Generation of anaglyph stereoscopic images

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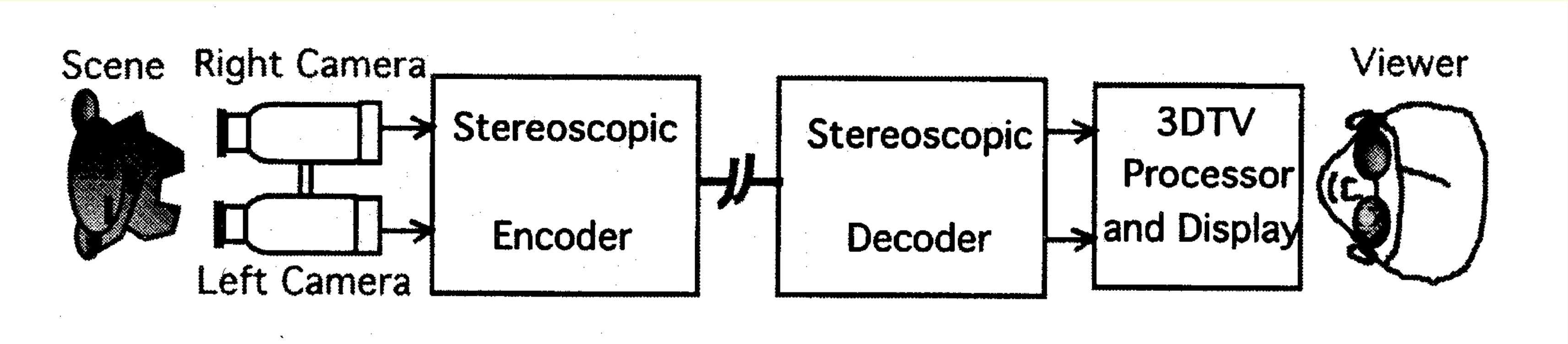
Today's Program

- Generalities on stereoscopic viewing
- What is anaglyph?
- Mathematical formulation of color stereo viewing
- Mathematical formulation of anaglyph viewing
- Method to generate anaglyphs by projection
- Some results
- What's left?

Stereoscopic image transmission and viewing



Stereoscopic Pair (Ray Hannisian)

Stereoscopic display methods

- Stereoscope (1832)
- Anaglyph (1853)
- Polarized glasses (1891)
- Liquid crystal shutter (time-sequential)
- Autostereoscopic (no glasses)

Wheatstone’s stereoscope

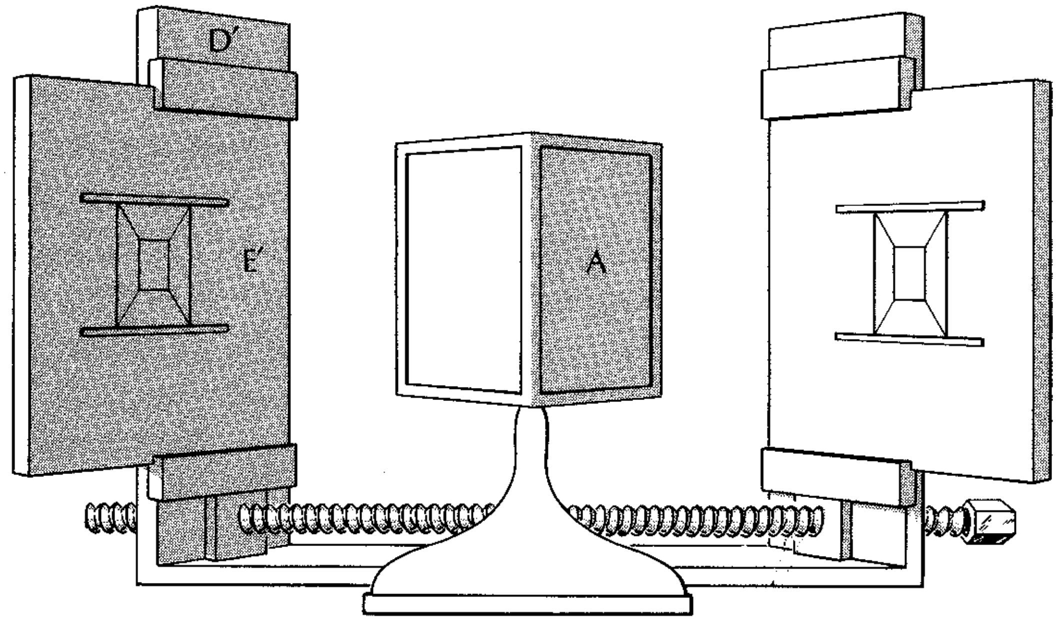
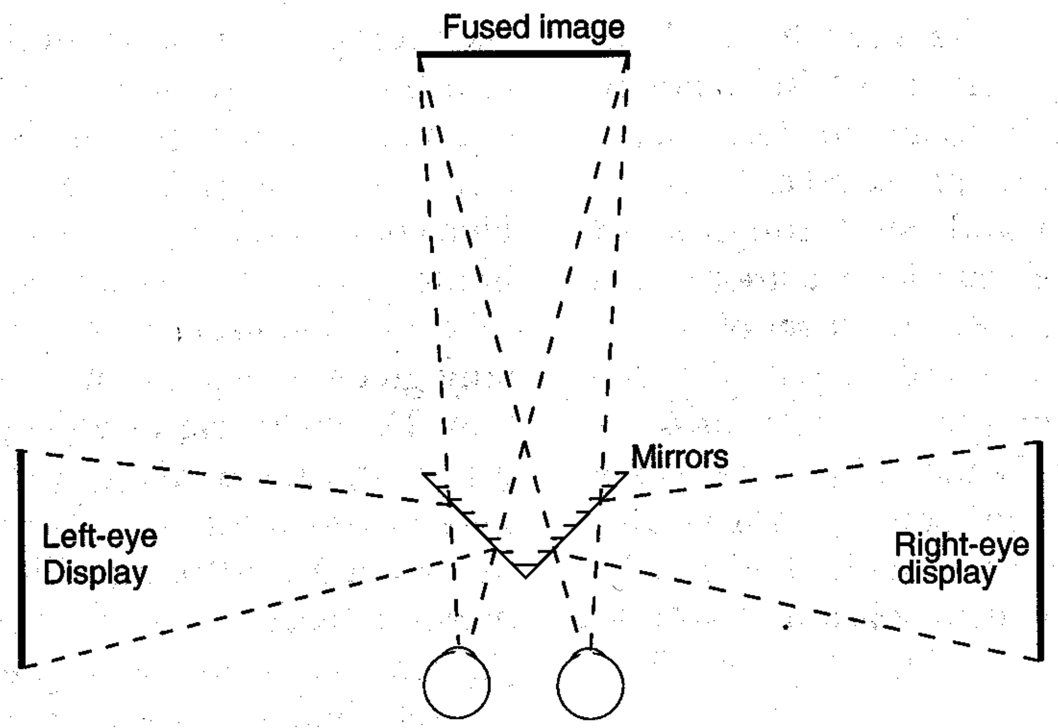


