

# Vector Semantics & Embeddings

## Word Embeddings

Adapted by Diana Inkpen, 2021 for csi5386 at the University of Ottawa

From Chapter 6 of Speech and Language Processing (3rd ed.), by Dan Jurafsky and James H. Martin.

# What do words mean?

## Introductory logic classes:

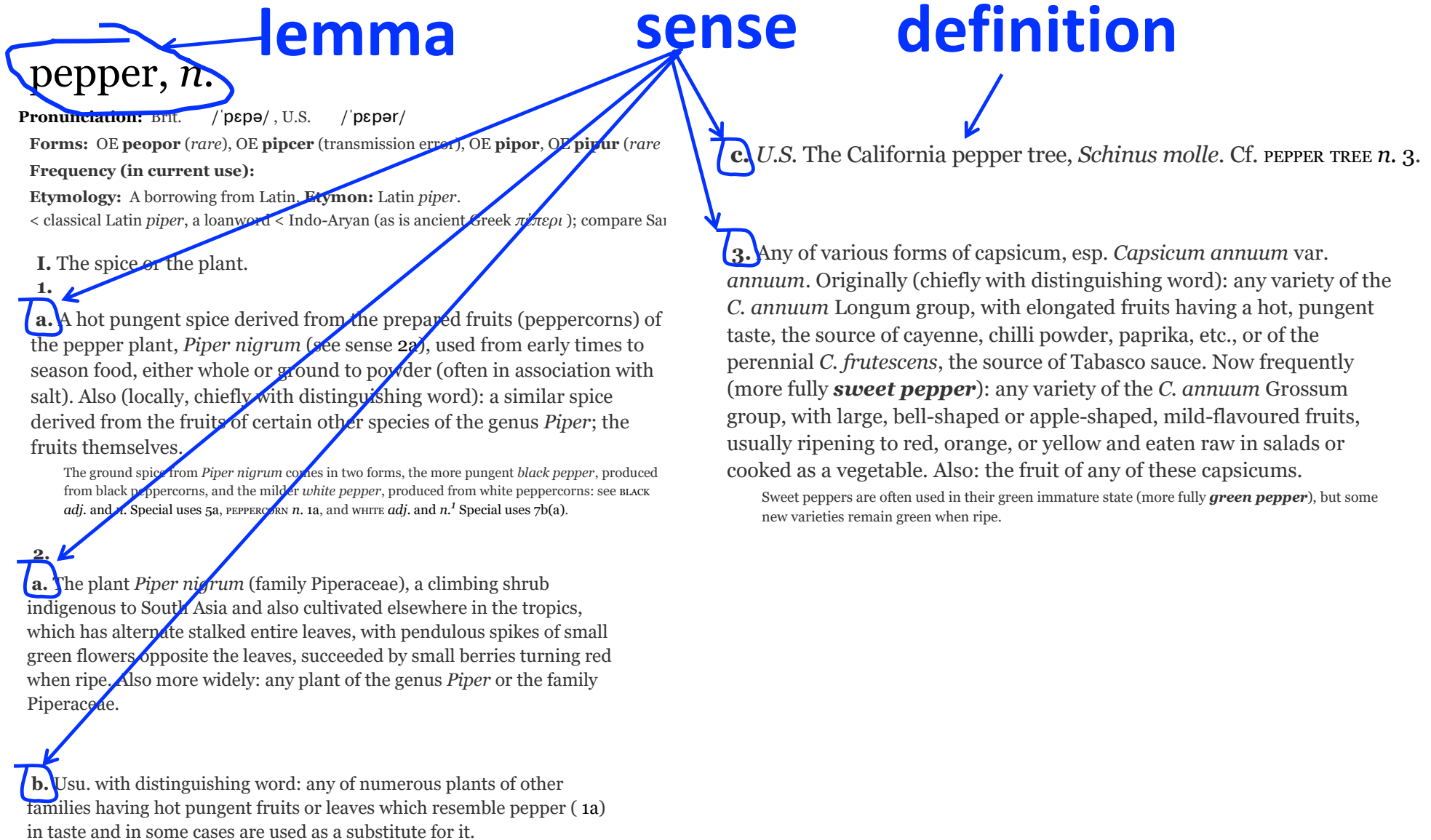
- The meaning of "dog" is DOG; cat is CAT

$$\forall x \text{ DOG}(x) \rightarrow \text{MAMMAL}(x)$$

Word senses: look in a dictionary

<http://www.oed.com/>

# Words, Lemmas, Senses, Definitions



# Lemma *pepper*

Sense 1: spice from pepper plant

Sense 2: the pepper plant itself

Sense 3: another similar plant (Jamaican pepper)

Sense 4: another plant with peppercorns (California pepper)

Sense 5: *capsicum* (i.e. chili, paprika, bell pepper, etc)

A **sense** or “**concept**” is the meaning component of a word

# Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / H<sub>2</sub>O

# Relation: Synonymy

Note that there are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may not preserve the acceptability based on notions of politeness, slang, register, genre, etc.

# Relation: Synonymy?

water/H<sub>2</sub>O

big/large

brave/courageous

# The Linguistic Principle of Contrast

Difference in form → difference in meaning



# Abbé Gabriel Girard 1718

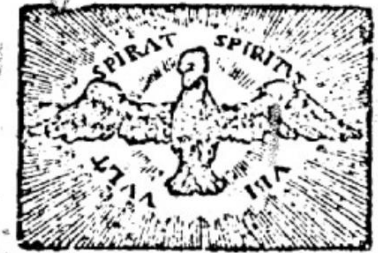
Re: "exact" synonyms

"je ne crois pas qu'il y ait de  
mot synonyme dans aucune  
Langue."

[I do not believe that there  
is a synonymous word in any  
language]

LA JUSTESSE  
DE LA  
LANGUE FRANÇOISE,  
OU  
LES DIFFERENTES SIGNIFICATIONS  
DES MOTS QUI PASSENT  
POUR  
SYNONIMES.

Par M. l'Abbé GIRARD C. D. M. D. D. F.



A PARIS,  
Chez LAURENT D'HOURY, Imprimeur-  
Libraire, au bas de la rue de la Harpe, vis-  
à vis la rue S. Severin, au Saint Esprit.

M. DCC. XVIII.

Avec Approbation & Privilège du Roy.

# Relation: Similarity

Words with similar meanings. Not synonyms, but sharing some element of meaning.

car, bicycle

cow, horse

# Ask humans how similar 2 words are

word1	word2	similarity
vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

# Evaluation measures

**Pearson correlation** between set of scores produced by the system and the expected values produced by the human judges.

$$r = \frac{\sum (x_i - \bar{x}) (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$r$  = correlation coefficient

$x_i$  = values of the x-variable in a sample

$\bar{x}$  = mean of the values of the x-variable

$y_i$  = values of the y-variable in a sample

$\bar{y}$  = mean of the values of the y-variable

**Spearman correlation** compares the ranks not the values.  
Range of values  $[-1,1]$ . Close to zero means no correlation.

# Relation: Word relatedness

Also called "word association"

Words can be related in any way, perhaps via a semantic frame or field

- car, bicycle: **similar**
- car, gasoline: **related**, not similar

# Semantic field

Words that

- cover a particular semantic domain
- bear structured relations with each other.

