ELG3125 Signal and System Analysis Lab

Lab4: Property and Combination of LTI Systems

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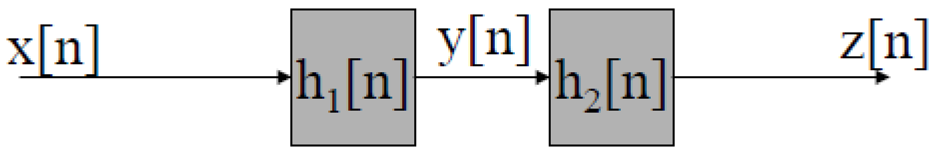
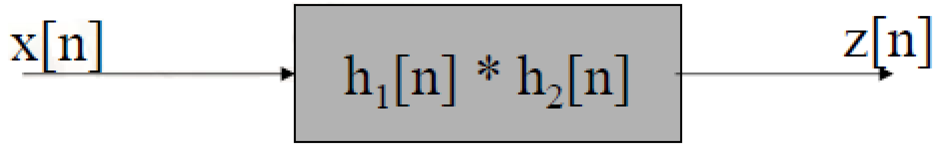
Combination of LTI Systems

- Combination of two LTI systems





**Also applicable for continuous-time systems.*

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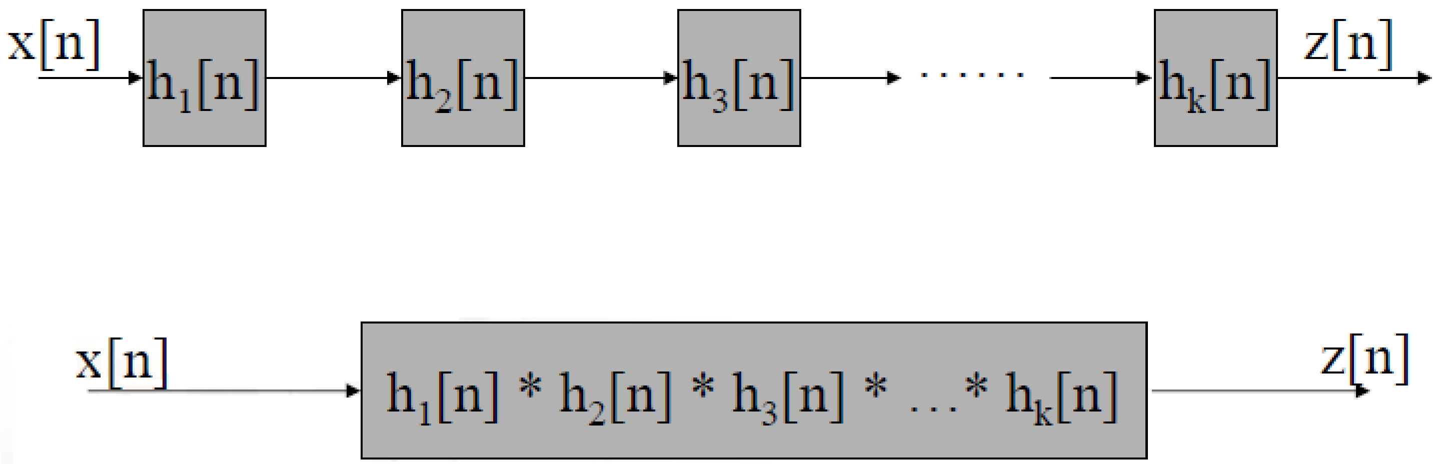
Combination of LTI Systems

- Combination of multiple LTI systems



**Also applicable for continuous-time systems.*

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Assignment – Question 1

1. A LTI discrete-time system has an input $x[n]$ and an output $y[n]$:

$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$, system is initially at rest.

The output $y[n]$ of this first system then becomes the input of a second system: $z[n]+\frac{9}{20}z[n-1]+\frac{1}{20}z[n-2]=y[n]$, system is initially at rest.

Compute and display the impulse response of the equivalent system between $x[n]$ and $z[n]$.