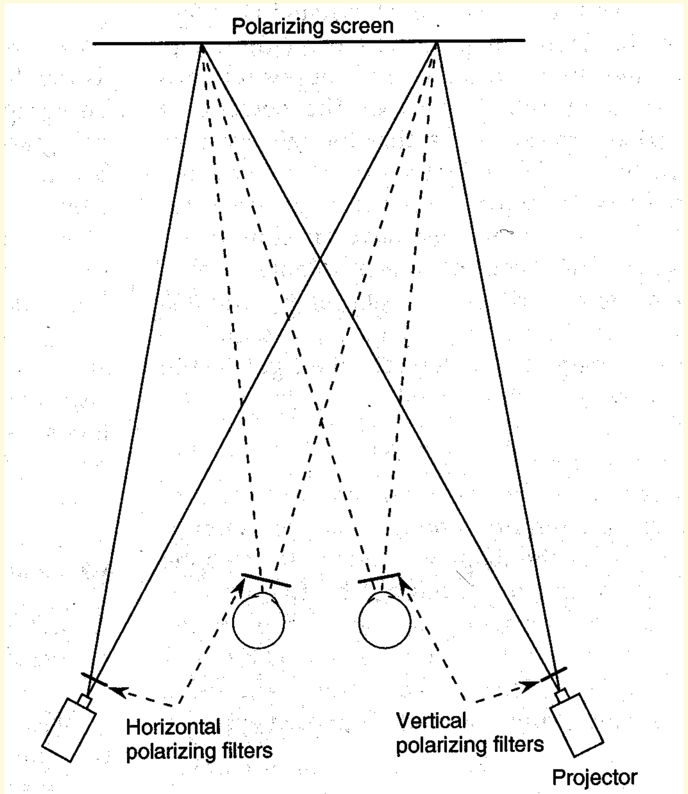
Figure 1.8. Wheastone's first mirror stereoscope. (From Wheatstone 1838.)

Stereoscope optics



(a) Wheastone's mirror stereoscope

Polarized stereoscopic display



(c) The polaroid projection stereoscope

What is ‘anaglyph’?

Anaglyph is a method to view stereoscopic images using colored spectacles. The method was patented in 1891 by Louis Ducos du Hauron, but similar methods had been demonstrated previously by W. Rollmann in 1853 and J.C. D’Almeida in 1858.

