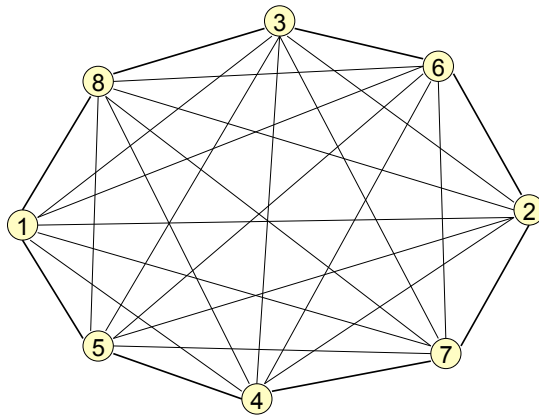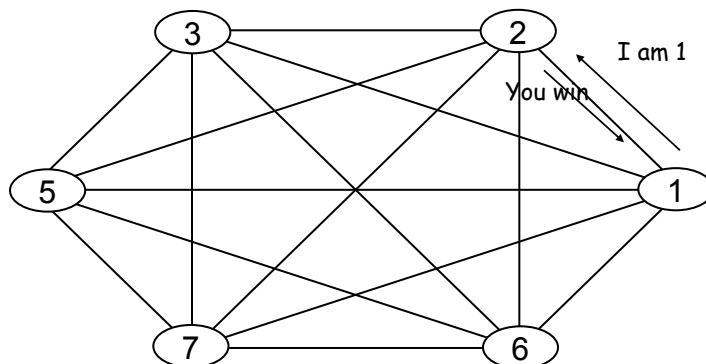# Election in the Complete Graph

Paola Flocchini

Trivial Algorithm.

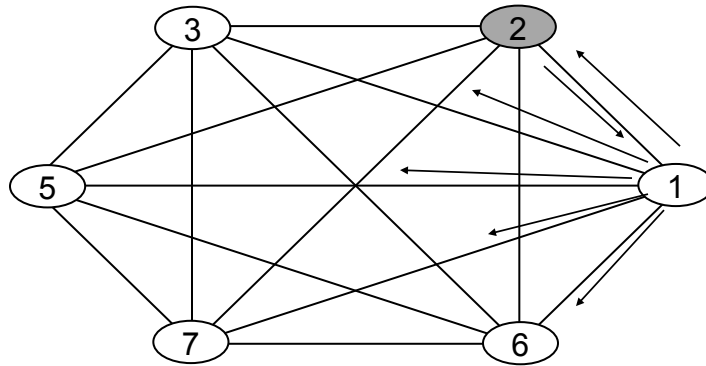Ask neighbours one at a time

I am 1

You win

Paola Flocchini

Trivial Algorithm.

Ask neighbours one at a time

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Message Complexity

$O(n^2)$

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Better Algorithm

Ideas:
- In stages
- Territory acquisition (capture neighbours)
    ensuring that a node is captured by at most
    one candidate in the same stage
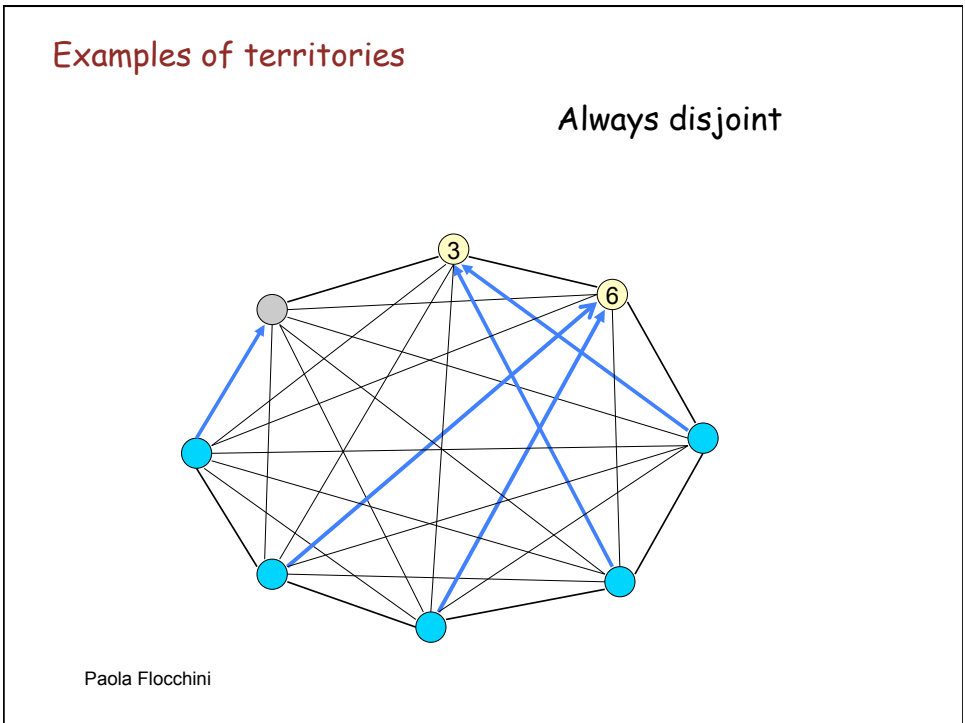- Disjoint territories

