

	$f(\mathbf{x}) = \int_{\mathbb{R}^D} F(\mathbf{u}) \exp(j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{u}$	$F(\mathbf{u}) = \int_{\mathbb{R}^D} f(\mathbf{x}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{x}$
(i)	$af_1(\mathbf{x}) + bf_2(\mathbf{x})$	$aF_1(\mathbf{u}) + bF_2(\mathbf{u})$
(ii)	$f(\mathbf{x} - \mathbf{x}_0)$	$F(\mathbf{u}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}_0)$
(iii)	$f(\mathbf{x}) \exp(j2\pi\mathbf{u}_0 \cdot \mathbf{x})$	$F(\mathbf{u} - \mathbf{u}_0)$
(iv)	$f^*(\mathbf{x})$	$F^*(-\mathbf{u})$
(v)	$f(\mathbf{A}\mathbf{x})$	$\frac{1}{ \det \mathbf{A} } F(\mathbf{A}^{-T}\mathbf{u})$
(vi)	$f_1(\mathbf{x}) * f_2(\mathbf{x})$	$F_1(\mathbf{u})F_2(\mathbf{u})$
(vii)	$f_1(\mathbf{x})f_2(\mathbf{x})$	$F_1(\mathbf{u}) * F_2(\mathbf{u})$
(viii)	$f_1(x)f_2(y)$	$F_1(u)F_2(v)$
(ix)	$\int_{\mathbb{R}^D}  f(\mathbf{x}) ^2 d\mathbf{x} = \int_{\mathbb{R}^D}  F(\mathbf{u}) ^2 d\mathbf{u}$	

Multidimensional Fourier transform properties.

$f(\mathbf{x}) = \int_{\mathbb{R}^D} F(\mathbf{u}) \exp(j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{u}$	$F(\mathbf{u}) = \int_{\mathbb{R}^D} f(\mathbf{x}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{x}$
$\text{rect}(x, y)$	$\frac{\sin \pi u}{\pi u} \frac{\sin \pi v}{\pi v}$
$\text{circ}(x, y)$	$\frac{1}{\sqrt{u^2+v^2}} J_1(2\pi\sqrt{u^2+v^2})$
$\exp(-(x^2 + y^2)/2r^2)$	$2\pi r^2 \exp(-2\pi^2(u^2 + v^2)r^2)$
$\cos(\pi(x^2 + y^2)/r^2)$	$r^2 \sin(\pi(u^2 + v^2)r^2)$
$\exp(j\pi(x^2 + y^2)/r^2)$	$jr^2 \exp(-j\pi(u^2 + v^2)r^2)$
$\delta(\mathbf{x})$	1

Multidimensional Fourier transform of selected functions.

	$f[\mathbf{x}] = d(\Lambda) \int_{\mathcal{P}^*} F(\mathbf{u}) \exp(j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{u}$	$F(\mathbf{u}) = \sum_{\mathbf{x} \in \Lambda} f[\mathbf{x}] \exp(-j2\pi\mathbf{u} \cdot \mathbf{x})$
(i)	$af_1[\mathbf{x}] + bf_2[\mathbf{x}]$	$aF_1(\mathbf{u}) + bF_2(\mathbf{u})$
(ii)	$f[\mathbf{x} - \mathbf{x}_0]$	$F(\mathbf{u}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}_0)$
(iii)	$f[\mathbf{x}] \exp(j2\pi\mathbf{u}_0 \cdot \mathbf{x})$	$F(\mathbf{u} - \mathbf{u}_0)$
(iv)	$f^*[\mathbf{x}]$	$F^*(-\mathbf{u})$
(v)	$f_1[\mathbf{x}] * f_2[\mathbf{x}]$	$F_1(\mathbf{u})F_2(\mathbf{u})$
(vi)	$f_1[\mathbf{x}]f_2[\mathbf{x}]$	$d(\Lambda) \int_{\mathcal{P}^*} F_1(\mathbf{r})F_2(\mathbf{u} - \mathbf{r}) d\mathbf{r}$
(vii)	$\sum_{\mathbf{x} \in \Lambda}  f[\mathbf{x}] ^2 = d(\Lambda) \int_{\mathcal{P}^*}  F(\mathbf{u}) ^2 d\mathbf{u}$	

Properties of the multidimensional Fourier transform over a lattice  $\Lambda$ .

### Formulas

$$\exp(jX) = \cos(X) + j \sin(X), \quad j = \sqrt{-1}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{rect}(x, y) = \begin{cases} 1, & \text{if } |x| \leq 0.5 \text{ and } |y| \leq 0.5; \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{circ}(x, y) = \begin{cases} 1, & \text{if } x^2 + y^2 \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

Affine transformation:  $\mathcal{Q}_{\mathbf{A}, \mathbf{d}} : \vec{\mathbf{g}} = \mathcal{Q}_{\mathbf{A}, \mathbf{d}} \vec{\mathbf{f}} : g(\mathbf{x}) = f(\mathbf{A}(\mathbf{x} - \mathbf{d}))$

If  $\Lambda = \text{LAT}(\mathbf{V})$ , then  $d(\Lambda) = |\det(\mathbf{V})|$ , and  $\Lambda^* = \text{LAT}(\mathbf{V}^{-T})$ .

#### Sampling

If  $f[\mathbf{x}] = f_c(\mathbf{x})$ ,  $\mathbf{x} \in \Lambda$  then

$$F(\mathbf{u}) = \frac{1}{d(\Lambda)} \sum_{\mathbf{r} \in \Lambda^*} F_c(\mathbf{u} + \mathbf{r})$$