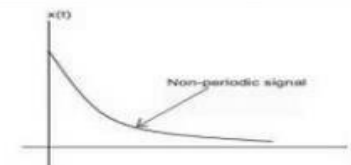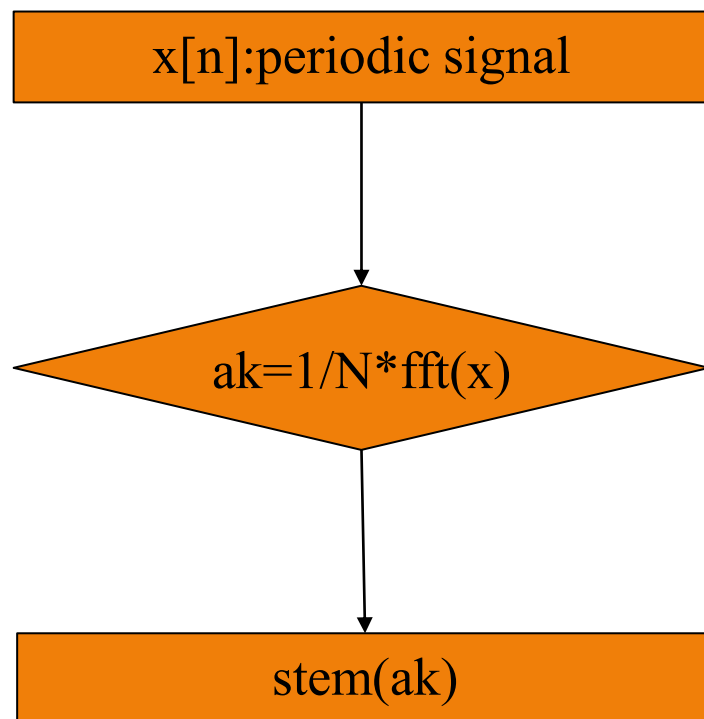
Aperiodic Signal

- A signal which does not repeat itself after a specific interval of time is called aperiodic signal.
- A signal that does not repeats its pattern over a period is called aperiodic signal or non periodic.
- They cannot be represented by any mathematical equation
- Their value cannot be determined with certainty at any given point of time
- They are random signals
- Example: sound signals from radio , all types of noise signals
- Figure:

uOttawa

1.Frequency Contents for Periodic Signal


```
x[n]:periodic signal
            |
            v
      ak=1/N*fft(x)
            |
            v
        stem(ak)
```

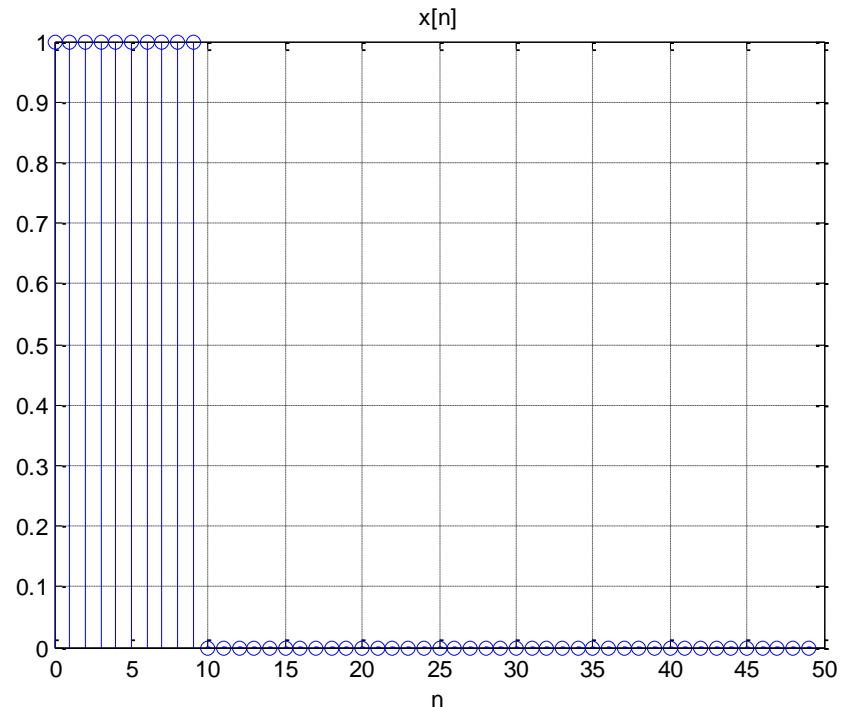*We use stem() because output of Fourier Series is always Discrete