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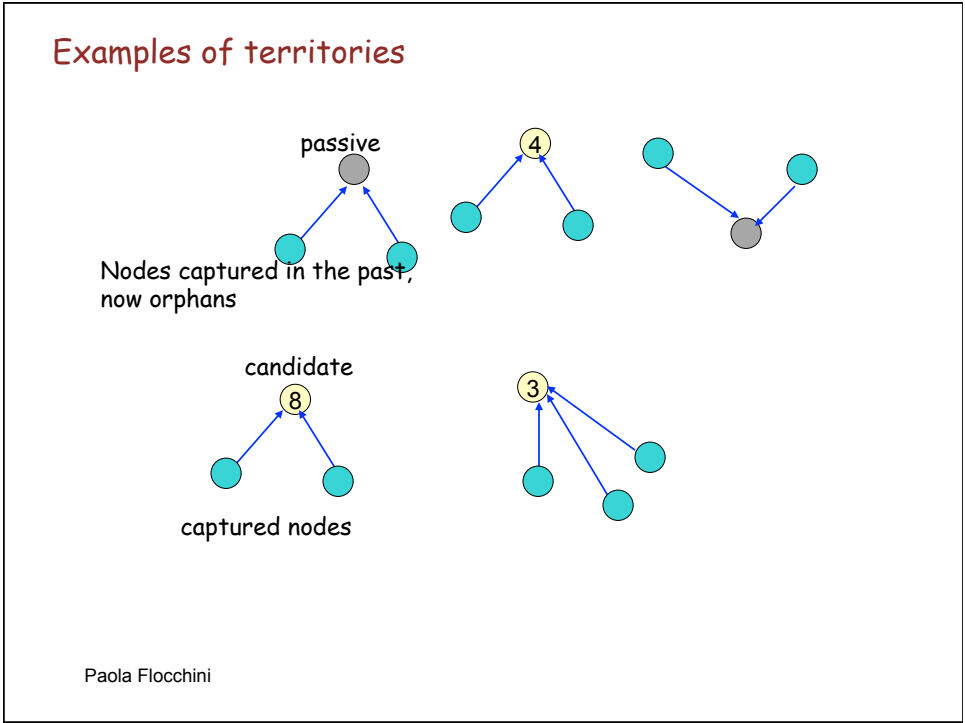A node attacks another node, if successful
it captures the node increasing the size of its territory
(= stage number)

Defeated nodes become captured (belonging
to a owner) and stop attacking

CANDIDATE: still playing trying to increase the territory
PASSIVE: transitional phase, will not attack anymore,
               will eventually become captured
CAPTURED: belong to a territory, owned by a candidate

Examples of territories

passive

4

Nodes captured in the past, now orphans

candidate

8

captured nodes

3

Examples of territories

Always disjoint

3

6

The Attack - general rule

Bigger territories win over smaller ones (i.e. higher stages)
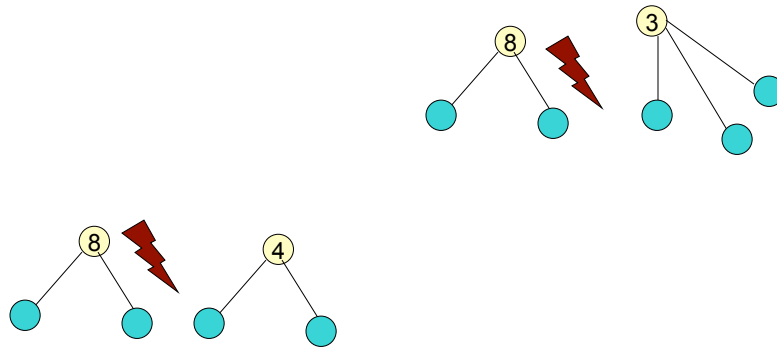
In case of tie, smaller Ids win

