	$f(\boldsymbol{x}) = \int_{\mathbb{R}^D} F(\boldsymbol{u}) \exp(j2\pi \boldsymbol{u} \cdot \boldsymbol{x})\, d\boldsymbol{u}$	$F(\boldsymbol{u}) = \int_{\mathbb{R}^D} f(\boldsymbol{x}) \exp(-j2\pi \boldsymbol{u} \cdot \boldsymbol{x})\, d\boldsymbol{x}$
(i)	$af_1(\boldsymbol{x}) + bf_2(\boldsymbol{x})$	$aF_1(\boldsymbol{u}) + bF_2(\boldsymbol{u})$
(ii)	$f(\boldsymbol{x} - \boldsymbol{x}_0)$	$F(\boldsymbol{u}) \exp(-j2\pi \boldsymbol{u} \cdot \boldsymbol{x}_0)$
(iii)	$f(\boldsymbol{x}) \exp(j2\pi \boldsymbol{u}_0 \cdot \boldsymbol{x})$	$F(\boldsymbol{u} - \boldsymbol{u}_0)$
(iv)	$f^*(\boldsymbol{x})$	$F^*(-\boldsymbol{u})$
(v)	$f(\mathbf{A}\boldsymbol{x})$	$\frac{1}{\lvert\det \mathbf{A}\rvert} F(\mathbf{A}^{-T}\boldsymbol{u})$
(vi)	$f_1(\boldsymbol{x}) * f_2(\boldsymbol{x})$	$F_1(\boldsymbol{u}) F_2(\boldsymbol{u})$
(vii)	$f_1(\boldsymbol{x}) f_2(\boldsymbol{x})$	$F_1(\boldsymbol{u}) * F_2(\boldsymbol{u})$
(viii)	$f_1(x) f_2(y)$	$F_1(u) F_2(v)$
(ix)	$\int_{\mathbb{R}^D} \lvert f(\boldsymbol{x})\rvert^2\, d\boldsymbol{x} = \int_{\mathbb{R}^D} \lvert F(\boldsymbol{u})\rvert^2\, d\boldsymbol{u}$	

Multidimensional Fourier transform properties.

$f(\boldsymbol{x}) = \int_{\mathbb{R}^D} F(\boldsymbol{u}) \exp(j2\pi \boldsymbol{u} \cdot \boldsymbol{x})\, d\boldsymbol{u}$	$F(\boldsymbol{u}) = \int_{\mathbb{R}^D} f(\boldsymbol{x}) \exp(-j2\pi \boldsymbol{u} \cdot \boldsymbol{x})\, d\boldsymbol{x}$
$\mathrm{rect}(x, y)$	$\frac{\sin \pi u}{\pi u} \frac{\sin \pi v}{\pi v}$
$\mathrm{circ}(x, y)$	$\frac{1}{\sqrt{u^2+v^2}} J_1(2\pi \sqrt{u^2 + v^2})$
$\exp(-(x^2 + y^2)/2r^2)$	$2\pi r^2 \exp(-2\pi^2 (u^2 + v^2) r^2)$
$\cos(\pi(x^2 + y^2)/r^2)$	$r^2 \sin(\pi(u^2 + v^2) r^2)$
$\exp(j\pi(x^2 + y^2)/r^2)$	$jr^2 \exp(-j\pi(u^2 + v^2) r^2)$
$\delta(\boldsymbol{x})$	1

Multidimensional Fourier transform of selected functions.

	$f[\boldsymbol{x}] = d(\Lambda)\int_{\mathcal{P}^*} F(\boldsymbol{u})\exp(j2\pi\boldsymbol{u}\cdot\boldsymbol{x})\,d\boldsymbol{u}$	$F(\boldsymbol{u}) = \sum_{\boldsymbol{x}\in\Lambda} f[\boldsymbol{x}]\exp(-j2\pi\boldsymbol{u}\cdot\boldsymbol{x})$
(i)	$af_1[\boldsymbol{x}] + bf_2[\boldsymbol{x}]$	$aF_1(\boldsymbol{u}) + bF_2(\boldsymbol{u})$
(ii)	$f[\boldsymbol{x} - \boldsymbol{x}_0]$	$F(\boldsymbol{u})\exp(-j2\pi\boldsymbol{u}\cdot\boldsymbol{x}_0)$
(iii)	$f[\boldsymbol{x}]\exp(j2\pi\boldsymbol{u}_0\cdot\boldsymbol{x})$	$F(\boldsymbol{u} - \boldsymbol{u}_0)$
(iv)	$f^*[\boldsymbol{x}]$	$F^*(-\boldsymbol{u})$
(v)	$f_1[\boldsymbol{x}] * f_2[\boldsymbol{x}]$	$F_1(\boldsymbol{u})F_2(\boldsymbol{u})$
(vi)	$f_1[\boldsymbol{x}]f_2[\boldsymbol{x}]$	$d(\Lambda)\int_{\mathcal{P}^*} F_1(\boldsymbol{r})F_2(\boldsymbol{u} - \boldsymbol{r})\,d\boldsymbol{r}$
(vii)	$\sum_{\boldsymbol{x}\in\Lambda} \lvert f[\boldsymbol{x}]\rvert^2 = d(\Lambda)\int_{\mathcal{P}^*} \lvert F(\boldsymbol{u})\rvert^2\,d\boldsymbol{u}$	