Example 1:

- Periodic signal with period N = 50

$$x[n] = \begin{cases} 1, & 0 \leq n < 10 \\ 0, & 10 \leq n < 50 \end{cases}$$

```
N=50;
n=linspace(0,N-1,N);
x=[ones(1,10) zeros(1,N-10)];
stem(n,x);
xlabel('n');
title('x[n]'); grid on;
```
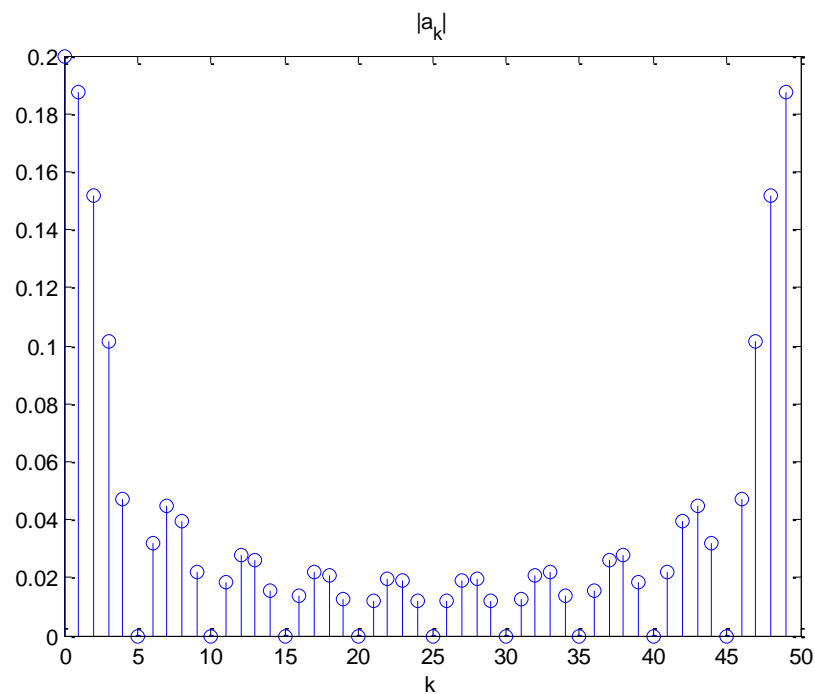

x[n]

uOttawa

Example 1 (Cont.)

- Frequency content without shift

```
ak=1/N*fft(x);
stem(n,abs(ak));
xlabel('k');
title('|a{_k}|');
```
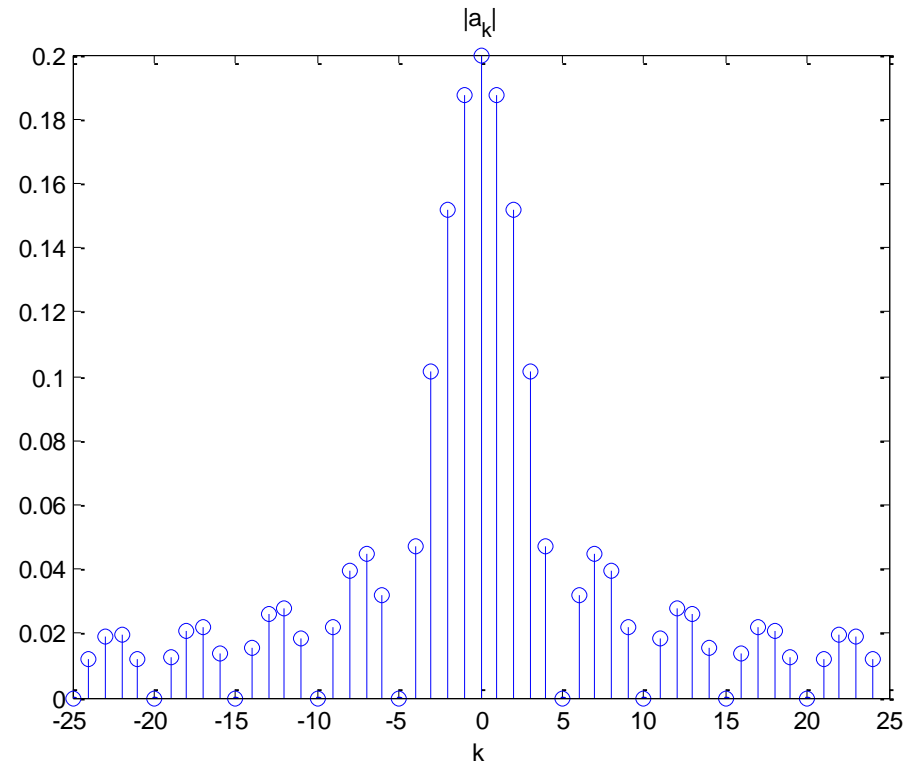
Example 1 (Cont.)

