ELG3125 Signal and System Analysis Lab

• Lab5: Fourier series: Synthesis of signals

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Continuous-time Fourier Series …

- Fourier series representation

  Synthesis equation:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

  Analysis equation:

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}$$

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Continuous-time Fourier Series …

- Synthesis equation
  The synthesis or reconstruction of signal x(t) from a summation of complex exponential terms (or from cosine terms) weighted by the Fourier Series coefficients can also be written by:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$= a_0 + 2\sum_{k=1}^{+\infty} |a_k| \cos(k\omega_0 t + \angle a_k)$$

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Continuous-time Fourier Series (Truncated version)

If instead of using an infinite amount of terms, the summation is truncated to N_a terms (with N_a odd here), we then obtain the following approximation.

$$x(t) \approx \hat{x}(t) = \sum_{k=-(N_a-1)/2}^{(N_a-1)/2} a_k e^{jk\omega_0 t}$$

$$= a_0 + 2 \sum_{k=1}^{(N_a-1)/2} |a_k| \cos(k\omega_0 t + \angle a_k)$$

$* \; N_a \rightarrow \infty \; then \; \hat{x}(t) \rightarrow x(t).$

Discrete-time Fourier Series

- Fourier Series representation

Synthesis equation:

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$$

Analysis equation:

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n}$$

* No need for truncated version because N is finite already.

Syntax Review

```
x=0;
for k=1:10
    x=x+k;
end
```

$$x = \sum_{k=1}^{10} k$$

```
abs(ak);
```

$$|a_k|$$

```
angle(ak);
```

$$\angle a_k$$

```
exp(-j*k*w0*n);
```

$$e^{-jk\omega_0 n}$$

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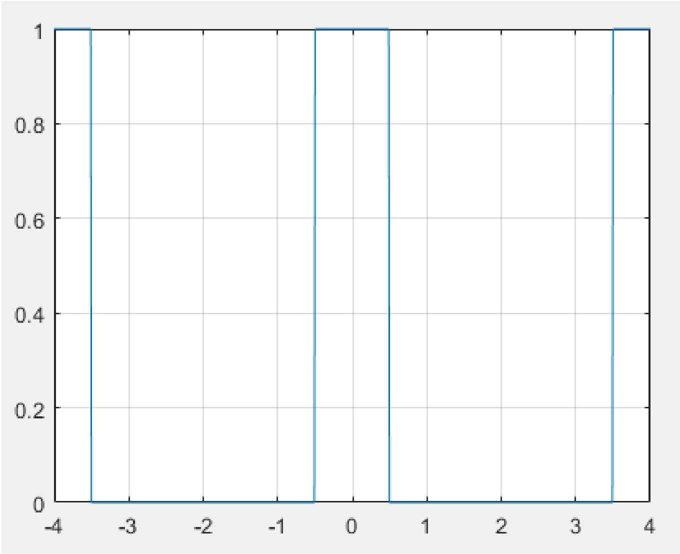
Example (Fourier series for a square wave)

- Square wave x(t) with period T.

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$T_1 = 0.5\,, T = 4$$



- Fourier Series coefficients

$$a_0 = \frac{2T_1}{T} \quad a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}, k \neq 0 \qquad \omega_0 = \frac{2\pi}{T}$$

- Now, let's rebuild x(t) from a_0 and a_k.

So, we implement:

$$x(t) = a_0 + 2 \sum_{k=1}^{(N_a-1)/2} |a_k| \cos(k\omega_0 t + \angle a_k)$$

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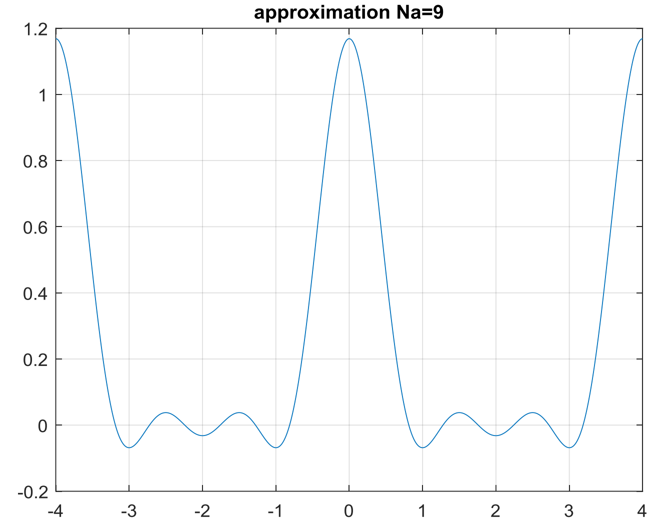
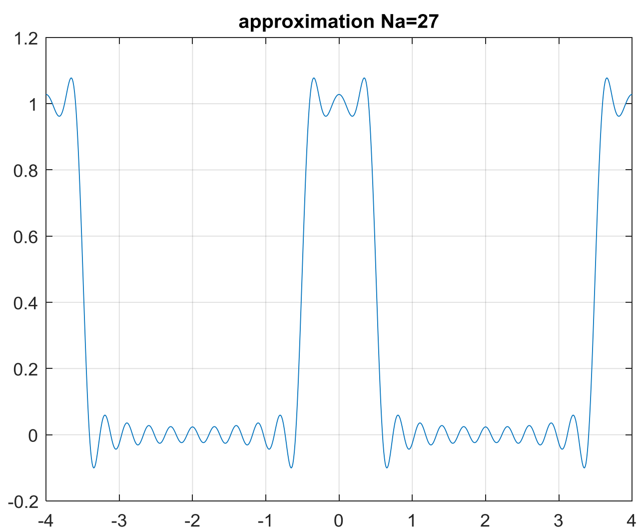
DO IT NOW

