

3. The primaries for the 1960 CIE Uniform Chromaticity Scale (UCS) color space are defined in terms of the standard CIE XYZ primaries $[X]$, $[Y]$ and $[Z]$ as follows:

$$[U] = 1.5[X] + 1.5[Z] \quad (1)$$

$$[V] = [Y] - 3[Z] \quad (2)$$

$$[W] = 2[Z] \quad (3)$$

An arbitrary color $[Q]$ is expressed in the XYZ space and the UVW space respectively as

$$[Q] = Q_X[X] + Q_Y[Y] + Q_Z[Z] \quad (4)$$

$$= Q_U[U] + Q_V[V] + Q_W[W] \quad (5)$$

- 3(a) The tristimulus values Q_U , Q_V and Q_W can be obtained from Q_X , Q_Y and Q_Z using the matrix equation

$$\begin{bmatrix} Q_U \\ Q_V \\ Q_W \end{bmatrix} = \mathbf{A} \begin{bmatrix} Q_X \\ Q_Y \\ Q_Z \end{bmatrix}$$

Using the information provide above, determine the matrix \mathbf{A} from first principles. Do not quote any unproved (by you) results from the notes.

Formula which may be useful:

$$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ f & e & c \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{d}{ab} & \frac{1}{b} & 0 \\ \frac{de-bf}{abc} & -\frac{e}{bc} & \frac{1}{c} \end{bmatrix}$$

- 3(b) Determine the XYZ chromaticities of the four colors $[U]$, $[V]$, $[W]$, and $[U] + [V] + [W]$ and plot them on the xy chromaticity diagram below. Which, if any, of the four colors are physically realizable? Explain.

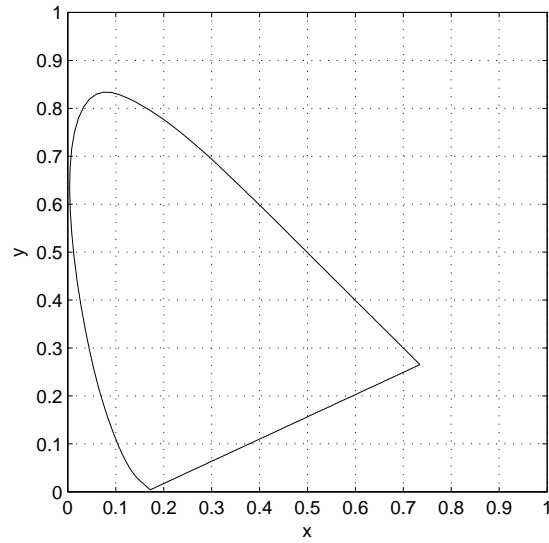


Figure 3: CIE XYZ chromaticity diagram.

3(c) A light $[Q]$ consisting of the superposition of two monochromatic (spectral) lights has the power density spectrum approximated by

$$Q(\lambda) = \delta(\lambda - 600) + 0.5\delta(\lambda - 520)$$

where λ is measured in nanometers (nm). Estimate the XYZ tristimulus values of $[Q(\lambda)]$ using the plotted XYZ color matching functions. What are the corresponding UVW tristimulus values. Give these values in equations with correct notation. Your calculations do not have to be very precise, and you may give answers as arithmetic expressions. Do explain what you are doing.

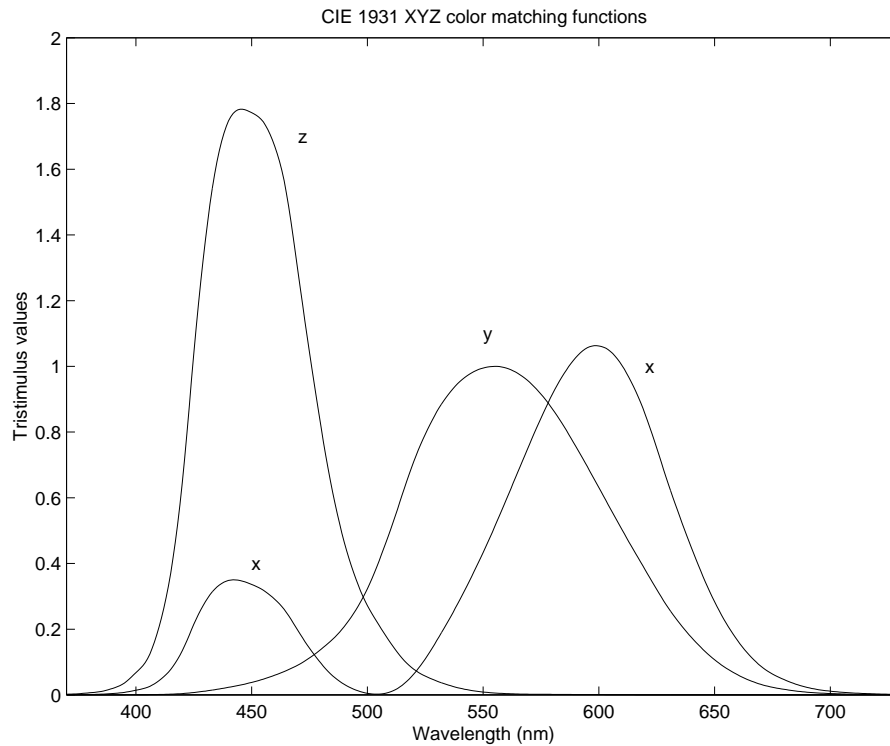


Figure 4: CIE XYZ color matching functions.