## **hospitals**

*surgeon, scalpel, nurse, anaesthetic, hospital*

## **restaurants**

*waiter, menu, plate, food, menu, chef*

## **houses**

*door, roof, kitchen, family, bed*

# Relation: Antonymy

Senses that are opposites with respect to only one feature of meaning

Otherwise, they are very similar!

dark/light      short/long      fast/slow      rise/fall  
hot/cold      up/down      in/out

More formally: antonyms can

- define a binary opposition or be at opposite ends of a scale
  - long/short, fast/slow
- Be *reversives*:
  - rise/fall, up/down

# Relation: Superordinate/ subordinate

One sense is a **subordinate** of another if the first sense is more specific, denoting a subclass of the other

- *car* is a subordinate of *vehicle*
- *mango* is a subordinate of *fruit*

Conversely **superordinate**

- *vehicle* is a superordinate of *car*
- *fruit* is a superordinate of *mango*

<b>Superordinate</b>	vehicle	fruit	furniture
<b>Subordinate</b>	car	mango	chair



# So far

## Concepts or word senses

- Have a complex many-to-many association with **words** (homonymy, multiple senses)

## Have relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Superordinate/subordinate, basic level
- Connotation

# Distributional Semantics

Let's define words by their usages

One way to define "usage":

words are defined by their environments (the words around them)

Zellig Harris (1954):

**If A and B have almost identical environments we say that they are synonyms.**

# What does recent English borrowing *ongchoi* mean?

Suppose you see these sentences:

- Ong choi is delicious **sautéed with garlic**.
- Ong choi is superb **over rice**
- Ong choi **leaves** with salty sauces

And you've also seen these:

- ...spinach **sautéed with garlic over rice**
- Chard stems and **leaves** are **delicious**
- Collard greens and other **salty** leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens

# Ongchoi: *Ipomoea aquatica* "Water Spinach"

空心菜

*kangkong*

rau muống

...



# A new model of meaning focusing on distributional similarity

Each word = a vector

- Not just "word" or word45.

Similar words are "nearby in space"



We define a word as a vector

Called an "embedding" because it's embedded into a space

The standard way to represent meaning in NLP

**Every modern NLP algorithm uses embeddings as the representation of word meaning**

Fine-grained model of meaning for similarity

# Intuition: why vectors?

Consider sentiment analysis:

- With **words**, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires **exact same word** to be in training and test
- With **embeddings**:
  - Feature is a word vector
  - 'The previous word was vector [35,22,17...]
  - Now in the test set we might see a similar vector [34,21,14]
  - We can generalize to **similar but unseen words!!!**

# We'll discuss 2 kinds of embeddings

## tf-idf

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

## Word2vec

- **Dense** vectors
- Representation is created by training a classifier to **predict** whether a word is likely to appear nearby
- In later chapters we'll discuss extensions called **contextual embeddings**

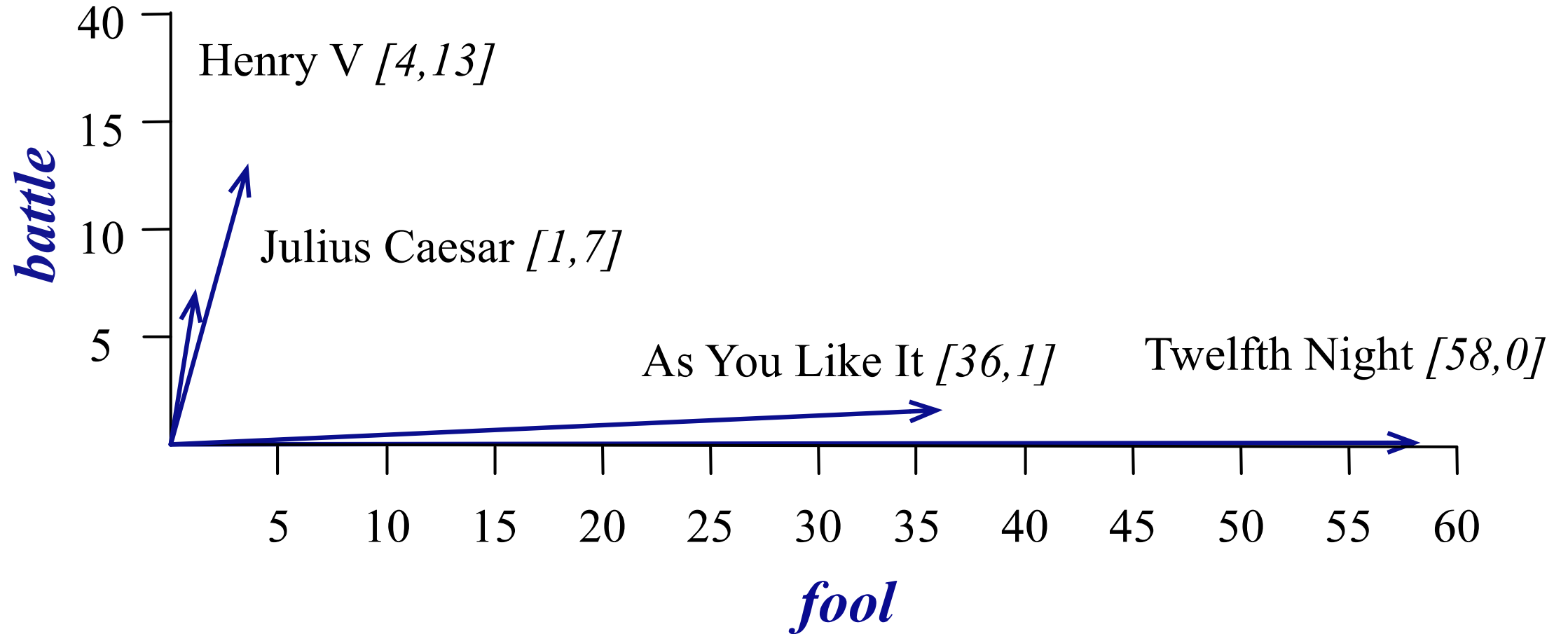


# Word vectors: Term-document matrix

Each document is represented by a vector of words

	<b>As You Like It</b>	<b>Twelfth Night</b>	<b>Julius Caesar</b>	<b>Henry V</b>
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

# Visualizing document vectors



# Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

Vectors are similar for the two comedies  
Different than the history

Comedies have more *fools* and *wit* and fewer *battles*.

# Idea for word meaning: Words can be vectors too!!!

	<b>As You Like It</b>	<b>Twelfth Night</b>	<b>Julius Caesar</b>	<b>Henry V</b>
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

*battle* is "the kind of word that occurs in Julius Caesar and Henry V"

*fool* is "the kind of word that occurs in comedies, especially Twelfth Night"