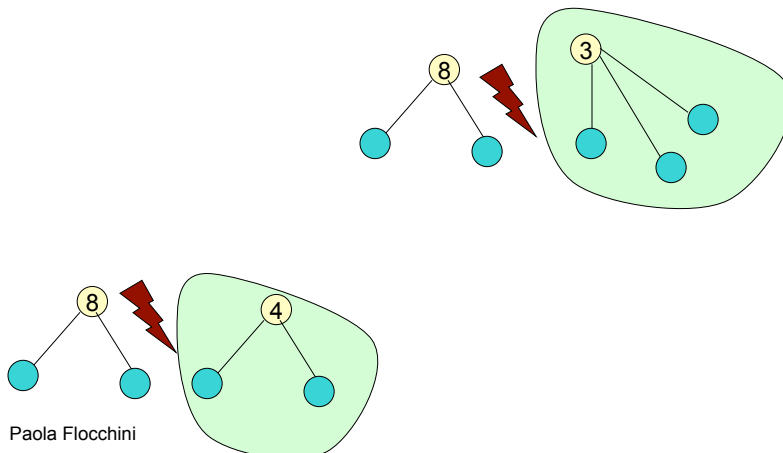Paola Flocchini

The Attack - general rule

Bigger territories win over smaller ones (i.e. higher stages)

In case of tie, smaller Ids win

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The Attack

Stage 0

(8)

An attack is always originated by a
CANDIDATE

A CANDIDATE knows the size of its territory
(the stage number, which is initially 0 and is increased
after each successful attack)

An attack could reach CANDIDATE PASSIVE CAPTURED

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The Attack

Stage 1

(8)

An attack is always originated by a
CANDIDATE

A CANDIDATE knows the size of its territory
(the stage number, which is initially 0 and is increased
after each successful attack)

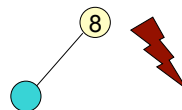An attack could reach CANDIDATE PASSIVE CAPTURED

Paola Flocchini

The Attack
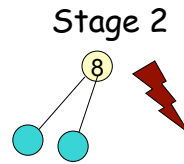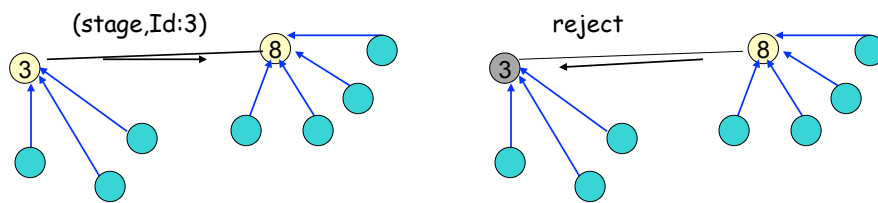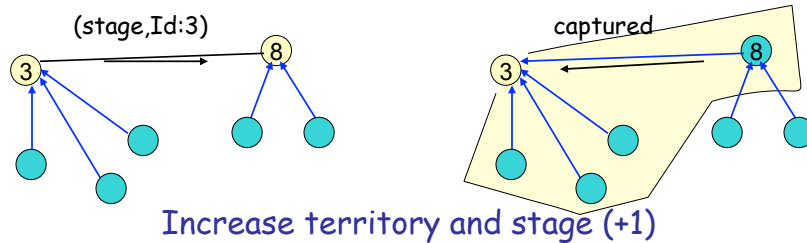
Stage 2