- Frequency content with shift

```
ak=fftshift(ak);
k=n-N/2;
stem(k,abs(ak));
xlabel('k');
title('|a{_k}|');
```

*fftshift() is a function to shift zero frequency to the center.

uOttawa

Exercise1: a) Draw the output of the following code.

```
N=30;
n=linspace(0,N-1,N);
x=[ones(1,8), 0.5*ones(1,5), zeros(1,N-13)];
stem(n,x);
xlabel('n');
title('x[n]'); grid on;
```


b) Assume the signal x-in section a- is a periodic signal, write a piece of code which shows the frequency content of the signal in a figure. Let the x-axis be frequency and y-axis be the magnitude of frequency content.

2.Frequency Contents for Non-periodic Signal

$x[n]: finite\ during\ 0 \leq n < N_x.$

$$X\left(e^{j\omega}\right) = \sum_{-\infty}^{+\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{N_x-1} x[n]\, e^{-j\omega n},\ -\pi \leq \omega < \pi$$

$$X\left(e^{jk\Delta\omega}\right) = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\Delta\omega n} = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\frac{2\pi}{N_\omega}n},\ \Delta\omega = \frac{2\pi}{N_\omega}$$

$$X(k) = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\frac{2\pi}{N_\omega}n},\ -\frac{N_\omega}{2} \leq k \leq \frac{N_\omega}{2} - 1$$

If N is chosen such that N=N_x =N_ω,

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n},\ -\frac{N}{2} \leq k < \frac{N}{2} - 1$$