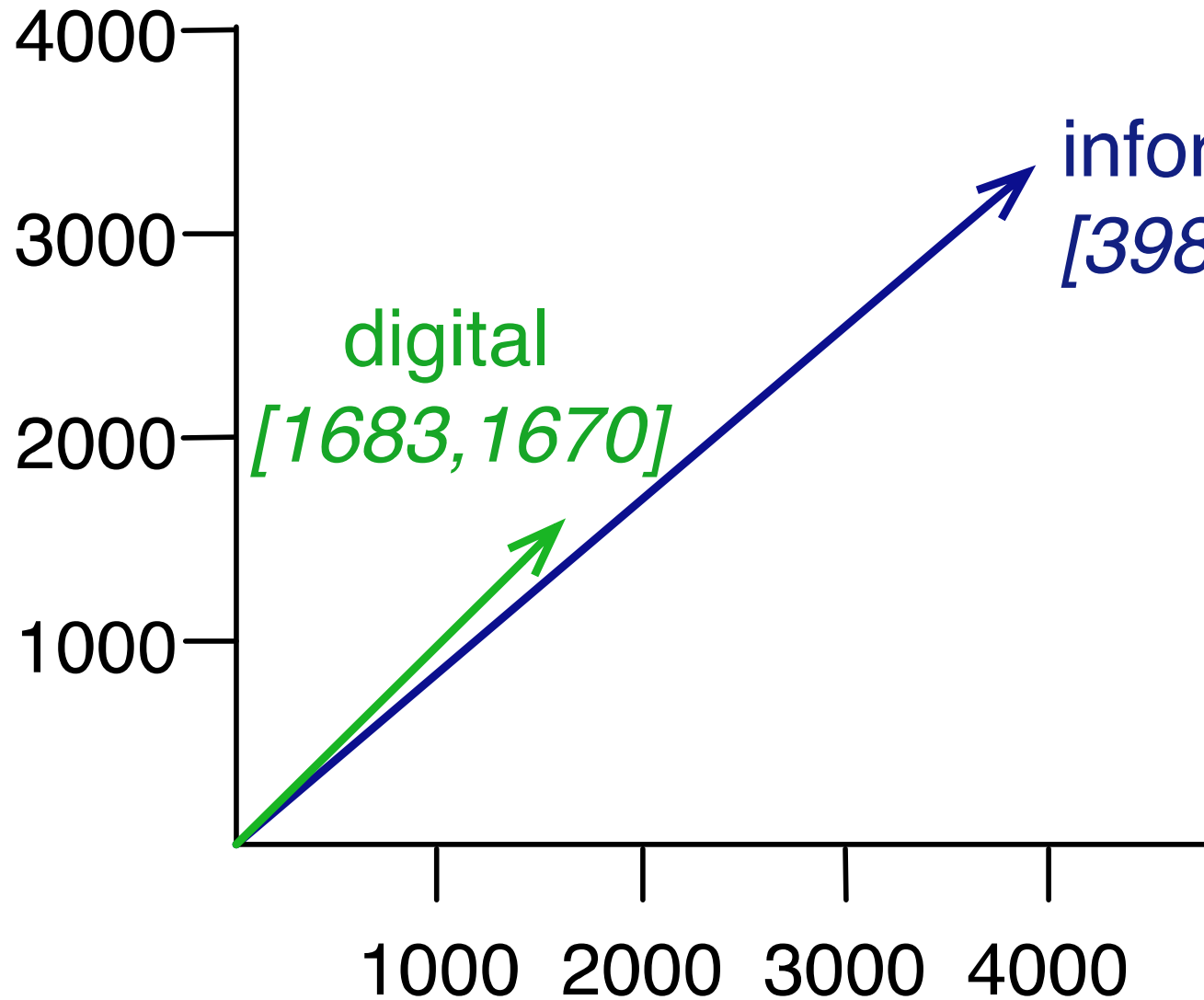
# More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by **cherry** pie, a traditional dessert  
often mixed, such as **strawberry** rhubarb pie. Apple pie  
computer peripherals and personal **digital** assistants. These devices usually  
a computer. This includes **information** available on the internet

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

***computer***



***data***

# Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^N v_i w_i = v_1 w_1 + v_2 w_2 + \dots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can be a similarity metric between vectors

# Problem with raw dot-product

Dot product favors long vectors

Dot product is higher if a vector is longer (has higher values in many dimension)

Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^N v_i^2}$$

Frequent words (of, the, you) have long vectors (since they occur many times with other words).

So dot product overly favors frequent words



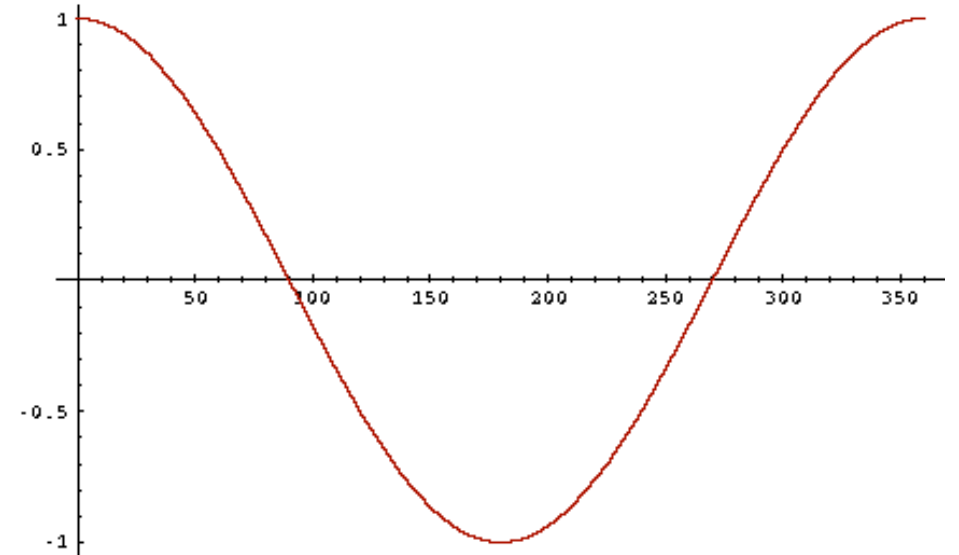
# Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

The same as using normalized vectors.

# Cosine as a similarity metric

- 1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

# Cosine examples

$$\cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\vec{v}}{|\vec{v}|} \cdot \frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

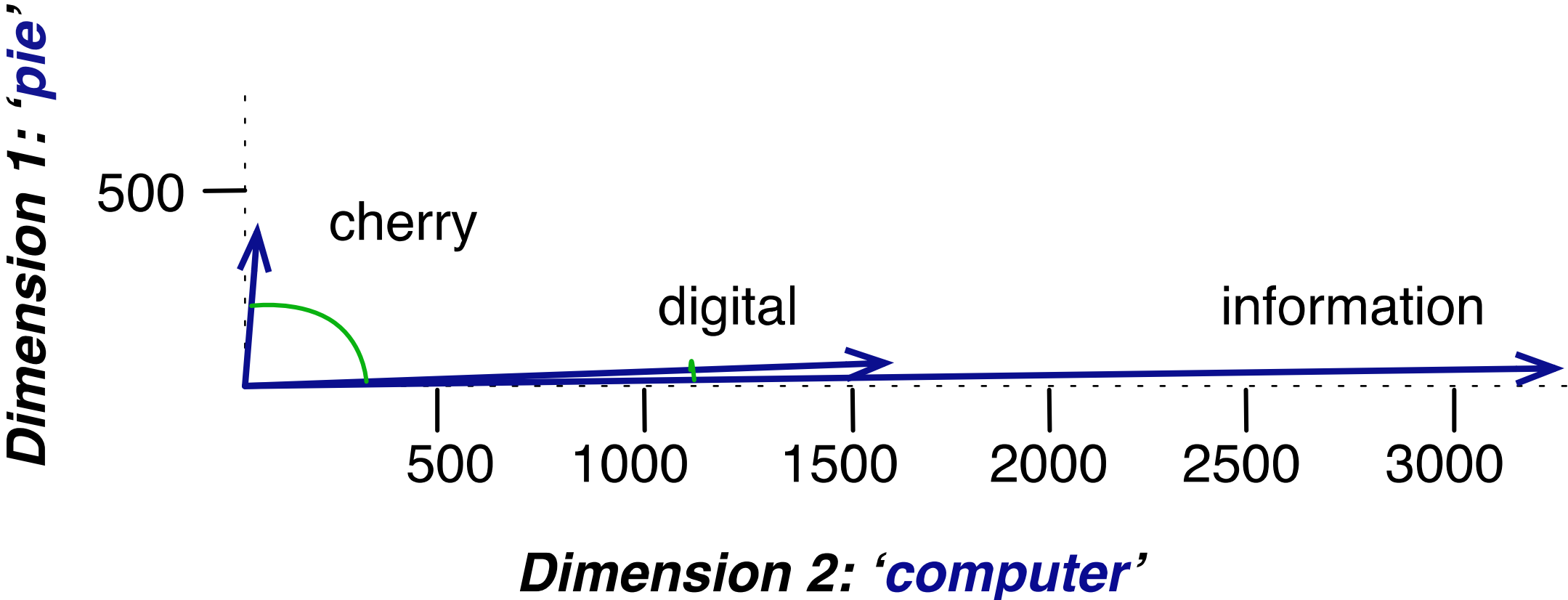
$$\cos(\text{cherry}, \text{information}) =$$