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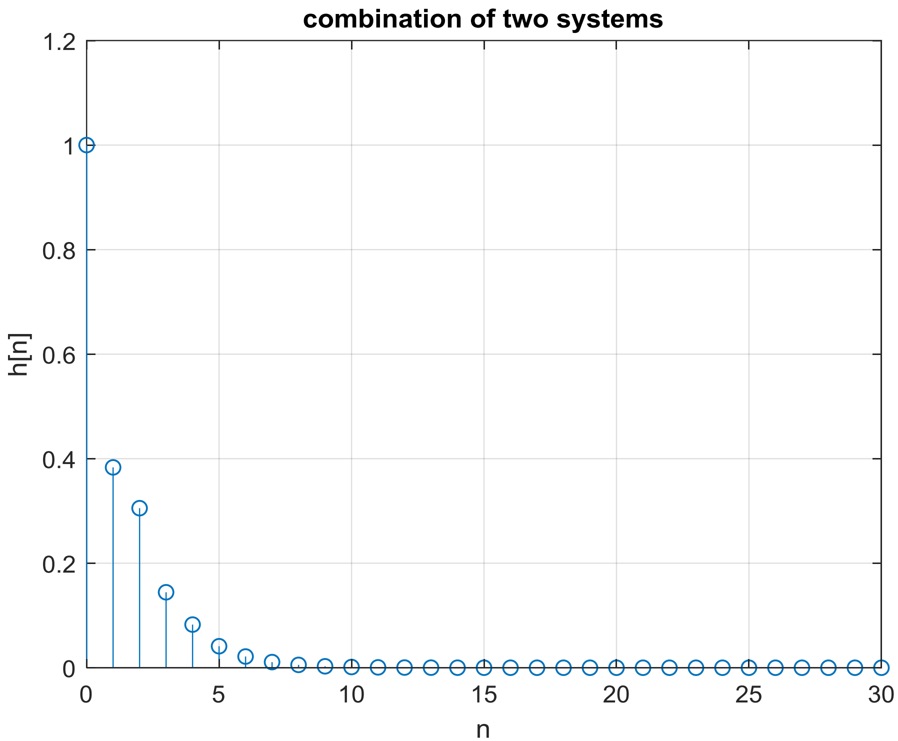
Assignment – Question 1 …

```
n=0:100;
%Impluse reponse of system
1
B1=[1 0 0];
A1=[1 -5/6 1/6];
h1=impz(B1,A1,n);
%Impluse reponse of system
2
B2=[1 0 0];
A2=[1 9/20 1/20];
h2=impz(B2,A2,n);
%combination of two systems
h=conv(h1,h2);
%plot
nh=0:length(h)-1;
stem(nh,h),grid;
axis([0,30,0,1.2]);
title('combination of two
systems')
xlabel('n'); ylabel('h[n]')
```

$$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$$

$$z[n]+\frac{9}{20}z[n-1]+\frac{1}{20}z[n-2]=y[n]$$



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System Stability

- For a discrete-time system, a sufficient condition to guarantee its stability is: if the impulse response is absolutely summable, i.e.,

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$

* Then, if the input x[n] is bounded, the output y[n] is bounded.

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System Stability …

- For a continuous-time system, a sufficient condition to guarantee its stability is: if the impulse response is absolutely integrable, i.e.,

$$\int_{\tau=-\infty}^{+\infty} |h(\tau)| < \infty$$

* Then, if the input x(t) is bounded the output y(t) is bounded.

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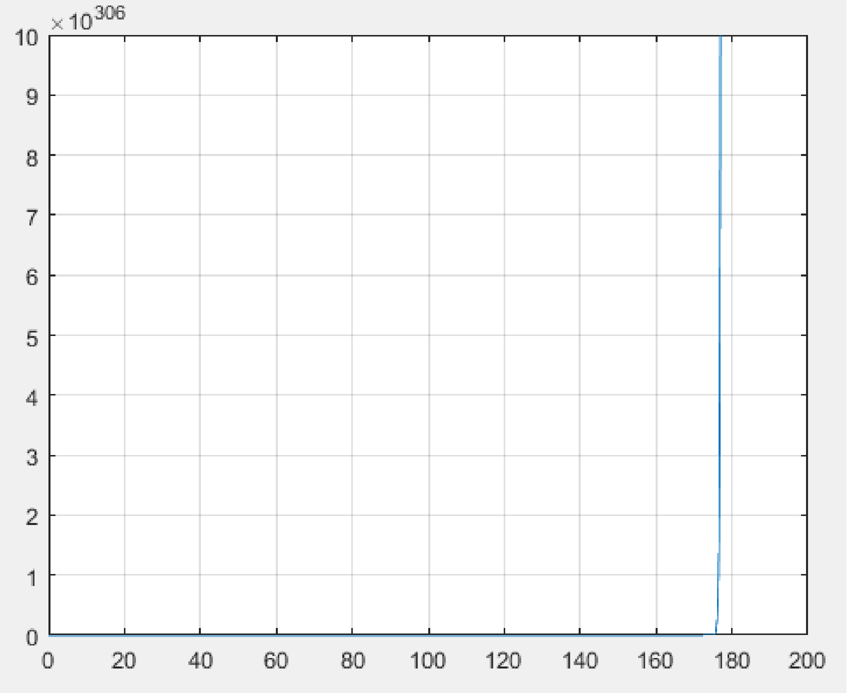
Assignment – Question 2

