Probabilistic IR Model

Probabilistic Model

- An initial set of documents is retrieved (somehow)
- User inspects these docs looking for the relevant ones (only top 10-20) (we see later that we eliminate this manual step in the actual probabilistic model)
- IR system uses this info to refine description of ideal answer set
- By repeting this process, description of the ideal answer set will improve
- Description of ideal answer set is modeled in probabilistic terms

Probabilistic Ranking Principle

- Given a user query q and a document d_j, the probabilistic model estimates the probability that the user will find the document d_j relevant.
- The model assumes that probability of relevance depends on the query and the document representations only.
- Ideal answer set is referred to as R.
- Documents in the set R are predicted to be relevant.
  - how to compute probabilities?
  - what is the sample space?

The Ranking

- Probabilistic ranking computed as:
  - $sim(q,d_j) = P(d_j \text{ relevant-to } q) / P(d_j \text{ non-relevant-to } q)$
    - How to read this? “Maximize the number of relevant documents, minimize the number of irrelevant documents”
  - This is the odds of the document dj being relevant
- Definition:
  - $w_{ij} \in \{0,1\}$
  - $P(R \mid d_j)$: probability that document d_j is relevant
  - $P(\neg R \mid d_j)$: probability that d_i is not relevant
  - Use Bayes Rule: $P(A|B)\ P(B) = P(B|A)P(A)$

The Ranking

- $$\begin{aligned} sim(d_j,q) &= P(R \mid d_j) \,/\, P(\neg R \mid d_j) \\ &= \frac{[P(d_j \mid R) * P(R)]}{[P(d_j \mid \neg R) * P(\neg R)]} \\ &\sim \frac{P(d_j \mid R)}{P(d_j \mid \neg R)} \end{aligned}$$
- $P(d_j \mid R)$: probability of randomly selecting the document d_j from the set R of relevant documents
- Note that $P(R)$ and $P(\neg R)$ are the same for all documents in the collection for the given query

The Ranking

- $sim(d_j,q) \sim \dfrac{P(d_j \mid R)}{P(d_j \mid \neg R)}$

  $\sim \dfrac{[\prod P(k_i \mid R)] \; * \; [\prod P(\neg k_i \mid R)]}{[\prod P(k_i \mid \neg R)] \; * \; [\prod P(\neg k_i \mid \neg R)]}$

- $P(k_i \mid R)$: probability that the index term k_i is present in a document randomly selected from the set R of relevant documents
- Based on independence assumption
  - Strong assumption!
    - In real life, does not always hold

The Ranking

- $$sim(dj,q) \sim \log \frac{[\prod P(k_i \mid R)] * [\prod P(\neg k_i \mid R)]}{[\prod P(k_i \mid \neg R)] * [\prod P(\neg k_i \mid \neg R)]}$$

$$\sim [\log \prod \frac{P(k_i \mid R)}{P(\neg k_i \mid R)} + \log \prod \frac{P(k_i \mid \neg R)}{P(\neg k_i \mid \neg R)}]$$

$$\sim \sum w_{iq} * w_{ij} * (\log \frac{P(k_i \mid R)}{P(\neg k_i \mid R)} + \log \frac{P(k_i \mid \neg R)}{P(\neg k_i \mid \neg R)})$$

where $P(\neg k_i \mid R) = 1 - P(k_i \mid R)$

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The Initial Ranking

- $sim(d_j,q) \sim$

  $$\sim \sum w_{iq} * w_{ij} * \left(\log \frac{P(k_i \mid R)}{P(\neg k_i \mid R)} + \log \frac{P(k_i \mid \neg R)}{P(\neg k_i \mid \neg R)}\right)$$

- Probabilities $P(k_i \mid R)$ and $P(k_i \mid \neg R)$?
- Estimates based on assumptions:
  - $P(k_i \mid R) = 0.5$
  - $P(k_i \mid \neg R) = n_i / N$

    where n_i is the number of docs that contain k_i
  - Use this initial guess to retrieve an initial ranking
  - Improve upon this initial ranking

Improving the Initial Ranking

- $sim(d_j,q) \sim$

$$\sim \sum w_{iq} * w_{ij} * \left(\log \frac{P(k_i \mid R)}{P(\neg k_i \mid R)} + \log \frac{P(k_i \mid \neg R)}{P(\neg k_i \mid \neg R)}\right)$$

  - V : set of docs initially retrieved
  - V_i : subset of docs retrieved that contain k_i
- Reevaluate estimates:
  - $P(k_i \mid R) = \frac{V_i}{V}$
  - $P(k_i \mid \neg R) = \frac{n_i - V_i}{N - V}$
- Repeat recursively

Improving the Initial Ranking

- $sim(d_j,q) \sim$

$$\sim \sum w_{iq} * w_{ij} * \left(\log \frac{P(k_i \mid R)}{P(\neg k_i \mid R)} + \log \frac{P(k_i \mid \neg R)}{P(\neg k_i \mid \neg R)}\right)$$

- To avoid problems with $V=1$ and $V_i=0$:
  - $P(k_i \mid R) = \dfrac{V_i + n_i/N}{V + 1}$
  - $P(k_i \mid \neg R) = \dfrac{n_i - V_i + n_i/N}{N - V + 1}$
  - (replace n_i/N with 0.5)

Okapi Formula (BM25) (Robertson and Sparck-Jones, 1976)

$$w_{i,j} = \frac{tf_{i,j} \log\left(\frac{N - df_i + 0.5}{dfi + 0.5}\right)}{k_1 \times ((1-b) + b\frac{dl}{avdl}) + tf_{i,j}}$$

N = number of documents in the collection

$tf_{i,j}$ = frequency of term i id document j

df_i = number of documents that contain term j

dl = length of document j

avdl = average length over documents

k1 and b are parameters

- Use this weight in VSM or plug in the probabilistic formula.