$$\frac{442 * 5 + 8 * 3982 + 2 * 3325}{\sqrt{442^2 + 8^2 + 2^2} \sqrt{5^2 + 3982^2 + 3325^2}} = .017$$

$$\cos(\text{digital}, \text{information}) =$$

$$\frac{5 * 5 + 1683 * 3982 + 1670 * 3325}{\sqrt{5^2 + 1683^2 + 1670^2} \sqrt{5^2 + 3982^2 + 3325^2}} = .996$$

# Visualizing cosines (well, angles)



# But raw frequency is a bad representation

- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it*, or *they* are not very informative about the context
- Need a function that resolves this frequency paradox!

# Two common solutions for word weighting

**tf-idf:** tf-idf value for word  $t$  in document  $d$ :

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Words like "the" or "good" have very low idf

**PMI:** (Pointwise mutual information)

- $\text{PMI}(w_1, w_2) = \log \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$

See if words like "good" appear more often with "great" than we would expect by chance

# Term frequency (tf)

$$tf_{t,d} = \text{count}(t,d)$$

Instead of using raw count, we squash a bit:

$$tf_{t,d} = \log_{10}(\text{count}(t,d)+1)$$

# Document frequency (df)

$df_t$  is the number of documents  $t$  occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	Document Frequency
Romeo	113	1
action	113	31

Important: documents can be **anything**; we can call each paragraph a document



# Inverse document frequency (idf)

$$\text{idf}_t = \log_{10} \left( \frac{N}{\text{df}_t} \right)$$

N is the total number of documents in the collection

<b>Word</b>	<b>df</b>	<b>idf</b>
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

# Final tf-idf weighted value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

Tf=idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	0.074	0	0.22	0.28
<b>good</b>	0	0	0	0
<b>fool</b>	0.019	0.021	0.0036	0.0083
<b>wit</b>	0.049	0.044	0.018	0.022

# Sparse versus dense vectors

tf-idf vectors are

- **long** (length  $|V| = 20,000$  to  $50,000$ )
- **sparse** (most elements are zero)

Alternative: learn vectors which are

- **short** (length 50-1000)
- **dense** (most elements are non-zero)

# Sparse versus **dense vectors**

## Why dense vectors?

- Short vectors may be easier to use as **features** in machine learning (fewer weights to tune)
- Dense vectors may **generalize** better than explicit counts
- They may do better at capturing synonymy:
  - *car* and *automobile* are synonyms; but are distinct dimensions
  - a word with *car* as a neighbor and a word with *automobile* as a neighbor should be similar, but aren't
- **In practice, they work better**

# Common methods for getting short dense vectors

## “Neural Language Model”-inspired models

- Word2vec (skipgram, CBOW), GloVe

## Singular Value Decomposition (SVD)

- A special case of this is called LSA – Latent Semantic Analysis

## Alternative to these "static embeddings":

- Contextual Embeddings (ELMo, BERT)
- Compute distinct embeddings for a word in its context
- Separate embeddings for parts of words (FastText)

# Embeddings you can download!

Word2vec (Mikolov et al)

<https://code.google.com/archive/p/word2vec/>

Glove (Pennington, Socher, Manning)

<http://nlp.stanford.edu/projects/glove/>

A lot more

Word2vec

Popular embedding method

Very fast to train

Code available on the web

Idea: **predict** rather than **count**

# Word2vec

Instead of **counting** how often each word  $w$  occurs near "*apricot*"

- Train a classifier on a binary **prediction** task:
  - Is  $w$  likely to show up near "*apricot*"?

We don't actually care about this task

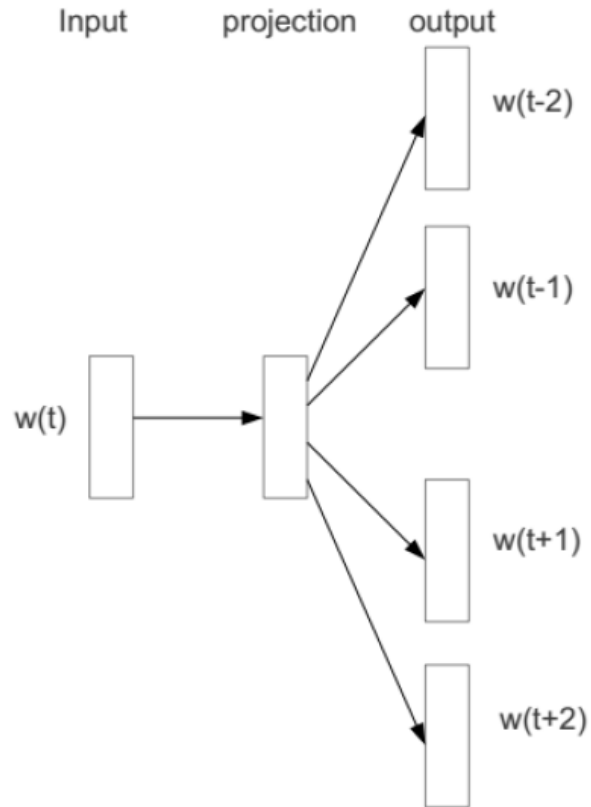
- But we'll take the learned classifier weights as the word embeddings

Big idea: **self-supervision**:

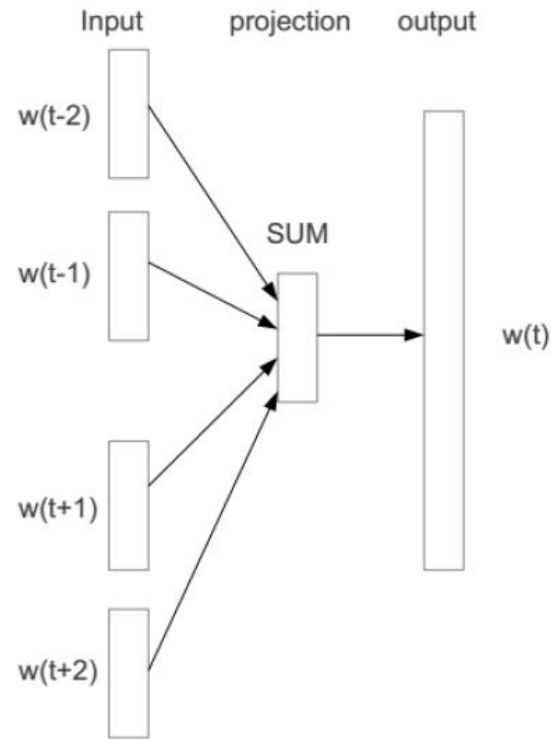
- A word  $c$  that occurs near *apricot* in the corpus acts as the gold "correct answer" for supervised learning
- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)