Origin of the word anaglyph

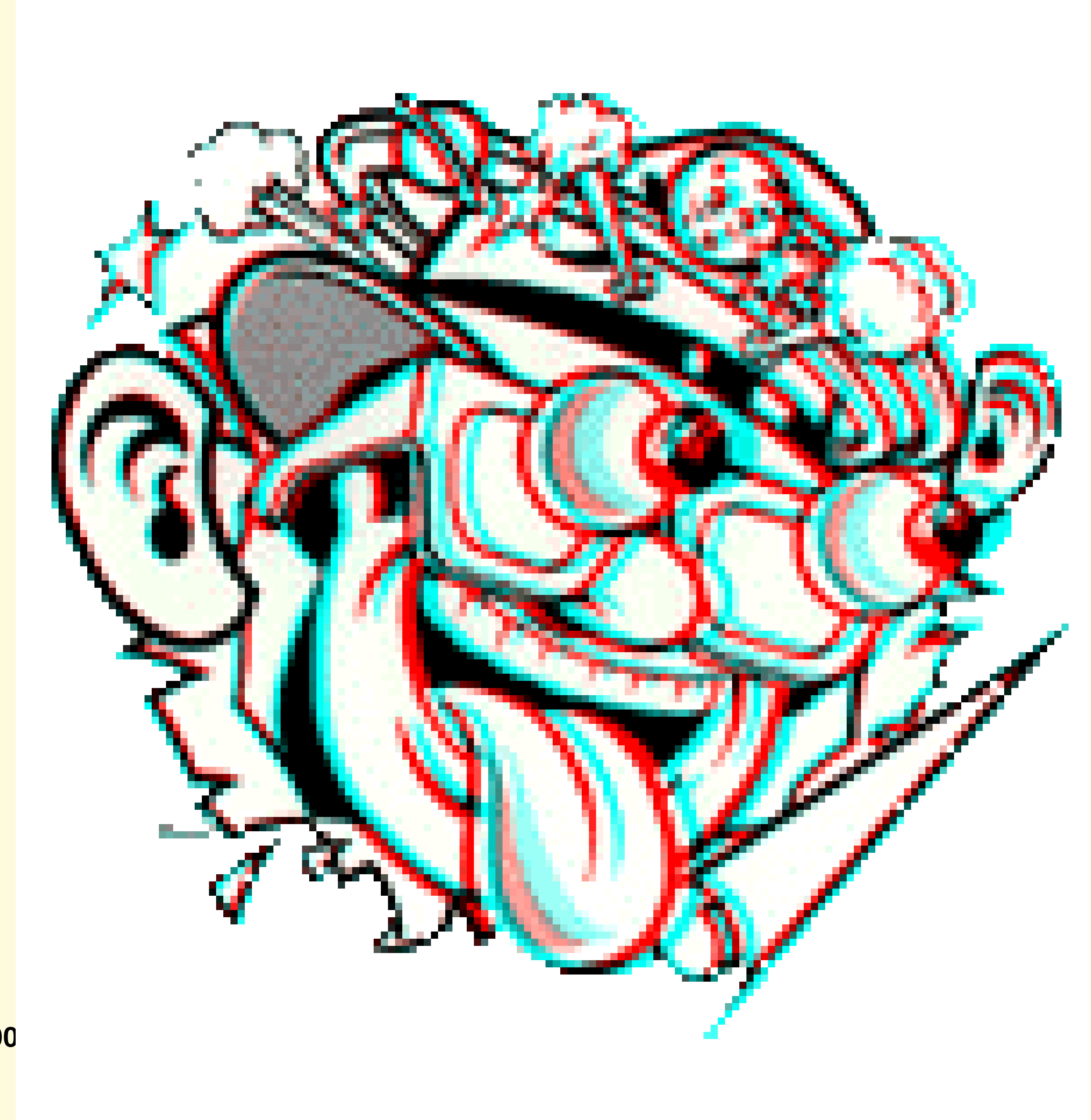
The word *anaglyph* is from the Greek

- ανα == again
- γλμφη == sculpture

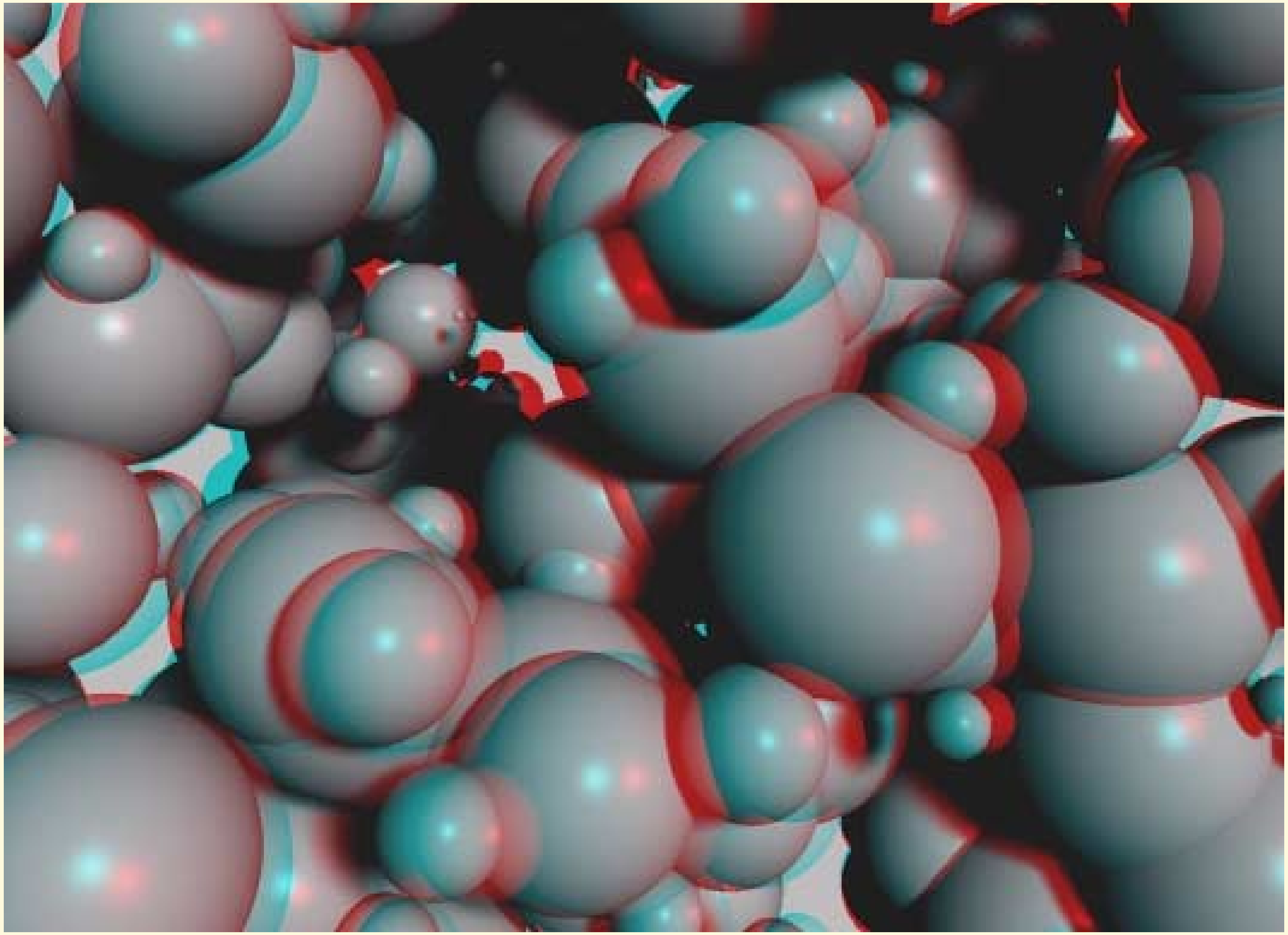
Classical method

For monochrome stereo images, the left view in blue (or green) is superimposed on the same image with the right view in red. When viewed through spectacles of corresponding colors but reversed, the three-dimensional effect is perceived.

3D drawing



Bubbles (André-Claude Latulippe)



Dinosaur (Henry Chung)

Ideal Stereoscopic Display

- Input left and right views:

$$V'_{lj}(\mathbf{x}), V'_{rj}(\mathbf{x}),\ j = 1,2,3;\ \mathbf{x} \in \Lambda$$

- The three components j=1,2,3 are gamma-corrected RGB (in that order) that can be directly displayed by a standard RGB display.
- Λ is the sampling raster for the image