uOttawa

Frequency Contents for Non-Periodic Signal

```
┌─────────────────────────────────┐
│    x[n]:non-periodic signal     │
└─────────────────────────────────┘
              │
              ▼
         ◇ X=fft(x) ◇
              │
              ▼
┌─────────────────────────────────┐
│            plot(X)              │
└─────────────────────────────────┘
```
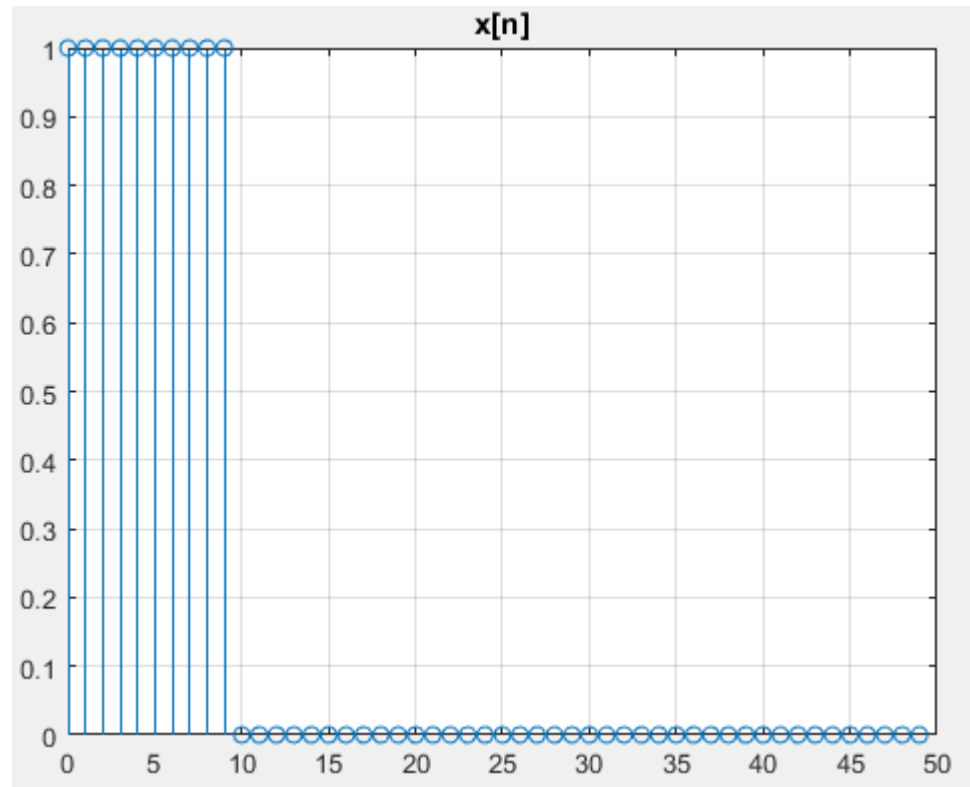
*We use plot() because output of Fourier Transform is always continuous.

uOttawa

Example 2:

- Finite-length signal with duration of N = 50

$$x[n] = \begin{cases} 1, & 0 \le n < 10 \\ 0, & 10 \le n < 50 \end{cases}$$
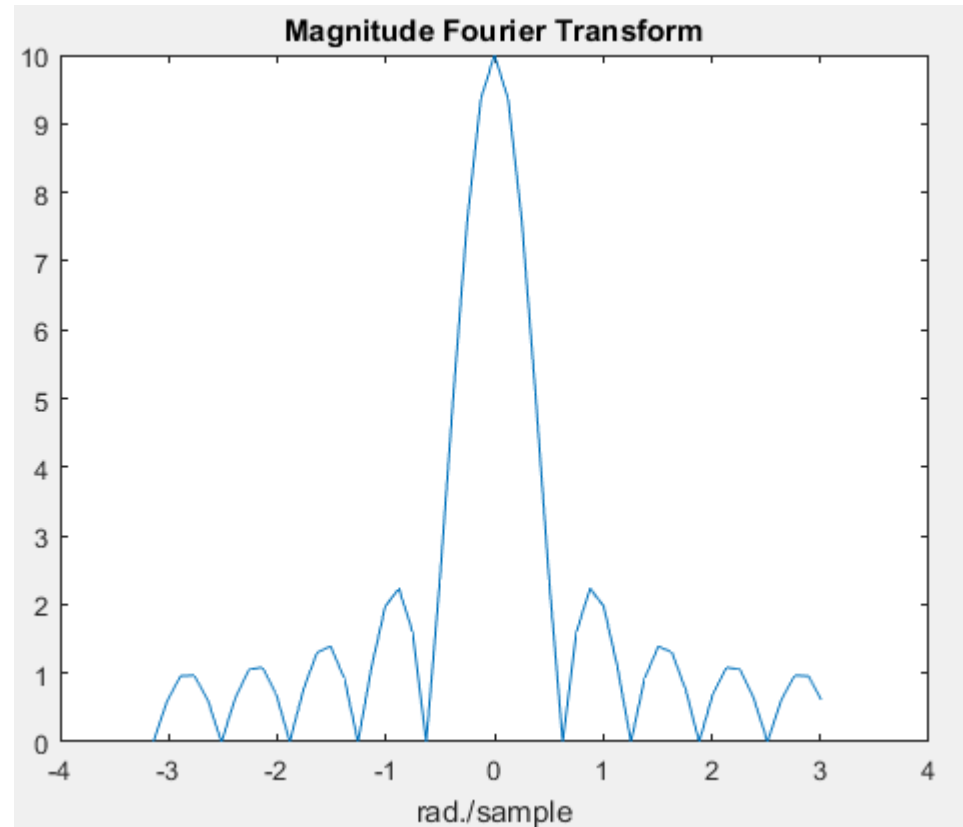
```
N=50;
n=linspace(0,N-1,N);
x=[ones(1,10) zeros(1,N-10)];
stem(n,x);
xlabel('n');
title('x[n]'); grid on;
```