Show by experiment if the following LTI systems are stable or not:

(1) $\frac{d^2y(t)}{dt^2} - 5\frac{dy(t)}{dt} + 4y(t) = x(t)$, system initially at rest.

```
t=0:0.1:200;
B1=[0 0 1];
A1=[1 -5 4];
h1=impulse(B1,A1,t);
plot(t,h1), grid on;
axis([0 200 0 10^307])
```



Not stable!

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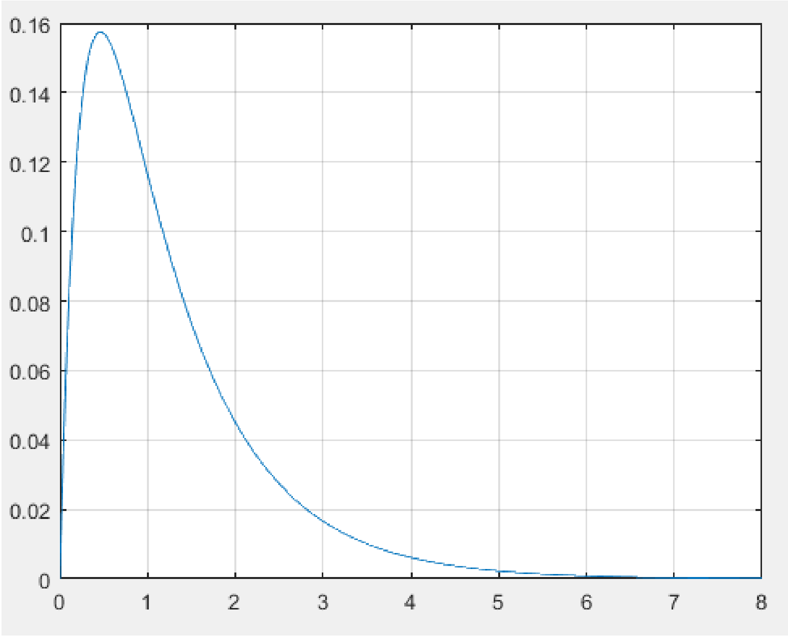
Assignment – Question 2 …

Show by experiment if the following LTI systems are stable or not:

(2) $\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+4y(t)=x(t)$, system initially at rest.

```
t=0:0.01:8;
B1=[0 0 1];
A1=[1 +5 4];
h1=impulse(B1,A1,t);
plot(t,h1), grid on;
```



Stable!

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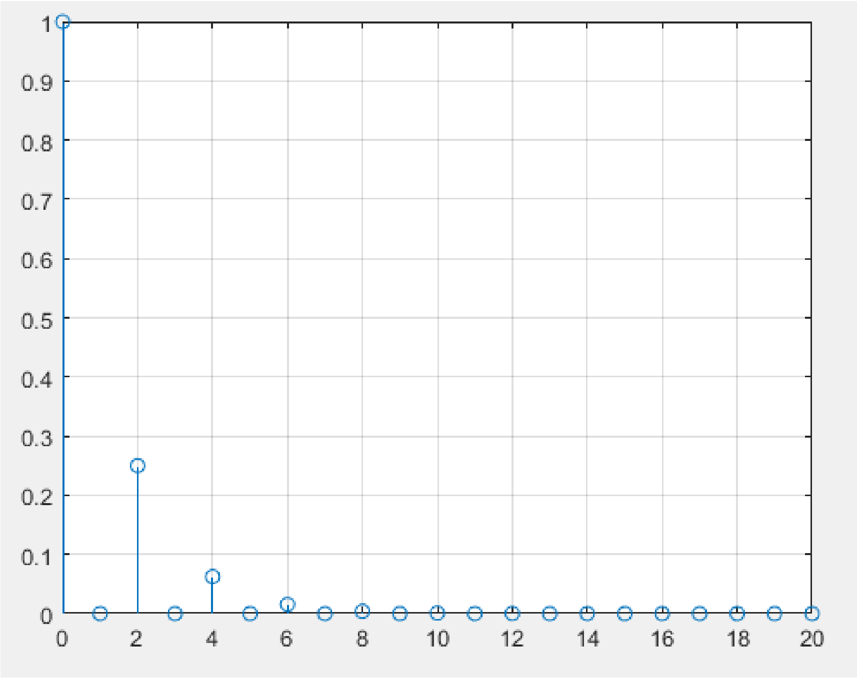
Assignment – Question 2 …

