1. Consider the crosshatch image shown in Fig. 1 below.

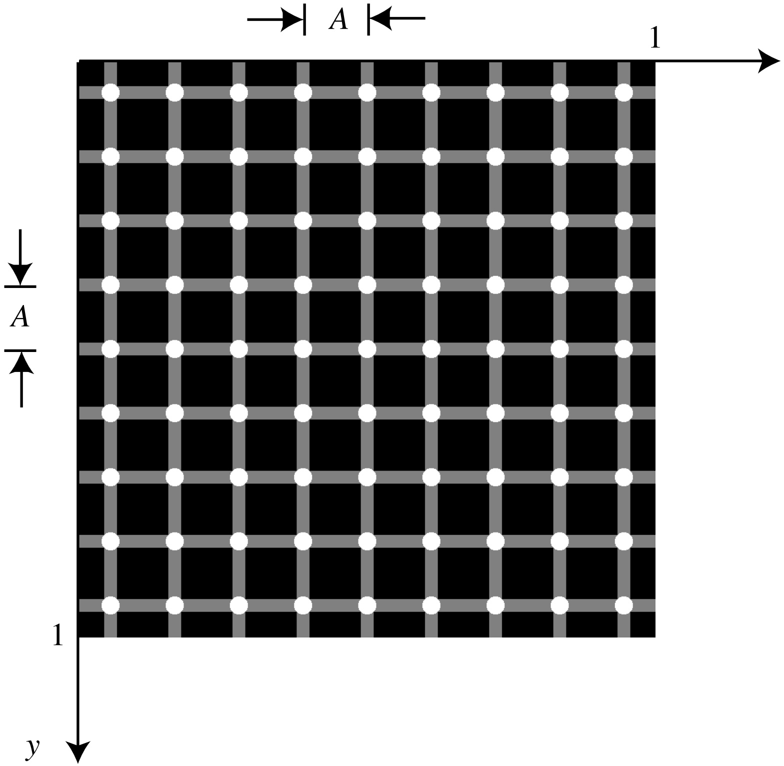

Figure 1: Crosshatch image.

1(a) Is this a still image or a time-varying image? Gaze generally at the image. What do you notice? Stare at one of the white dots. What do you notice now? Comment on your observations. Don't look at the image longer than you have to and hide it if it bothers you.

1(b) This image, considered as a continuous-space image, can be generated by repeating a basic function $f_A(x, y)$ on the points of a rectangular lattice with horizontal and vertical spacing A. The function $f_A(x, y)$ is illustrated in Fig. 2.

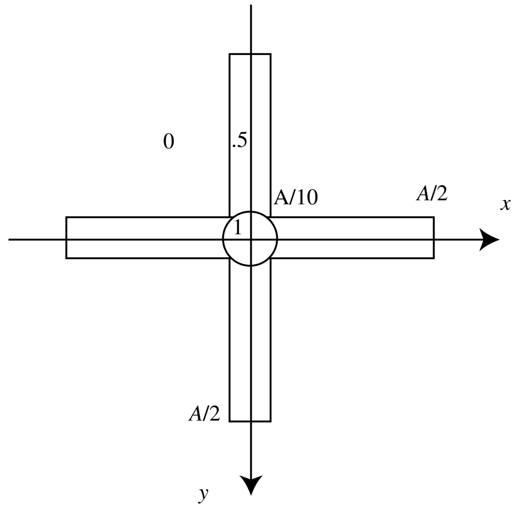

Figure 2: Basic function $f_A(x, y)$.

Note that the circle is completely inscribed within the plus-shaped figure and touches it at the four points $(\pm A/10, \pm A/10)$. Express this function $f_A(x, y)$ in terms of the standard rect function and circ function. There are several ways to do this, but a good way will give a solution of the form

$$f_A(x, y) = a_1 \operatorname{rect}(c_{11}x, c_{12}y) + a_2 \operatorname{rect}(c_{21}x, c_{22}y) - a_3 \operatorname{rect}(c_{31}x, c_{32}y) + a_4 \operatorname{circ}(c_{41}x, c_{42}y)$$

for some choice of these parameters a_i and c_{ij}. Clearly explain your reasoning. Express your solution in terms of the parameter A. From Fig. 1, what is the value of A?

1(c) Determine the Fourier transform $F_A(u, v)$ of the basic function $f_A(x, y)$, using the attached tables. Explain clearly all steps and properties that you have used.

1. Let $f(x, y) = 0.5\,\mathrm{rect}(3(x-0.25), 3(y-0.5))$ and $h(x, y) = \mathrm{rect}(15x, 15y)$, where x and y are in ph.

(a) Sketch the region of support of $f(x, y)$ and $h(x, y)$ in the XY-plane (i.e., the area where these two signals are nonzero).

(b) Compute the two-dimensional convolution $f(x, y) * h(x, y)$ from the definition using integration in the spatial domain.

(c) Suppose that $f(x, y)$ is the input to a two-dimensional system, and the output of this system is computed as in (b). What can we say about this system?

(d) Determine the continuous-space Fourier transforms $F(u, v)$, $H(u, v)$ and $G(u, v)$ of the above three signals. Make liberal use of Fourier transform properties. What are the units of u and v?

(e) Continuing with question (c), what is the interpretation of $H(u, v)$?

1. A two-dimensional continuous-space zero-one function

$$p_A(x,y)=\begin{cases}1, & (x,y)\in A;\\ 0, & \text{otherwise,}\end{cases}$$

has an elliptical region of support A with semi-minor axis X and semi-major axis 2X, oriented at 45°, as shown in Fig. 1.

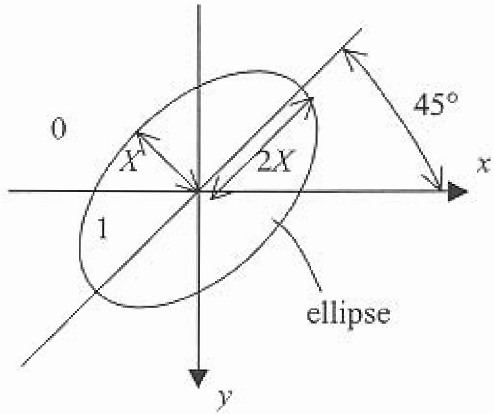

Fig. 1 Region of support of $p_A(x,y)$

(a) Show how $p_A(x,y)$ can be obtained from circ(x,y) by an affine transformation $Q_{\mathbf{A},\mathbf{d}}$. Clearly exhibit **A** and **d**. Recall that the equation of an ellipse with semi-major axis a in the horizontal direction and semi-minor axis b in the vertical direction is

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1.$$

(b) Determine $P_A(u,v)$, the Fourier transform of $p_A(x,y)$, using the Fourier transform properties.