Ideal Stereoscopic Display

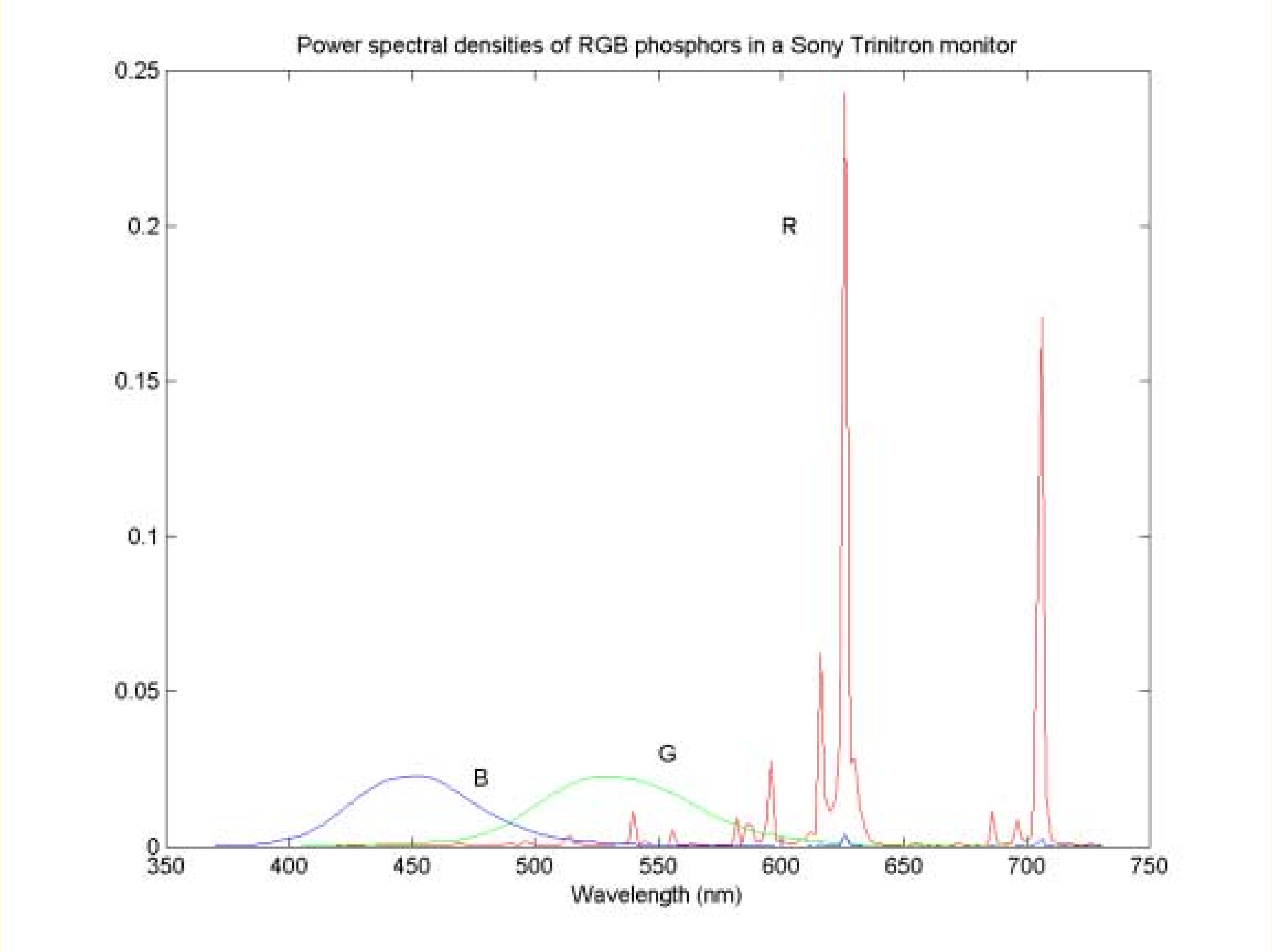
- The three components go through the display gamma, denoted g, and excite the display RGB phosphors,

$$V_{\cdot j}(\mathbf{x}) = g\left(V'_{\cdot j}(\mathbf{x})\right), \cdot = l, r;\ j = 1,2,3$$

- The spectral density functions of the RGB display phosphors are denoted $d_j(\lambda),\ j = 1,2,3$
- The light emanating from point **x** in left and right images is given by

$$Q_l(\lambda, \mathbf{x}) = \sum_{j=1}^{3} V_{lj}(\mathbf{x}) d_j(\lambda)$$

$$Q_r(\lambda, \mathbf{x}) = \sum_{j=1}^{3} V_{rj}(\mathbf{x}) d_j(\lambda)$$


Power spectral densities of RGB phosphors in a Sony Trinitron monitor
0.25
0.2
0.15
0.1
0.05
0
350
400
450
500
550
600
650
700
750
Wavelength (nm)
R
G
B

Perceived stereo pair

- The color perceived at point **x** in the left and right images is determined by the projection of $Q_l(\lambda, \mathbf{x})$ and $Q_r(\lambda, \mathbf{x})$ onto the visual subspace using color matching functions $\bar{p}_k(\lambda)$

$$
\begin{aligned}
\tilde{V}_{lk}(\mathbf{x}) &= \int Q_l(\lambda, \mathbf{x}) \bar{p}_k(\lambda) d\lambda \\
&= \sum_{j=1}^{3} V_{lj}(\mathbf{x}) \int \bar{p}_k(\lambda) d_j(\lambda) d\lambda \\
&= \sum_{j=1}^{3} c_{kj} V_{lj}(\mathbf{x}), \quad k = 1,2,3
\end{aligned}
$$

Perceived stereo pair

- In matrix notation, $\tilde{\mathbf{V}}_l(\mathbf{x}) = \mathbf{C}\mathbf{V}_l(\mathbf{x})$

$$[\mathbf{C}]_{kj} = c_{kj} = \int \bar{p}_k(\lambda) d_j(\lambda) d\lambda$$

- Similarly, $\tilde{\mathbf{V}}_r(\mathbf{x}) = \mathbf{C}\mathbf{V}_r(\mathbf{x})$
- With the standard XYZ color space,

$$\mathbf{C} = \begin{bmatrix} 0.4641 & 0.3055 & 0.1808 \\ 0.2597 & 0.6592 & 0.0811 \\ 0.0357 & 0.1421 & 0.9109 \end{bmatrix}$$