Show by experiment if the following LTI systems are stable or not:

(3) $y[n]-\frac{1}{4}y[n-2]=x[n]$, system initially at rest.

```
n=0:20;
B1=[1 0 0];
A1=[1 0 -0.25];
h1=impz(B1,A1,n);
stem(n,h1), grid on;
```



Stable!

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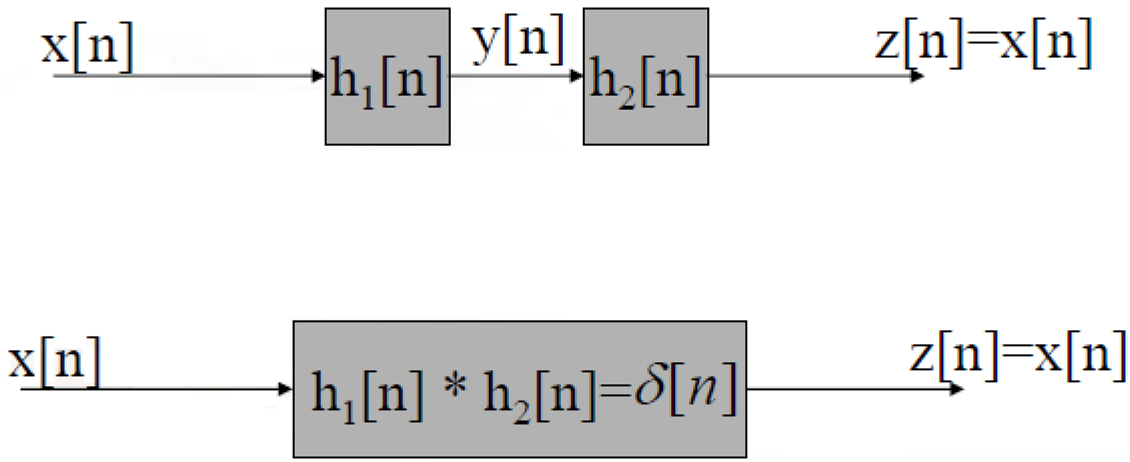
Invertibility of LTI Systems

- To see if a LTI system is the inverse of the other LTI system, check if the convolution of their impulse response is a delta function.



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Assignment – Question 3

Show by experiment which of the systems listed below (a, b, or c) is the inverse of the following LTI discrete time system:

$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$, system initially at rest.

(1) $y[n]=\frac{1}{6}x[n]-\frac{5}{6}x[n-1]+x[n-2]$, system initially at rest.

(2) $y[n]=x[n]-\frac{5}{6}x[n-1]+\frac{1}{6}x[n-2]$, system initially at rest.

(3) $\frac{1}{6}y[n]-\frac{5}{6}y[n-1]+y[n-2]=x[n]$, system initially at rest.