x[n]

- Codes to obtain frequency contents(respect to ω): $\omega = k\dfrac{2\pi}{N}$

```
Xw=fft(x);
Xw=fftshift(Xw);
k=n-N/2;
w=2*k*pi/N;
plot(w,abs(Xw));
xlabel('rad./sample');
title('Magnitude Fourier Transform');
```


Magnitude Fourier Transform

uOttawa

1. MatLab Computation of the Frequency Content of a Periodic Discrete-Time Signal

For a periodic discrete time signal, where the values of x[n], $0 \leq n \leq N-1$ are known over a period N, it is possible to visualize the frequency content using the codes in the example 1 of the slides. Unlike the analysis equation for the Fourier series coefficients a_k, these frequency components are computed numerically using the Matlab function $fft(x)$, where x is a vector corresponding to the values of $x[n]$ over one period. The function fft uses the Fast Fourier Transform, whose theory will not be covered in the course ELG 3125, but you can read this for fun from such text as "J.G. Proakis and D.G. Manolakis, *Digital Signal Processing*, Prentice Hall, *3/e*, 1996."

For convenience, the codes to get the frequency contents are systematically written as follows.

```
N=length(x);        %MatLab code to generate Fourier series for periodic signal x[n] with a period N
ak=1/N*fft(x);
ak=fftshift(ak);
k=-(N-1)/2:1:(N-1)/2;    % N odd assumed here in this code
w=k*2*pi/N;
figure(1)
subplot(2,1,1)
stem(w,abs(ak));        % Magnitude
title('magnitude ak coeffs.')
xlabel('rad./sample')
subplot(2,1,2)
stem(w,angle(ak));       % Phase
title('phase ak coeffs.')
xlabel('rad/sample')
```

1) Build a periodic signal $x[n]$ of length 21 samples with a slow variation, and observe the frequency content.

2) Repeat 1) but with a fast variation and, and observe the frequency content.

3) Verify that slow variations and fast variation in the signal correspond to low frequency content and high frequency content, respectively.

Exercise1:

Solution

uOttawa

Exercise2:

Solution

MatLab Computation of the Frequency Content of a Non-Periodic Discrete-Time Signal

for a non-periodic signals having a finite duration (or length) in the interval $0 \leq n \leq N-1$, it is possible to evaluate the discrete time Fourier transform $X(e^{j\omega})$ at some frequencies by using again the Matlab function $fft()$ as done in the following.

```matlab
%code for Fourier transform of a non-periodic signal x[n].
Xw=fft(x,max(1001,length(x)));  % minimum of 1001 values computed,
                                % to provide impression of continuous transform
Xw=fftshift(Xw);
Nfft=length(Xw);
k=-(Nfft-1)/2:1:(Nfft-1)/2;    % with Nfft odd here in this code
w=k*2*pi/Nfft;
figure(1)
subplot(2,1,1)
plot(w,abs(Xw));               % Magnitude
title('magnitude Fourier transf.')
xlabel('rad./sample')
subplot(2,1,2)
plot(w,angle(Xw));             % Phase
title('phase Fourier transf.')
xlabel('rad./sample')
```

1) Build a signal $x[n]$ of finite length 21 samples with a slow variation, and observe the frequency content.
2) Repeat 1) but with a fast variation and, and observe the frequency content.
3) Discuss the similarities and the differences between the frequency contents you observed in this part and those obtained in the last part with the periodic signals.

uOttawa

The END

uOttawa