CEG4316 Tutorial Sept. 27, 2013

1. A class of discrete space images is defined on the lattice

$$\Lambda = \mathrm{LAT}\left(\begin{bmatrix} 4X & 2X \\ 0 & 1.5X \end{bmatrix}\right)$$

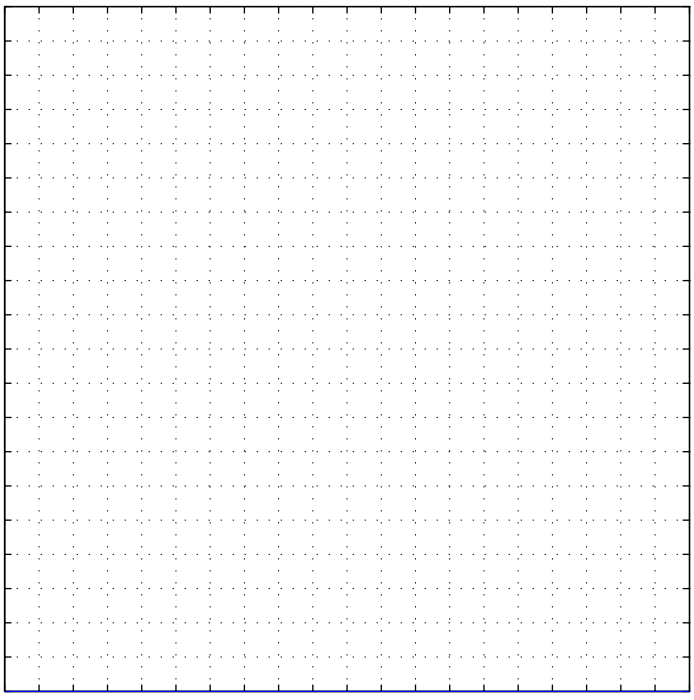
where $X = \frac{1}{100}$ ph and the aspect ratio is $ar = \frac{4}{3}$.

1(a) What is the sampling density and the approximate number of samples in one image?

1(b) A moving average filter defined on this lattice is given by

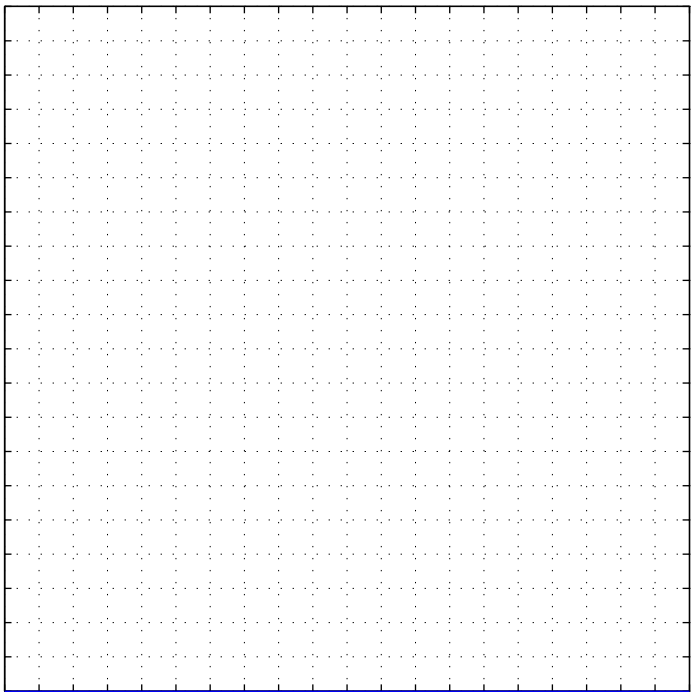
$$h[x,y] = \begin{cases} \frac{1}{5} & (x,y) = (0,0) \quad \text{or} \quad (x,y) = (\pm 2X, \pm 1.5X) \\ 0 & \text{otherwise.} \end{cases}$$

There are five non-zero coefficients. Sketch the points of Λ for $-5X \leq x \leq 5X$, $-5X \leq y \leq 5X$. Indicate $h[x,y]$ on your figure in the usual way.



1(c) Compute the frequency response $H(u, v)$ of the filter and express it in real form. What is the DC gain of this filter?

1(d) Sketch the points of the reciprocal lattice Λ^* for $-\frac{1}{X} \leq u, v \leq \frac{1}{X}$ and show the region corresponding to one period of $H(u, v)$ given by the Voronoi unit cell of Λ^*.



2. The two-dimensional discrete-space signal f is defined on a rectangular lattice with equal horizontal and vertical sample spacing $X = 1$. The signal f is applied to a linear shift-invariant system with unit-sample response $h[n_1, n_2]$ to give the output signal g. Assume that

$$f[n_1, n_2] = \begin{cases} 1 & \text{for all } (n_1, n_2) \text{ such that } 0 \leq n_1 \leq 2 \text{ AND } -1 \leq n_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$h[n_1, n_2] = \begin{cases} 1/2 & (n_1, n_2) = (0, 0) \\ 1/8 & (n_1, n_2) = (\pm 1, 0), (n_1, n_2) = (0, \pm 1) \\ 0 & \text{otherwise} \end{cases}$$

$f[n_1, n_2]$ has nine non-zero samples and $h[n_1, n_2]$ has five non-zero samples.

(a) What are the indices (n_1, n_2) for which $\mathbf{g}[n_1, n_2]$ is non-zero? (Hint: It may help in (a) and (b) to visualize the situation graphically.)

(b) Compute $g[n_1, n_2]$ using the convolution formula. Specify the result completely (i.e. for all (n_1, n_2))using the method of your choice.

(c) State (and justify) whether the unit sample response satisfies

1. separability
2. linear phase
3. quadrantal symmetry

(d) Determine the frequency response $H(u_1, u_2)$ of the filter. Sketch a contour plot of $H(u_1, u_2)$ for $-1 \leq u_1, u_2 \leq 1$ c/ph. (Just show a few labeled contours, approximately but neatly, demonstrating clearly any periodicity. It will help to tabulate a few values first.)