Perceived Stereo Image

- The value of the stereo image at each point x can be considered to be an element of a six-dimensional vector space S_6

$$\tilde{\mathbf{V}}(\mathbf{x}) = \begin{bmatrix} \tilde{V}_{l1}(\mathbf{x}) \\ \tilde{V}_{l2}(\mathbf{x}) \\ \tilde{V}_{l3}(\mathbf{x}) \\ \tilde{V}_{r1}(\mathbf{x}) \\ \tilde{V}_{r2}(\mathbf{x}) \\ \tilde{V}_{r3}(\mathbf{x}) \end{bmatrix}$$

Visualization of an Anaglyph Image

- The anaglyph image is denoted

$$V'_{aj}(\mathbf{x}),\ j = 1,2,3; \quad \mathbf{x} \in \mathrm{L}$$

- The light emitted from the screen at point **x** is

$$Q_a(\lambda, \mathbf{x}) = \sum_{j=1}^{3} V_{aj}(\mathbf{x}) d_j(\lambda)$$

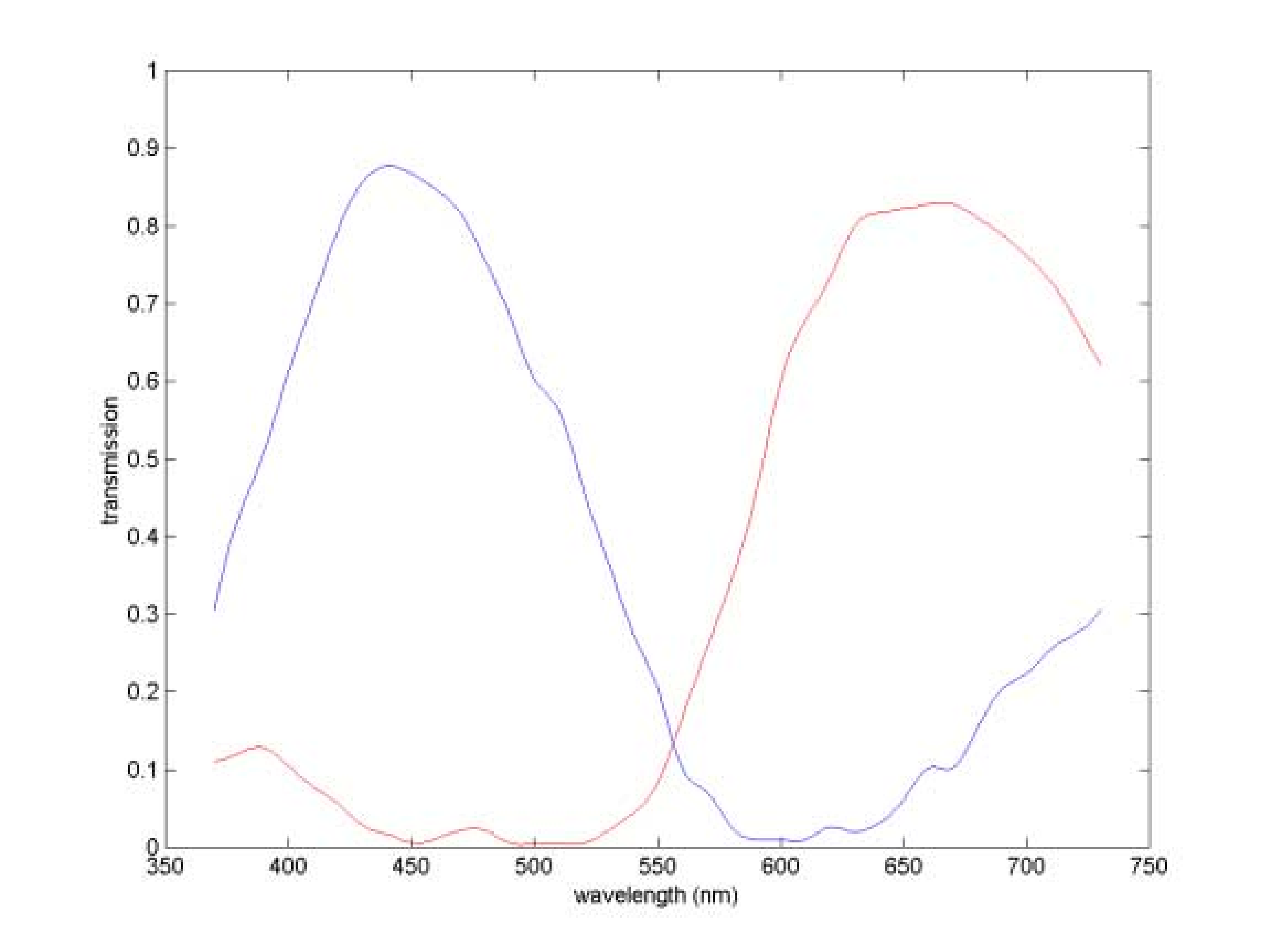
- where $V_{aj}(\mathbf{x}) = g\left(V'_{aj}(\mathbf{x})\right)$

Anaglyph Image

Perceived Anaglyph Image with Glasses

- The light from the CRT passes through two filters with spectral absorption functions $f_l(\lambda)$ and $f_r(\lambda)$ before arriving at the left and right eyes.
- The light spectral density at the left and right eyes is $Q_a(\lambda,\mathbf{x})f_l(\lambda)$ and $Q_a(\lambda,\mathbf{x})f_r(\lambda)$ respectively.