An attack is always originated by a CANDIDATE

A CANDIDATE knows the size of its territory
(the stage number, which is initially 0 and is increased after each successful attack)
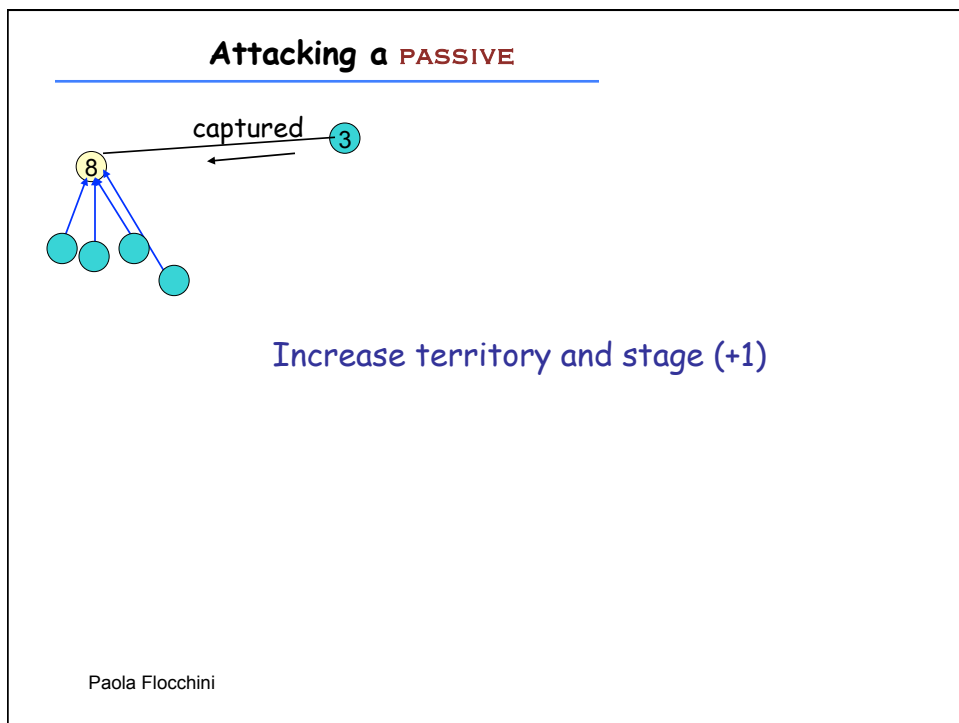
An attack could reach CANDIDATE PASSIVE CAPTURED