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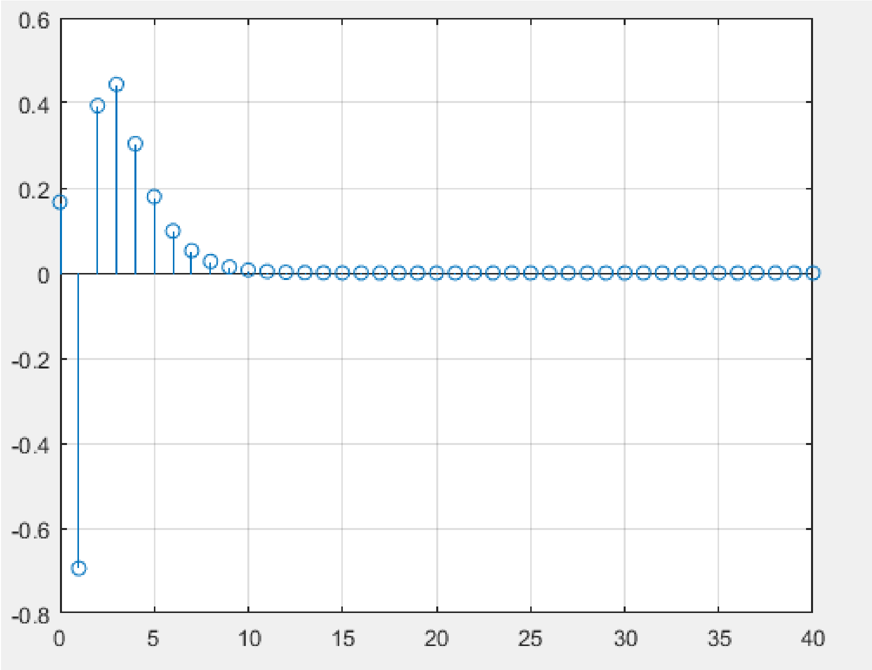
Assignment – Question 3 … (1)

- To consider if $y[n]=\frac{1}{6}x[n]-\frac{5}{6}x[n-1]+x[n-2]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A1=[1 0 0];
B1=[1/6 -5/6 1];
h1=impz(B1,A1,n);
ht=conv(h,h1);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



No, it is not.

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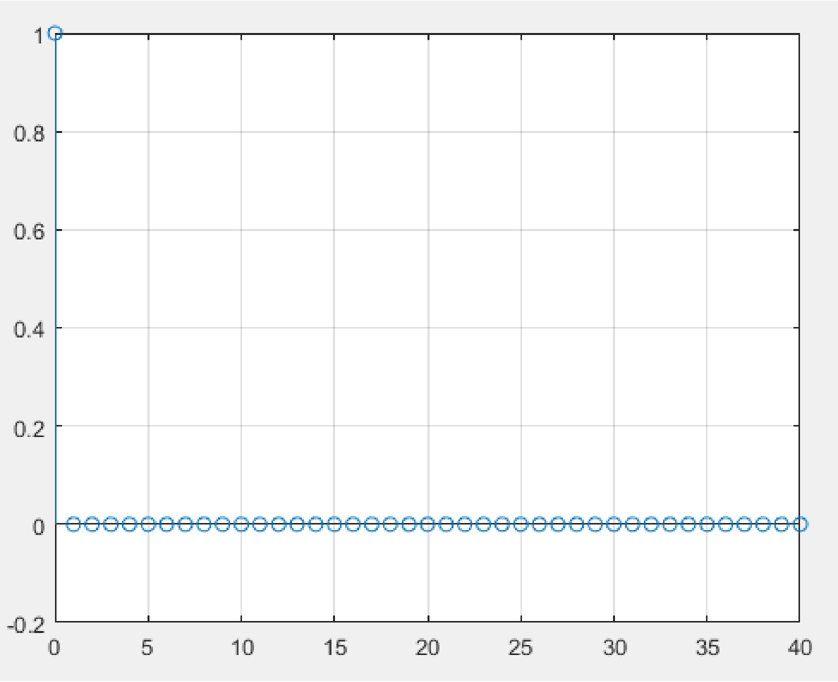
Assignment – Question 3 …(2)

- To consider if $y[n]=x[n]-\frac{5}{6}x[n-1]+\frac{1}{6}x[n-2]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A2=[1 0 0];
B2=[1 -5/6 1/6];
h2=impz(B1,A1,n);
ht=conv(h,h2);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



Yes, it is.