Transmission of red and blue filters



Perceived Anaglyph Image with Glasses

- The tristimulus values for the left and right views through the glasses are

$$\tilde{U}_{lk}(\mathbf{x}) = \int Q(\lambda, x) f_l(\lambda) \overline{p}_k(\lambda)\, d\lambda$$

$$= \sum_{j=1}^{3} V_{aj}(\mathbf{x}) \int \overline{p}_k(\lambda)\, d_j(\lambda) f_l(\lambda) d\lambda$$

$$= \sum_{j=1}^{3} a_{lkj} V_{aj}(\mathbf{x}), \quad k = 1,2,3$$

- or $\tilde{\mathbf{U}}_l(\mathbf{x}) = \mathbf{A}_l \mathbf{V}_a(\mathbf{x})$ Similarly, $\tilde{\mathbf{U}}_r(\mathbf{x}) = \mathbf{A}_r \mathbf{V}_a(\mathbf{x})$

Goal of Anaglyph reproduction

We want the anaglyph image to appear as similar as possible to the ideal stereoscopic image

$$\tilde{\mathbf{U}}(\mathbf{x}) = \mathbf{R}\mathbf{V}_a(\mathbf{x}) = \begin{bmatrix} \tilde{U}_{l1}(\mathbf{x}) \\ \tilde{U}_{l2}(\mathbf{x}) \\ \tilde{U}_{l3}(\mathbf{x}) \\ \tilde{U}_{r1}(\mathbf{x}) \\ \tilde{U}_{r2}(\mathbf{x}) \\ \tilde{U}_{r3}(\mathbf{x}) \end{bmatrix} \qquad \tilde{\mathbf{V}}(\mathbf{x}) = \begin{bmatrix} \tilde{V}_{l1}(\mathbf{x}) \\ \tilde{V}_{l2}(\mathbf{x}) \\ \tilde{V}_{l3}(\mathbf{x}) \\ \tilde{V}_{r1}(\mathbf{x}) \\ \tilde{V}_{r2}(\mathbf{x}) \\ \tilde{V}_{r3}(\mathbf{x}) \end{bmatrix}$$

Anaglyph
3D

Ideal
6D

Generation of Anaglyph Image

- For each pixel location, we seek an element of the three-dimensional anaglyph subspace as close as possible to the original stereo image value which is in a six-dimensional space.
- If we have a suitable distance metric, we can choose the value in the three-dimensional space that is closest to the original six-dimensional value.
- This achieved by ***Projection***

Assumptions

- The approximation is carried out independently at each sample location
- The error metric at each point is a weighted squared error between $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$

$$\left\|\tilde{\mathbf{U}}-\tilde{\mathbf{V}}\right\|^2=\left(\tilde{\mathbf{U}}-\tilde{\mathbf{V}}\right)^{\mathrm{T}}\mathbf{W}\left(\tilde{\mathbf{U}}-\tilde{\mathbf{V}}\right)$$

- A global scaling of the V_{aj} is used to account for the attenuation of the filters.

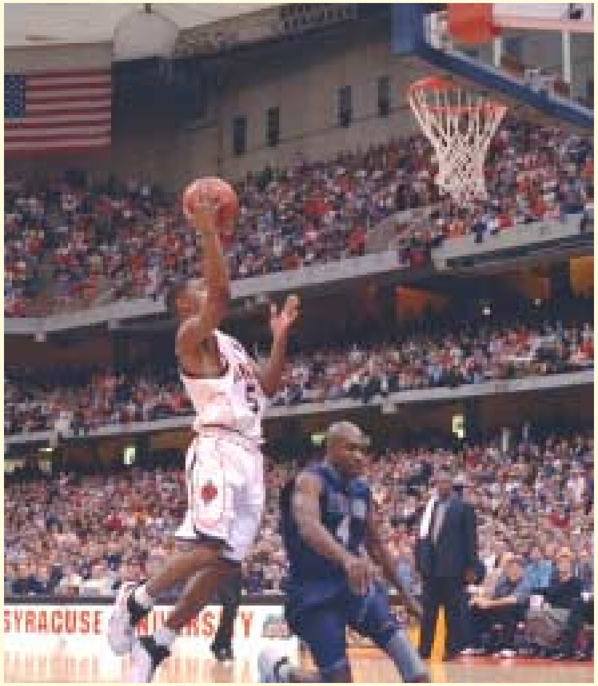
Projection

- Applying standard projection theory gives

$$\hat{\mathbf{V}}_a(\mathbf{x}) = \left(\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{R}\right)^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{W}\tilde{\mathbf{V}}(\mathbf{x})$$
$$= \left(\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{R}\right)^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{C}_2\mathbf{V}(\mathbf{x})$$