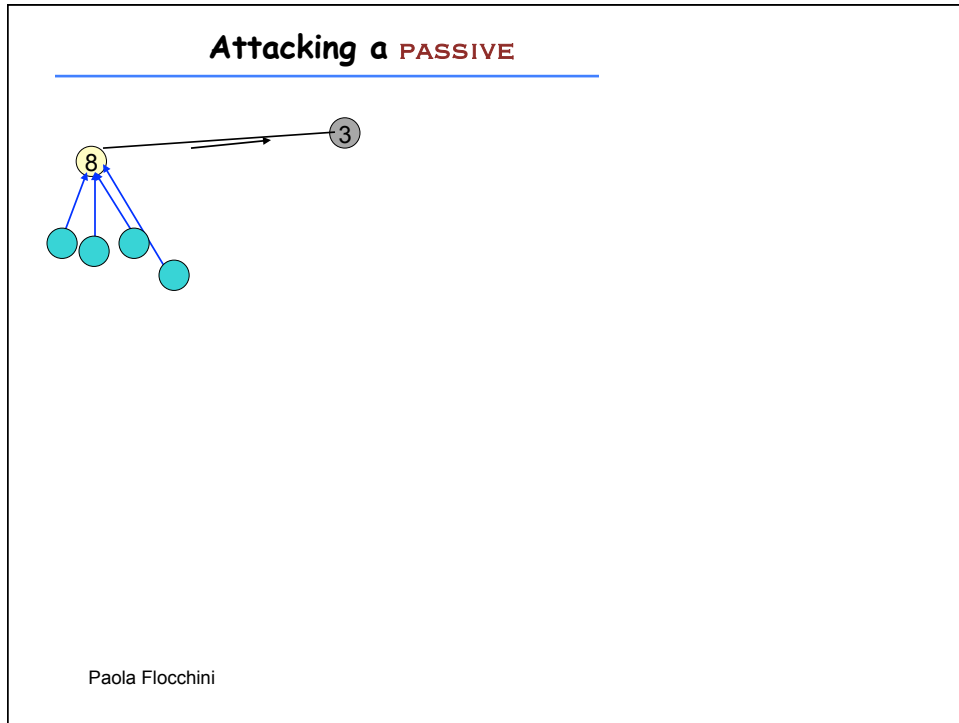
Attacking a CANDIDATE

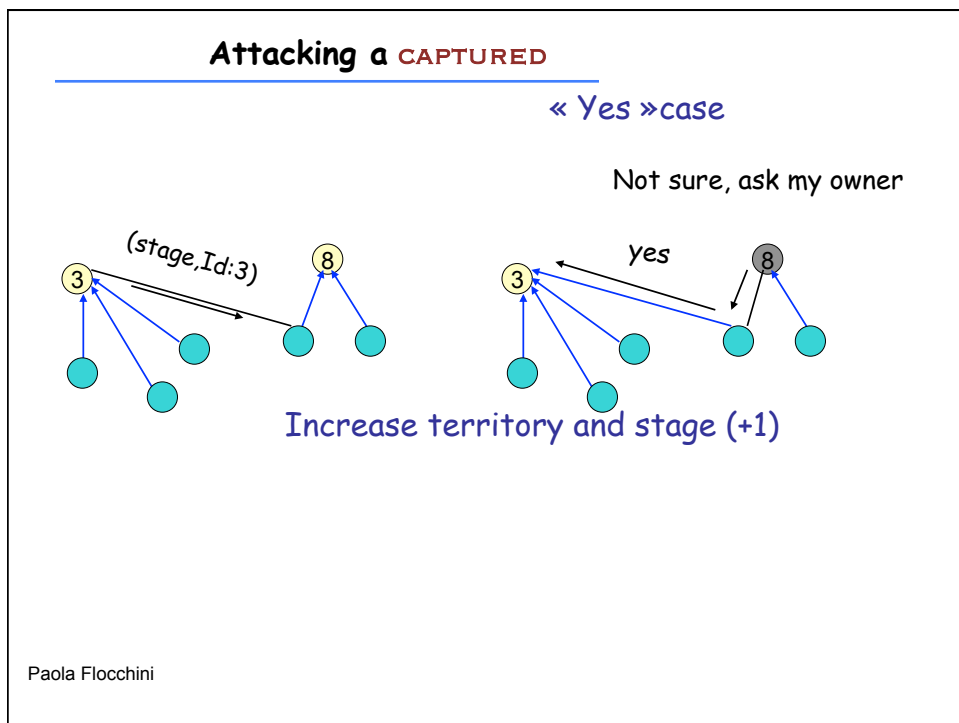Send capture message to one neighbour

(stage,Id:3)

captured

Increase territory and stage (+1)

(stage,Id:3)

reject

If still candidate, 3 becomes passive

Attacking a PASSIVE

Attacking a PASSIVE

captured

Increase territory and stage (+1)

Attacking a CAPTURED

Not sure, ask my owner

(stage,Id:3)

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Attacking a CAPTURED

« Yes »case

Not sure, ask my owner

(stage,Id:3)

yes

Increase territory and stage (+1)

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Attacking a CAPTURED

« No »case

Not sure, ask my owner

(stage,Id:3)

no

3 becomes passive, if still candidate

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PASSIVE

CANDIDATE

CAPTURED

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When to terminate ?

When does a candidate become a leader ?

When it captures more than n/2 nodes

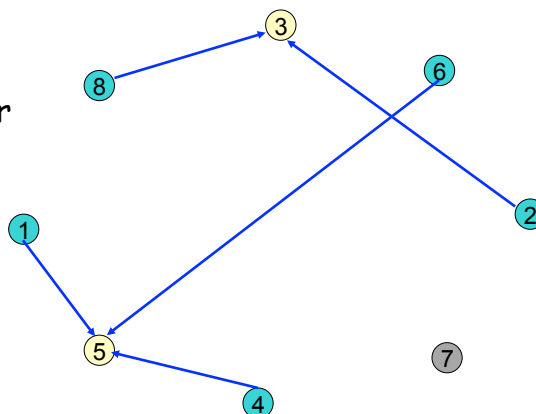If a candidate has captured more than n/2 nodes nobody else can become leader