# Word2Vec: Skip-grams vs. CBOW (Mikolov, 2014 Tutorial)



Skip-grams



CBOW

# Word2Vec: Skip-Grams

Word2vec provides a variety of options.  
We'll do:

**skip-grams with negative sampling (SGNS)**

Approach: predict if candidate word  $c$  is a "neighbor"

1. Treat the target word  $t$  and a neighboring context word  $c$  as **positive examples**.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

# Skip-Gram Training Data

Assume a +/- 2 word window, given training sentence:

...lemon, a [tablespoon of apricot jam, a] pinch...

c1                      c2 [target]      c3      c4

# Skip-Gram Classifier

(assuming a +/- 2 word window)

...lemon, a [tablespoon of apricot jam, a] pinch...

c1                      c2 [target]    c3    c4

Goal: train a classifier that is given a candidate (**w**ord, **c**ontext) pair

(apricot, tablespoon)

(apricot, aardvark)

...

And assigns each pair a probability:

$$P(+ | w, c)$$

# Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- $\text{Similarity}(w,c) \propto w \cdot c$

We'll need to normalize to get a probability

- (cosine isn't a probability either)

# Turning dot products into probabilities

$$\text{Sim}(w, c) \approx w \cdot c$$

To turn this into a probability

We'll use the sigmoid from logistic regression:

$$P(+|w, c) = \sigma(c \cdot w) = \frac{1}{1 + \exp(-c \cdot w)}$$

$$\begin{aligned} P(-|w, c) &= 1 - P(+|w, c) \\ &= \sigma(-c \cdot w) = \frac{1}{1 + \exp(c \cdot w)} \end{aligned}$$

# How Skip-Gram Classifier computes $P(+ | w, c)$

$$P(+ | w, c) = \sigma(c \cdot w) = \frac{1}{1 + \exp(-c \cdot w)}$$

This is for one context word, but we have lots of context words. We'll assume independence and just multiply them:

$$P(+ | w, c_{1:L}) = \prod_{i=1}^L \sigma(c_i \cdot w)$$
$$\log P(+ | w, c_{1:L}) = \sum_{i=1}^L \log \sigma(c_i \cdot w)$$



# Skip-gram classifier: summary

A probabilistic classifier that,

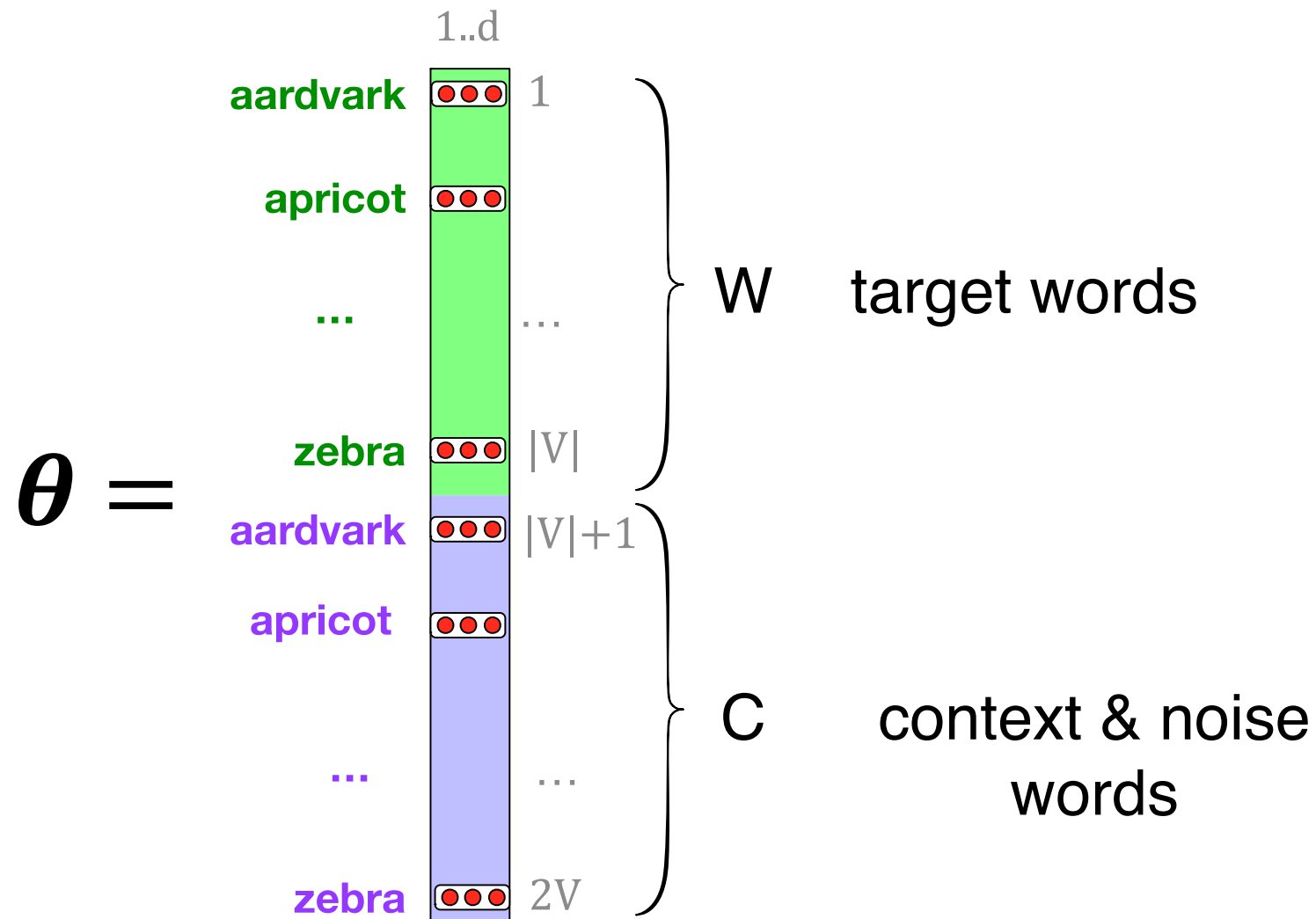
given a test target word  $w$

its context window of  $L$  words  $c_{1:L}$ ,

assigns a probability that  $w$  occurs in this window.

To compute this, we just need embeddings for all the words.