Properties of the multidimensional Fourier transform over a lattice Λ.

Formulas

$$\exp(jX) = \cos(X) + j\sin(X), \qquad j = \sqrt{-1}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\mathrm{rect}(x, y) = \begin{cases} 1, & \text{if } |x| \le 0.5 \text{ and } |y| \le 0.5; \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathrm{circ}(x, y) = \begin{cases} 1, & \text{if } x^2 + y^2 \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

Affine transformation: $\mathcal{Q}_{\mathbf{A},\boldsymbol{d}} : \vec{\boldsymbol{g}} = \mathcal{Q}_{\mathbf{A},\boldsymbol{d}}\vec{\boldsymbol{f}} : g(\boldsymbol{x}) = f(\mathbf{A}(\boldsymbol{x} - \boldsymbol{d}))$

If $\Lambda = \mathrm{LAT}(\mathbf{V})$, then $d(\Lambda) = |\det(\mathbf{V})|$, and $\Lambda^* = \mathrm{LAT}(\mathbf{V}^{-T})$.

Sampling

If $f[\boldsymbol{x}] = f_c(\boldsymbol{x}), \quad \boldsymbol{x} \in \Lambda$ then

$$F(\boldsymbol{u}) = \frac{1}{d(\Lambda)} \sum_{r \in \Lambda^*} F_c(\boldsymbol{u} + \boldsymbol{r})$$

Tristimulus values

$$C_i = \int_{\lambda_{\min}}^{\lambda_{\max}} C(\lambda) \bar{p}_i(\lambda)\, d\lambda.$$

where $\bar{p}_i(\lambda)$ are the color matching functions of the primaries $[P_i]$, $i = 1, 2, 3$.

Chromaticities

$$c_i = \frac{C_i}{C_1 + C_2 + C_3}$$

Luminance

$$C_L = C_1 P_{1L} + C_2 P_{2L} + C_3 P_{3L}$$

Obtaining tristimulus values from luminance and chromaticities

$$C_i = \frac{C_L c_i}{c_1 P_{1L} + c_2 P_{2L} + c_3 P_{3L}}$$

Representation by orthogonal basis vectors

$$\vec{\boldsymbol{f}} = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f_b[i,j] \vec{\boldsymbol{b}}_{ij}$$

$$f_b[k,l] = \frac{\langle \vec{\boldsymbol{f}} \mid \vec{\boldsymbol{b}}_{kl} \rangle}{\|\vec{\boldsymbol{b}}_{kl}\|^2}.$$

One-dimensional DFT

$$f[m] = \frac{1}{M} \sum_{k=0}^{M-1} f_b[k] e^{j2\pi km/M}$$

$$f_b[k] = \sum_{m=0}^{M-1} f[m] e^{-j2\pi km/M}$$

One-dimensional DCT

$$f[m] = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \sqrt{\beta_k} f_b[k] \cos\left(\frac{(2m+1)k\pi}{2M}\right)$$

$$f_b[k] = \sqrt{\frac{\beta_k}{M}} \sum_{m=0}^{M-1} f[m] \cos\left(\frac{(2m+1)k\pi}{2M}\right)$$

$$\beta_k = \begin{cases} 1 & \text{if } k = 0, \\ 2 & \text{if } k = 1, \ldots, M-1. \end{cases}$$

Quantization

$$Q(f) = q_i \quad \text{if} \quad d_{i-1} < f \leq d_i, \quad i = 1, 2, \ldots, L$$

$$\mathcal{E}(f) = i \quad \text{if} \quad d_{i-1} < f \leq d_i, \quad i = 1, 2, \ldots, L$$

$$\mathcal{D}(i) = q_i \quad i = 1, 2, \ldots, L$$

Variable length coding

$$\bar{b} = \sum_{k=1}^{L} p_k b_k \quad \text{average codeword length}$$

$$H = -\sum_{k=1}^{L} p_k \log_2 p_k \quad \text{entropy}$$

$$\bar{b} \geq H$$