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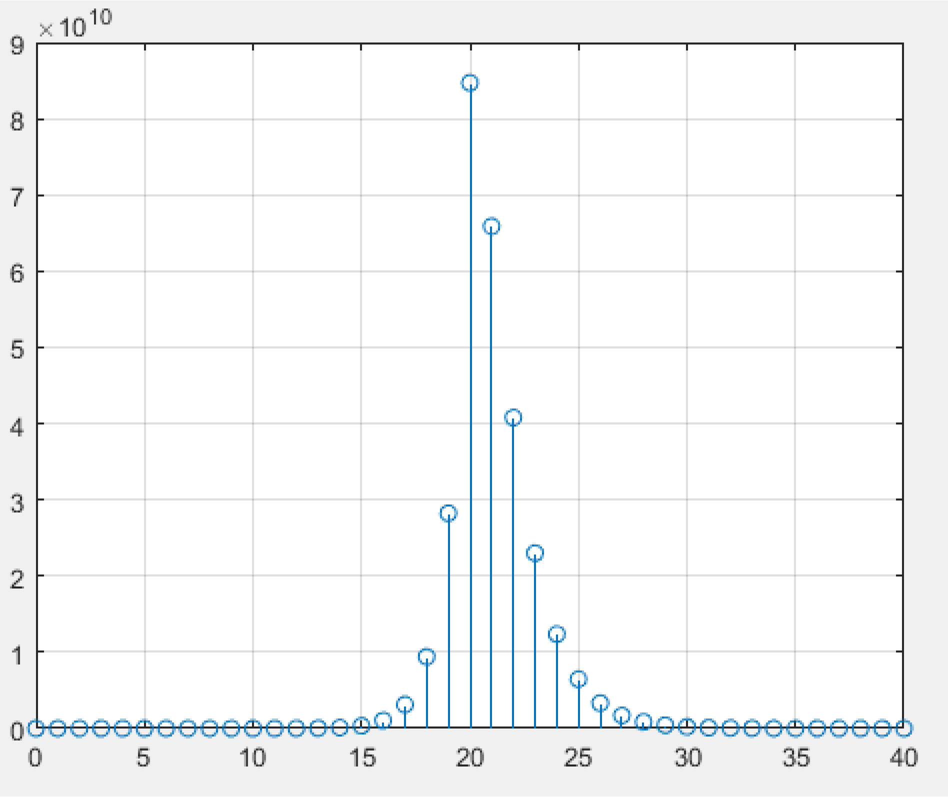
Assignment – Question 3 …(3)

- To consider if $\frac{1}{6}y[n]-\frac{5}{6}y[n-1]+y[n-2]=x[n]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A3=[1/6 -5/6 1];
B3=[1 0 0];
h3=impz(B1,A1,n);
ht=conv(h,h3);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



No, it is not.

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Verification of an inverse system with an audio signal

In the previous lab, the file "Audio1.wav" was applied to a discrete time LTI system whose impulse response was given by:

$$h[n] = 0.1 * (0.99)^n \quad 0 \le n \le 1000$$

Now, the following system is the inverse of that previous system:

$$y[n] = 10x[n] - \frac{99}{10}x[n-1]$$ system initially at rest.

Verify this
a) by using the impulse responses of the two systems;
b) by applying the output of the first system (as obtained in the previous lab) to the input of the inverse system. The output of the inverse system should then be quasi-identical to the original signal in "Audio1.wav". Visualize the resulting signals and listen to the resulting signals.