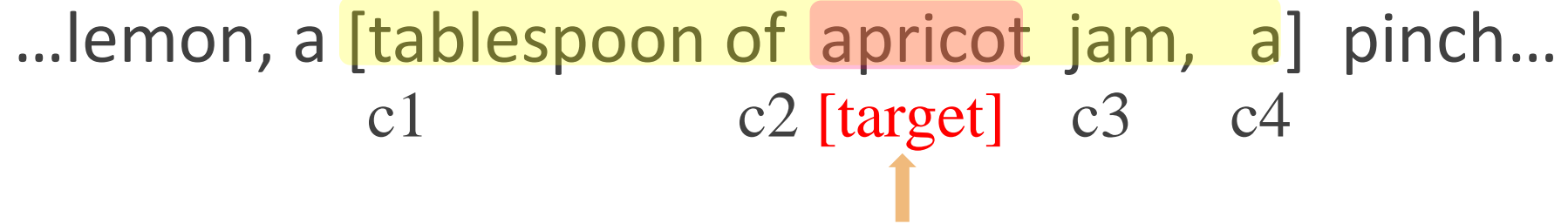
These embeddings we'll need: a set for  $w$ , a set for  $c$



# Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...

c1                      c2 [target]      c3      c4



## positive examples +

t

c

---

apricot    tablespoon

apricot    of

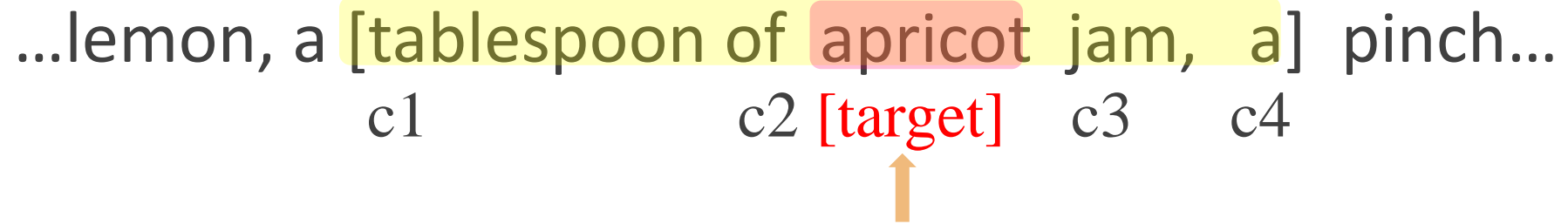
apricot    jam

apricot    a

# Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...

c1                      c2 [target]    c3    c4



## positive examples +

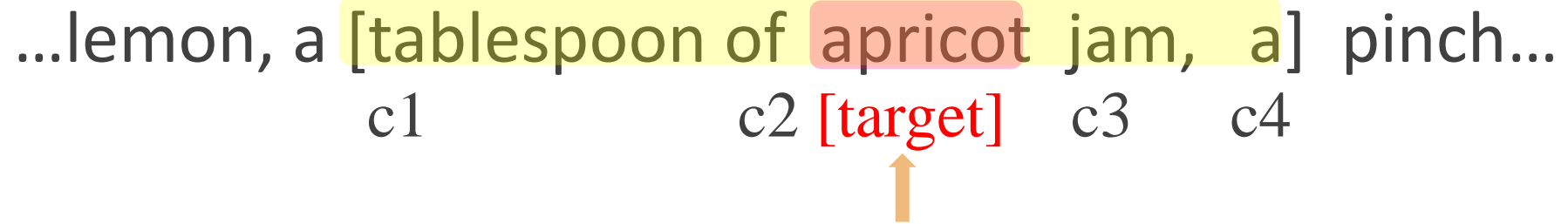
t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

For each positive example we'll grab k negative examples, sampling by frequency

# Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...

c1                      c2 [target]      c3      c4



## positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

## negative examples -

t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

# Word2vec: how to learn vectors

Given the set of positive and negative training instances, and an initial set of embedding vectors

The goal of learning is to adjust those word vectors such that we:

- **Maximize** the similarity of the **target word, context word** pairs  $(w, c_{\text{pos}})$  drawn from the positive data
- **Minimize** the similarity of the  $(w, c_{\text{neg}})$  pairs drawn from the negative data.

# Loss function for one $w$ with $c_{pos}$ , $c_{neg1} \dots c_{negk}$

Maximize the dot product of the word with the actual context words, and minimize the dot products of the word with the  $k$  negative sampled non-neighbor words.

$$\begin{aligned} L_{CE} &= -\log \left[ P(+|w, c_{pos}) \prod_{i=1}^k P(-|w, c_{neg_i}) \right] \\ &= - \left[ \log P(+|w, c_{pos}) + \sum_{i=1}^k \log P(-|w, c_{neg_i}) \right] \\ &= - \left[ \log P(+|w, c_{pos}) + \sum_{i=1}^k \log (1 - P(+|w, c_{neg_i})) \right] \\ &= - \left[ \log \sigma(c_{pos} \cdot w) + \sum_{i=1}^k \log \sigma(-c_{neg_i} \cdot w) \right] \end{aligned}$$

# Learning the classifier

How to learn?

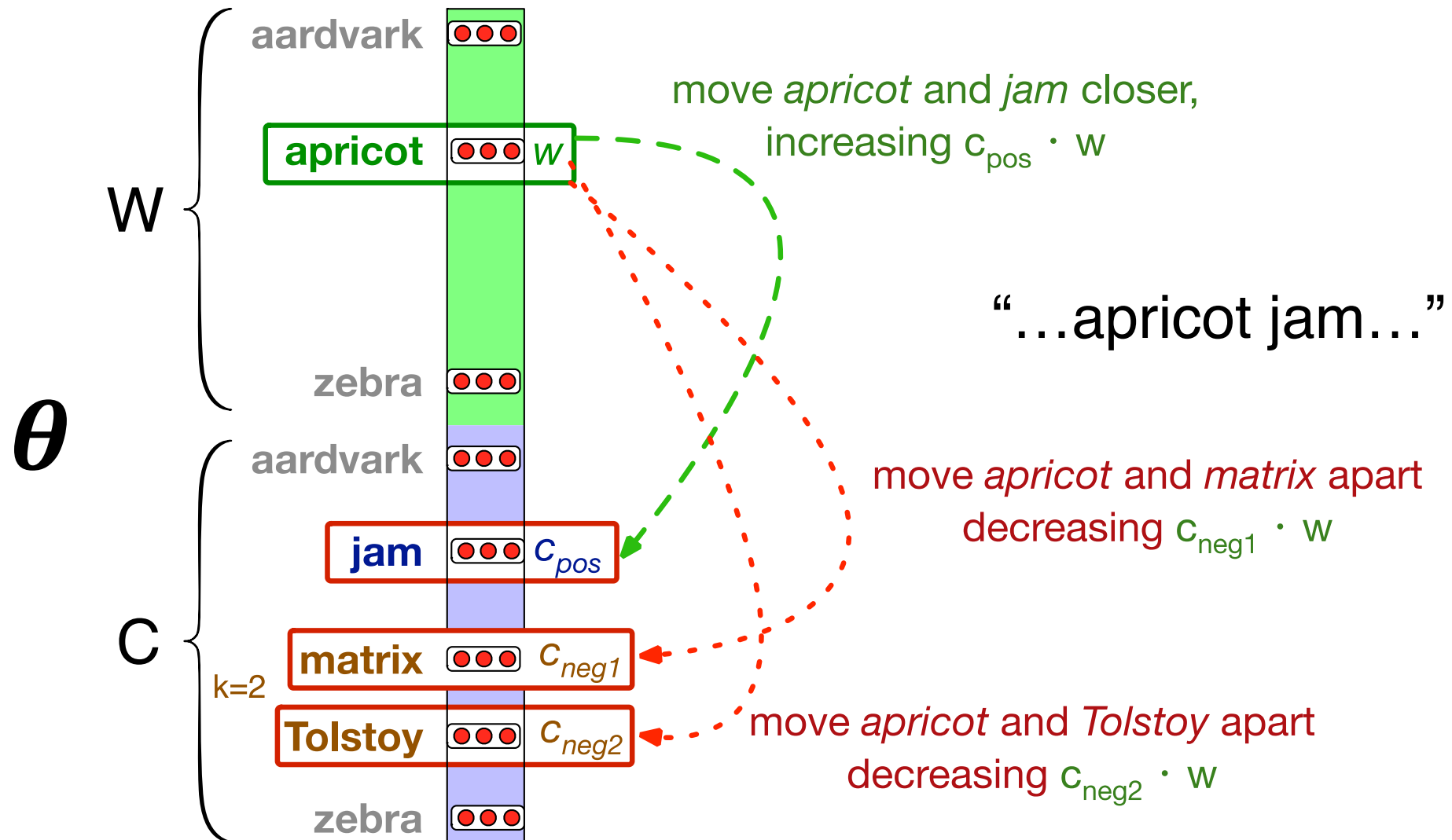
- Stochastic gradient descent!

