CEG4316 Image Processing

Fall 2011

Problem Set 3

Due 3:00 pm, December 7, 2011 in CEG4316 assignment box

1. Consider the following 16×8 data block from the image atrium_Y used in lab 3, representing two horizontally adjacent blocks $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$:

```
100 121 164 174 137 119 115 135 167 145 106 152 166 185 204 207
161 165 166 171 127 122 158 169 176 145 102 151 169 168 159 176
153 163 159 172 143 158 174 166 175 146 101 147 169 176 167 158
117 150 162 169 149 158 167 168 172 146 101 142 167 180 183 193
154 166 158 172 146 150 167 168 169 150  98 141 167 171 177 195
197 179 157 173 148 155 167 164 170 156  99 135 169 176 178 185
175 153 158 172 147 153 170 168 168 160 103 142 170 174 176 188
112 123 158 170 151 155 169 172 166 157 104 142 176 170 178 191
```

The MATLAB statements to generate the two blocks are:

```
f = imread('atrium_Y.tif');
f1 = double(f(301:308,601:608));
f2 = double(f(301:308,609:616));
```

In the following questions, you can use MATLAB wherever useful. If you do so, include the MATLAB statements used as part of your solution.

(a) Compute the two-dimensional 8×8 DCTs of the two horizontally adjacent 8×8 blocks (minus 128) and display them to integer precision. Verify for each block that $\sum_{m=0}^{7} \sum_{n=0}^{7} (f^{(i)}[m,n] - 128)^2 = \sum_{k=0}^{7} \sum_{l=0}^{7} f_b^{(i)}[k,l]^2$. This holds for the original f_b, not the rounded ones.

(b) We want to quantize this block with a set of uniform quantizers whose step sizes are given by the matrix $qf\,\boldsymbol{\Delta}_0$, where $qf = 1.4$ and $\boldsymbol{\Delta}_0$ is given on page 9-9 of the course notes. Display the output of the quantizer encoders $\mathcal{E}_{kl}$ and the output of the quantizer decoders $\mathcal{D}_{kl}$ as matrices for the two blocks of DCT coefficients.

(c) Compute the inverse DCT of the quantized DCT coefficients for the two blocks, and display them to integer precision (don't forget to add 128). Compute the squared error between the original image and the quantized image in the space domain and in the transform domain, and verify that they are equal (as per pg. 8-7 of the notes). Compute the PSNR for each block.

(d) We now want to do the code assignment for the output of the quantizer encoders, as found in (b) for the second block $\mathbf{f}^{(2)}$. Give the list of symbols to be encoded for the block in zigzag order (chapter 9, pg. 9-10). The possible symbol names are:

$C_i, i = 0, \ldots, 15$ (the category from table B), $(Z)_i, i = 1, \ldots, 15$ where Z is a signed integer and $2^{i-1} \leq |Z| < 2^i$, and R/C (for Run/Category, called Run/Size in Table K.5), and EOB (Table K.5). For example, the third element in the row of Table A for category 4 is denoted $(-13)_4$.

(e) Give the coded binary stream for the list of symbols you found in (d). Show the codeword boundaries to make it easy to check your result. Explain why the overall code that is used for a DCT block is uniquely decodable. How many bits are needed to represent the coded block. What is the compression ratio for this one block?