- The 3 x 6 matrix $\left(\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{R}\right)^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{C}_2$ is fixed and can be precomputed

Basketball Image

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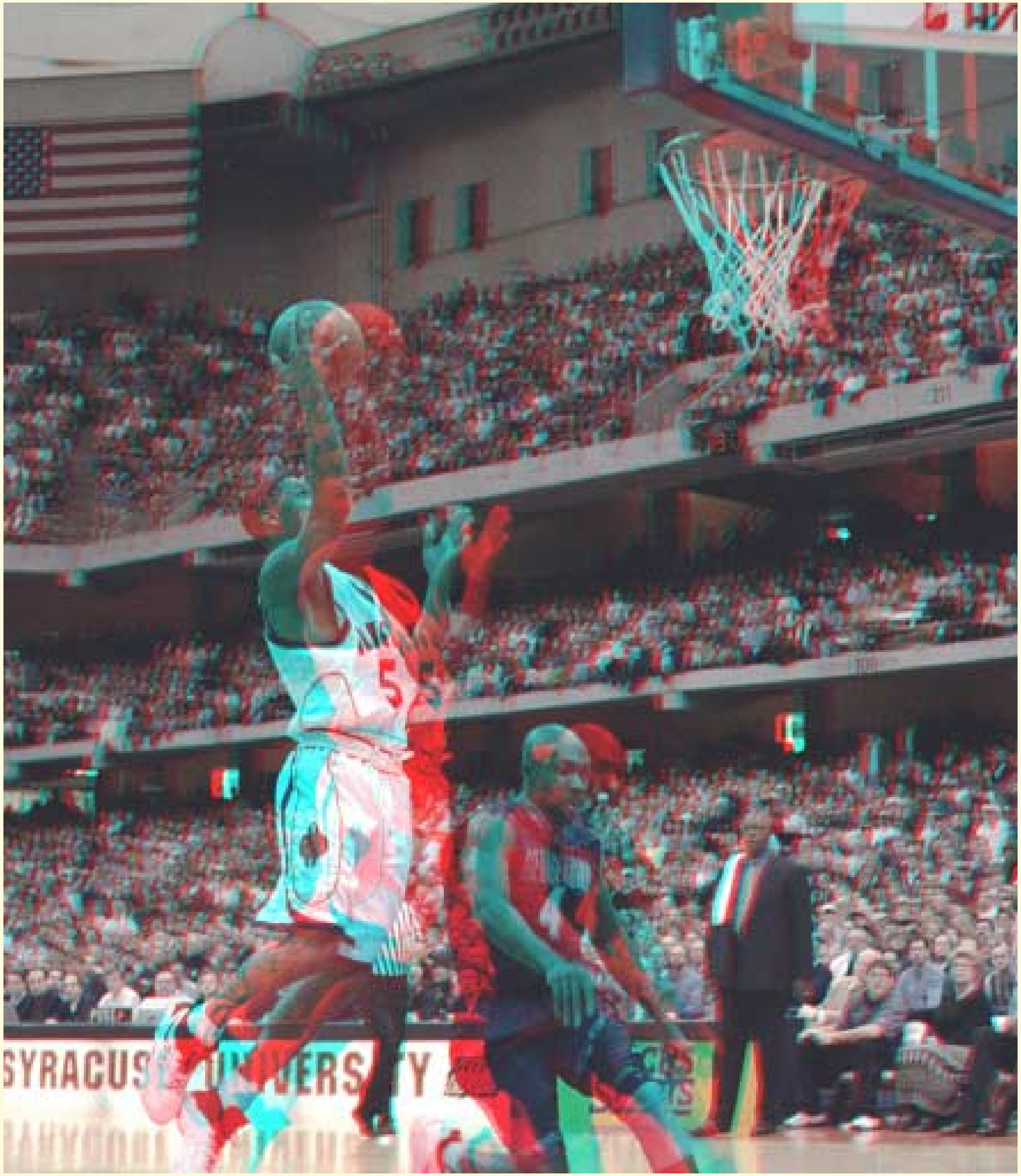
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Conclusion

- A method to generate anaglyphs based on the mathematical description of the image display and perception process has been developed.
- Some color reproduction is possible.
- Better stereo rendition than conventional approaches.

What Else?

- How to choose filters to give best results?
- Can color rendition be improved?
- Can the method be applied to time-shuttered stereo?

Thanks for coming!