We'll adjust the word weights to

- make the positive pairs more likely
- and the negative pairs less likely,
- over the entire training set.



# Intuition of one step of gradient descent



# The derivatives of the loss function

$$L_{CE} = - \left[ \log \sigma(c_{pos} \cdot w) + \sum_{i=1}^k \log \sigma(-c_{neg_i} \cdot w) \right]$$

$$\frac{\partial L_{CE}}{\partial c_{pos}} = [\sigma(c_{pos} \cdot w) - 1]w$$

$$\frac{\partial L_{CE}}{\partial c_{neg}} = [\sigma(c_{neg} \cdot w)]w$$

$$\frac{\partial L_{CE}}{\partial w} = [\sigma(c_{pos} \cdot w) - 1]c_{pos} + \sum_{i=1}^k [\sigma(c_{neg_i} \cdot w)]c_{neg_i}$$

# Update equation in SGD

Start with randomly initialized C and W matrices, then incrementally do updates

$$c_{pos}^{t+1} = c_{pos}^t - \eta [\sigma(c_{pos}^t \cdot w) - 1] w$$

$$c_{neg}^{t+1} = c_{neg}^t - \eta [\sigma(c_{neg}^t \cdot w)] w$$

$$w^{t+1} = w^t - \eta [\sigma(c_{pos} \cdot w^t) - 1] c_{pos} + \sum_{i=1}^k [\sigma(c_{neg_i} \cdot w^t)] c_{neg_i}$$

# Two sets of embeddings

SGNS learns two sets of embeddings

Target embeddings matrix  $W$

Context embedding matrix  $C$

It's common to just add them together,  
representing word  $i$  as the vector  $w_i + c_i$

# Summary: How to learn word2vec (skip-gram) embeddings

Start with  $V$  random  $d$ -dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
- Throw away the classifier code and keep the embeddings.

# Properties of Embeddings

**Large windows** ( $C = +/- 5$ ) : nearest words are related words in same semantic field

- *Hogwarts* nearest neighbors are Harry Potter world:
- *Dumbledore, Half-blood, Malfoy*

**Small windows** ( $C = +/- 2$ ) : nearest words are similar nouns, words in same taxonomy

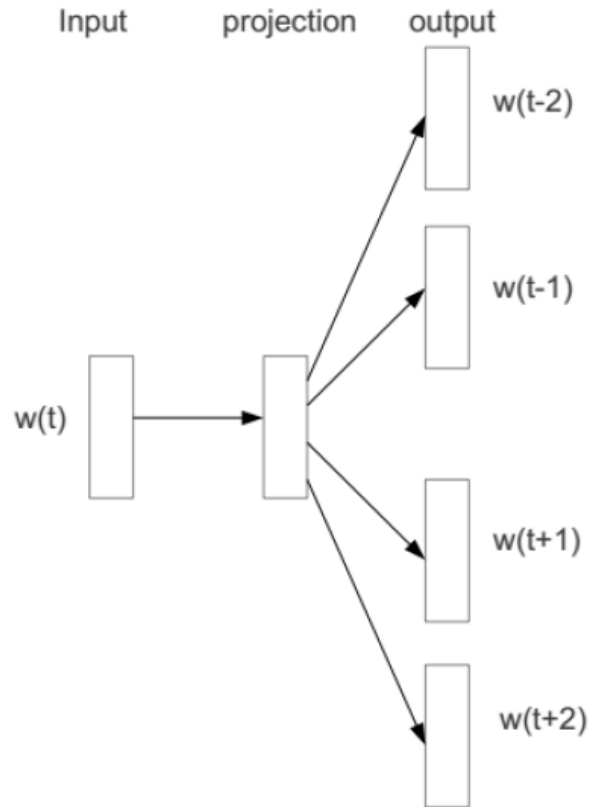
- *Hogwarts* nearest neighbors are other fictional schools
- *Sunnydale, Evernight, Blandings*

# Word2vec: CBOW

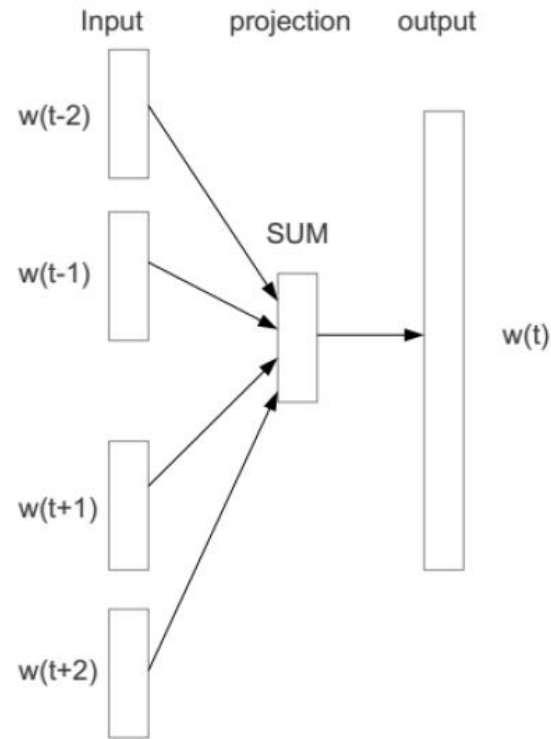
- The 'continuous bag-of-words model' (CBOW) adds inputs from words within short window to predict the current word.
- The weights for different positions are shared.
- The hidden layer is linear.

(Mikolov, 2014 Tutorial)

# Skip-grams vs. CBOW (Mikolov, 2014 Tutorial)



Skip-grams



CBOW

**Skip-gram:** work well with a small amount of the training data, represent well even rare words or phrases.

**CBOW:** faster to train, slightly **better** accuracy for the frequent words.



# FastText Embeddings

- FastText is another word embedding method that is an extension of the word2vec model.
- Instead of learning vectors for words directly, fastText represents each word as an n-gram of characters.
- This helps capture the meaning of shorter words and allows the embeddings to understand suffixes and prefixes.