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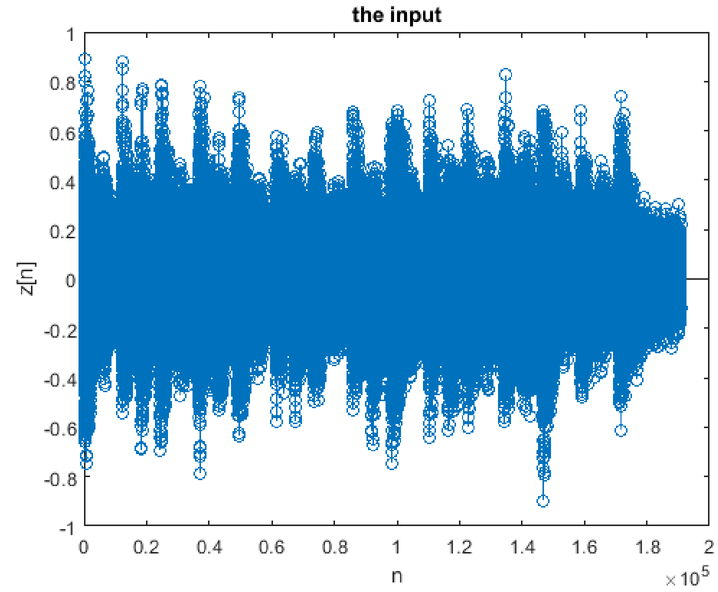
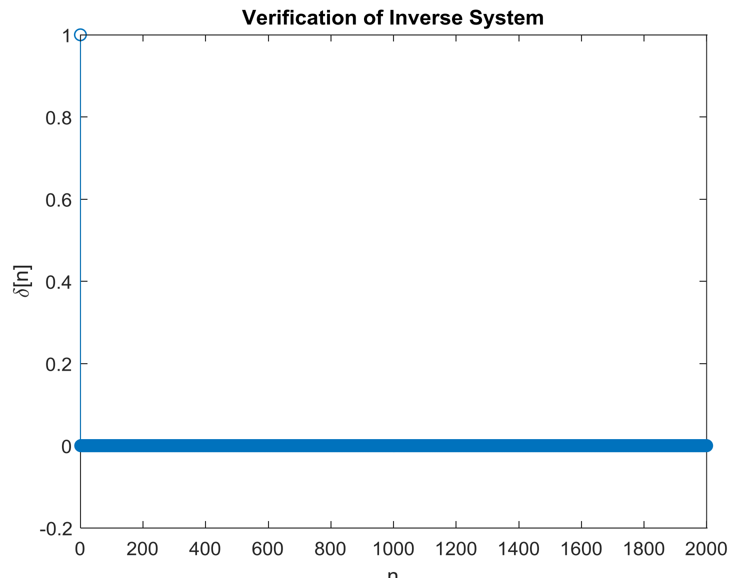
```
clear all; close all;
%******Save it from webpage******
webpage = 'http://www.site.uottawa.ca/~hjlee103/';
url = [webpage 'courses/ELG3125/Audio1.wav'];
websave('Audio1.wav',url);
%*************Done*************
[x,fs]=audioread('Audio1.wav');
n_x=0:length(x)-1;
figure(1); stem(n_x,x);
xlabel('n');ylabel('z[n]');
title('the input');
sound(x,fs);
```




```
n_h=0:1000;
a=1; b=0.1*(0.99.^n_h);
h=b; % (h=b when a=1) OR h=impz(b,a);
y_n=filter(b,a,x);
%************(1)*************
a_inv=[1 0];
b_inv=[10 -99/10];
h0=impz(b_inv,a_inv,n_h);
delta=conv(h,h0);
d_x=0:length(delta)-1;
figure(2); stem(d_x,delta);
xlabel('n'); ylabel('\delta[n]');
title('Verification of Inverse System');
```



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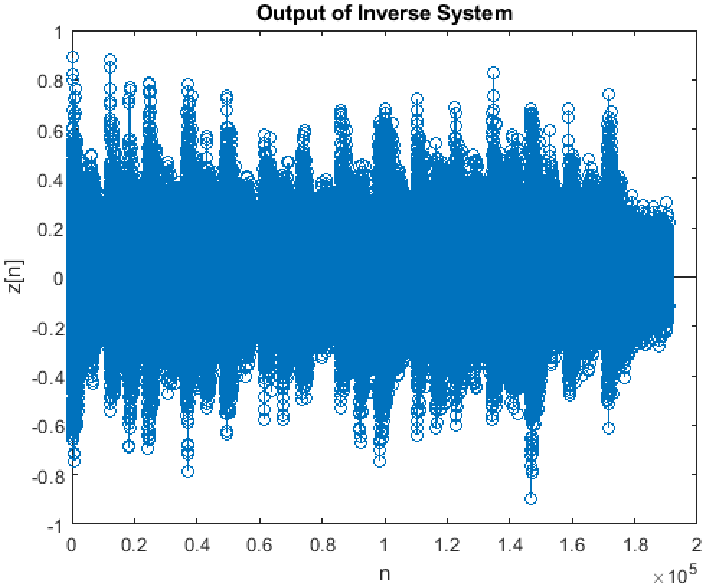
```
%************(2)*************
z=filter(b_inv,a_inv,y_n);
figure(3);
stem(n_x,z);
xlabel('n'); ylabel('z[n]');
title('Output of Inverse
System');
sound(z,fs);
```



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The END