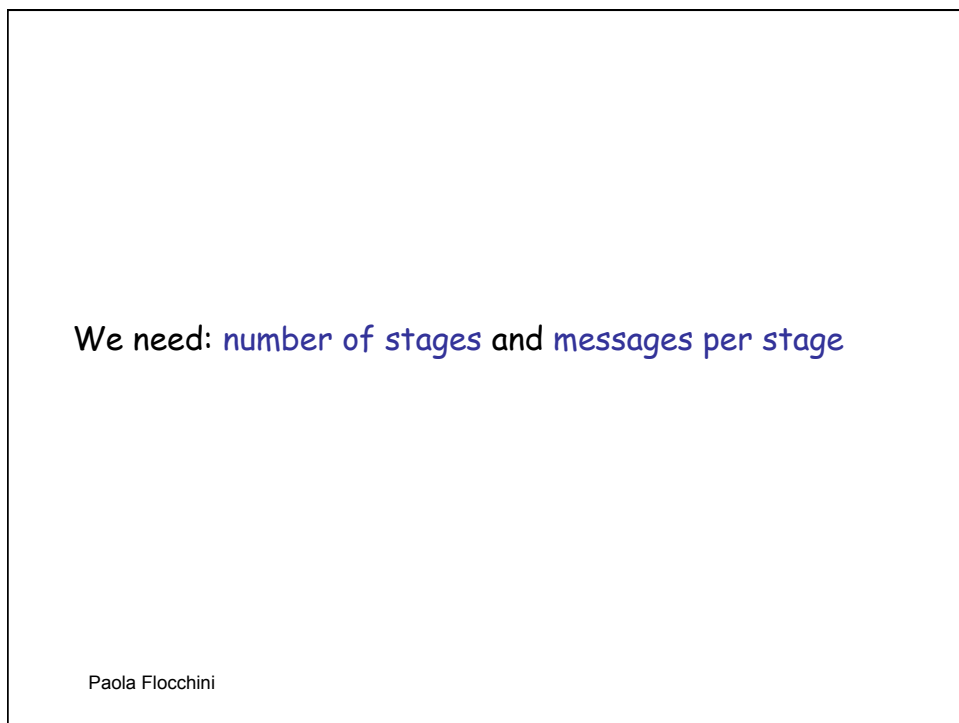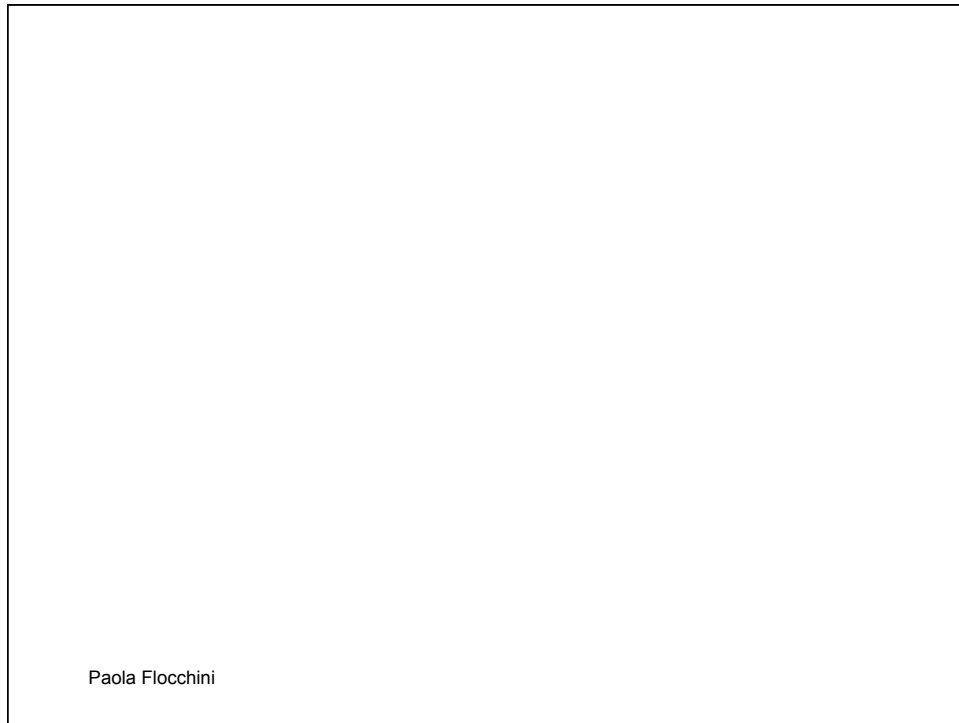Paola Flocchini

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Important

The territories of any two candidates are disjoint

Because at any time, any node has only ONE owner.

Each territory is
rooted in its owner

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We need: number of stages and messages per stage

Messages per attack

candidate --- candidate   2 msgs

candidate --- passive   2 msgs

candidate --- captured   4 msgs

At most 4 messages per attack

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Number of stages

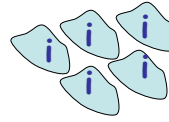A candidate with n/2 +1 captured nodes becomes leader and notify

n/2 +1 stages

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How many candidates in each stage ?

Stage i ---> territory of size i

With disjoint territories

There cannot be more than **n/i** candidates in stage i

$$n_i \leq n/i$$

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Message Complexity

$n_i \leq n/i$

At most 4 messages per attack

Messages in stage i $\leq$ 4 n/i

Harmonic number $H_{n/2}$

$= O(\log n)$

$$O\left(\sum_{1}^{n/2} 4\, n/i \right) = O\left(4\, n \sum_{1}^{n/2} 1/i \right)$$

M(completeElect) = O(n log n)

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