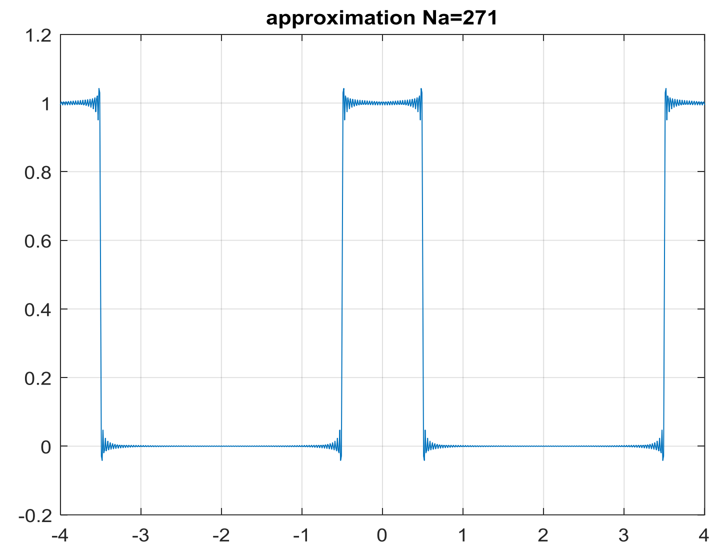
Note: T=4; T1=0.5; w0=2*pi/T;

w0=2*pi/T; a0=2*T1/T; ak=sin(k*w0*T1)/(k*pi);

```
T=4;T1=0.5; w0=2*pi/T;
t=-4:0.01:4; a0=2*T1/T;
Na=271; % Na=9, 27 or 271
sum=0;
for k=1:(Na-1)/2
    ak=sin(k*w0*T1)/(k*pi);
    sum=sum+abs(ak)*...
        cos(k*w0.*t+angle(ak));
end
x_approx=a0+2*sum;
plot(t,x_approx); grid on;
title(sprintf('approximation Na=%d',Na))
```





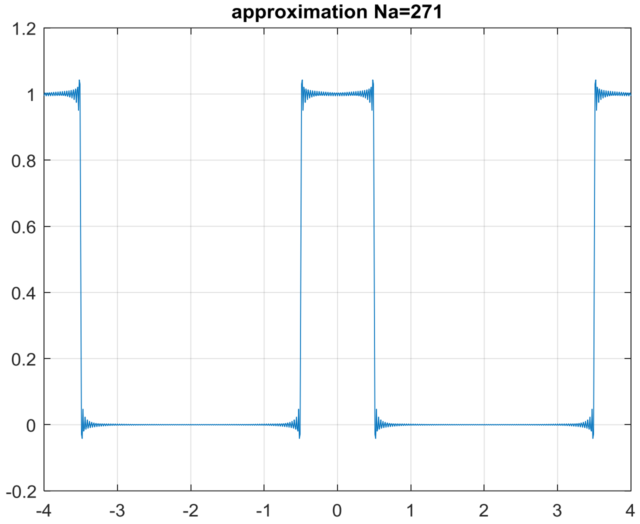


```
T=4;T1=0.5; w0=2*pi/T;
t=-4:0.01:4;
Na=271; % Na=9, 27 or 271
x_approx=0;
for k=-(Na-1)/2:(Na-1)/2
    if (k==0)
        ak=2*T1/T;
    else
        ak=sin(k*w0*T1)/(k*pi);
    end
    x_approx=x_approx+ak*...
        exp(1j*k*w0.*t);
end

plot(t,real(x_approx)); grid on;
title(sprintf('approximation
Na=%d',Na))
```

Using exponential form:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$



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