# GloVe Embedding's (Pennington et al. 2014)

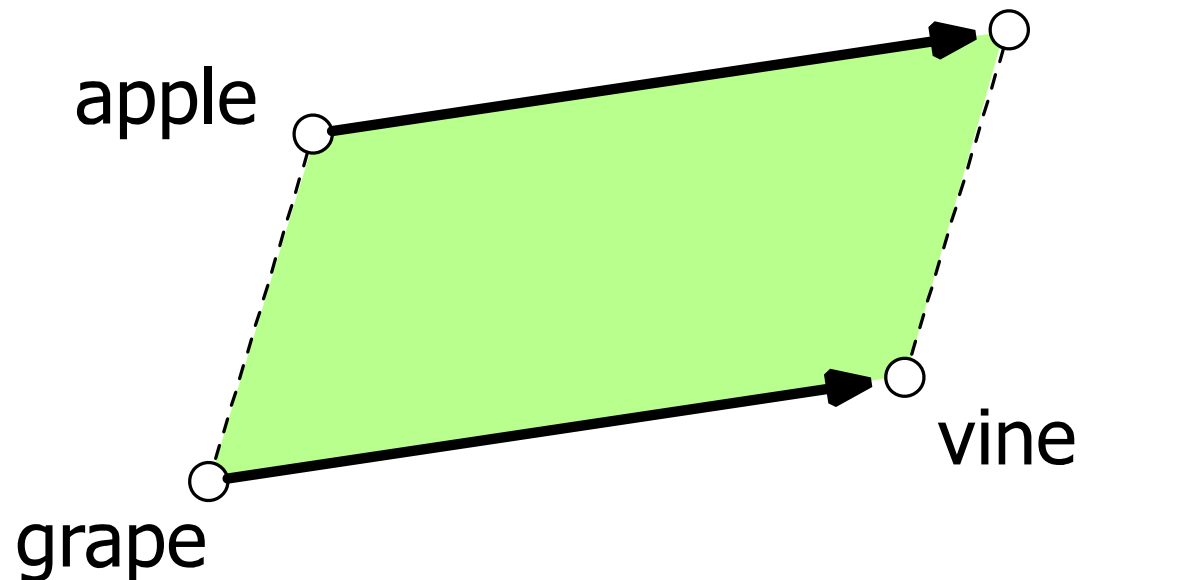
- Count-based method not neural network.
- Very large corpus.
- Log-bilinear model with a weighted least-squares objective.
- Focus on encoding vector differences.

# Analogy relations

The classic parallelogram model of analogical reasoning  
(Rumelhart and Abrahamson 1973)

To solve: "apple is to tree as grape is to \_\_\_\_\_"

Add  $\overrightarrow{\text{apple}} - \overrightarrow{\text{tree}}$  to  $\overrightarrow{\text{grape}}$  to get **vine**



Evaluation measure:  
Accuracy over the set  
of analogy questions.

# Analogy relations via parallelogram

The parallelogram method can solve analogies with both sparse and dense embeddings (Turney and Littman 2005, Mikolov et al. 2013b)

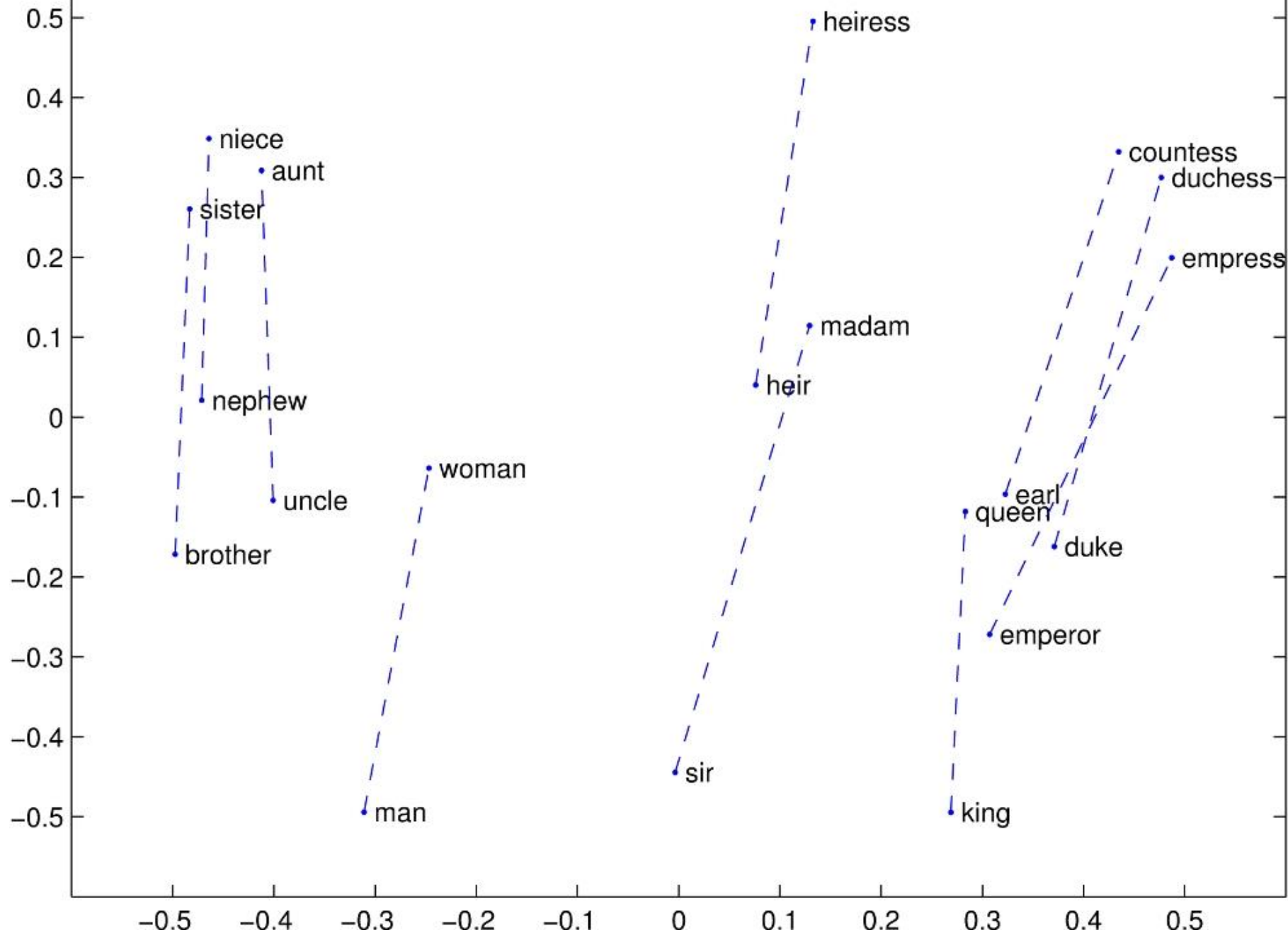
$\vec{\text{king}} - \vec{\text{man}} + \vec{\text{woman}}$  is close to  $\vec{\text{queen}}$

$\vec{\text{Paris}} - \vec{\text{France}} + \vec{\text{Italy}}$  is close to  $\vec{\text{Rome}}$

For a problem  $a:a^*::b:b^*$ , the parallelogram method is:

$$\hat{b}^* = \underset{x}{\operatorname{argmax}} \operatorname{distance}(x, a^* - a + b)$$

# Structure in GloVe Embedding space



# Caveats with the parallelogram method

It only seems to work for frequent words, small distances and certain relations (relating countries to capitals, or parts of speech), but not others. (Linzen 2016, Gladkova et al. 2016, Ethayarajh et al. 2019a)

Understanding analogy is an open area of research (Peterson et al. 2020)