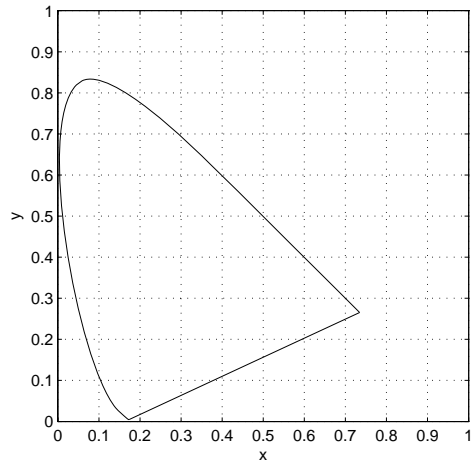
3. The recommendation 709 RGB primaries can be expressed in terms of the CIE XYZ primaries by

$$\begin{bmatrix} [R] \\ [G] \\ [B] \end{bmatrix} = \begin{bmatrix} 0.41 & 0.21 & 0.02 \\ 0.36 & 0.72 & 0.12 \\ 0.18 & 0.07 & 0.95 \end{bmatrix} \begin{bmatrix} [X] \\ [Y] \\ [Z] \end{bmatrix}.$$

Consider the cyan, magenta and yellow (CMY) primaries used in printing. These are given by $[C] = [B] + [G]$, $[M] = [R] + [B]$ and $[YE] = [R] + [G]$.

- (a) Determine the *tristimulus values* of $[C]$, $[M]$ and $[YE]$ with respect to the XYZ primaries. Compute the XYZ *chromaticities* of $[C]$, $[M]$ and $[YE]$ and plot them on the xy chromaticity diagram on the next page. Comment on the suitability of cyan, magenta and yellow as primaries for an additive color display device like a cathode ray tube (CRT).

3(a) Continued.



- (b) Suppose that $[C]$, $[M]$ and $[YE]$ as in part (a) are taken as primaries in a color system. Determine the tristimulus values of a monochromatic light $\delta(\lambda - 650nm)$ with respect to these primaries. You will need to use the XYZ color matching functions tabulated on the next page. Carefully explain all steps. Can the given light be *physically* synthesized as a sum of a positive quantity of the $[C]$, $[M]$ and $[YE]$ primaries? You may use the expression

$$\begin{bmatrix} [X] \\ [Y] \\ [Z] \end{bmatrix} = \begin{bmatrix} -2.08 & 2.13 & 1.12 \\ 1.60 & -1.80 & 0.26 \\ 0.80 & 0.26 & -0.76 \end{bmatrix} \begin{bmatrix} [C] \\ [M] \\ [YE] \end{bmatrix}$$

but explain how you obtain this equation from the results in part (a) of this question.

\bar{x}	\bar{y}	\bar{z}	λ
0.0143	0.0004	0.0679	400 nm
0.0435	0.0012	0.2074	
0.1344	0.0040	0.6456	
0.2839	0.0116	1.3856	
0.3483	0.0230	1.7471	
0.3362	0.0380	1.7721	450 nm
0.2908	0.0600	1.6692	
0.1954	0.0910	1.2876	
0.0956	0.1390	0.8130	
0.0320	0.2080	0.4652	
0.0049	0.3230	0.2720	500 nm
0.0093	0.5030	0.1582	
0.0633	0.7100	0.0782	
0.1655	0.8620	0.0422	
0.2904	0.9540	0.0203	
0.4334	0.9950	0.0087	550 nm
0.5945	0.9950	0.0039	
0.7621	0.9520	0.0021	
0.9163	0.8700	0.0017	
1.0263	0.7570	0.0011	
1.0622	0.6310	0.0008	600 nm
1.0026	0.5030	0.0003	
0.8544	0.3810	0.0002	
0.6424	0.2650	0.0000	
0.4479	0.1750	0.0000	
0.2835	0.1070	0.0000	650 nm
0.1649	0.0610	0.0000	
0.0874	0.0320	0.0000	
0.0468	0.0170	0.0000	
0.0227	0.0082	0.0000	
0.0114	0.0041	0.0000	700 nm

Color matching functions of XYZ primaries.