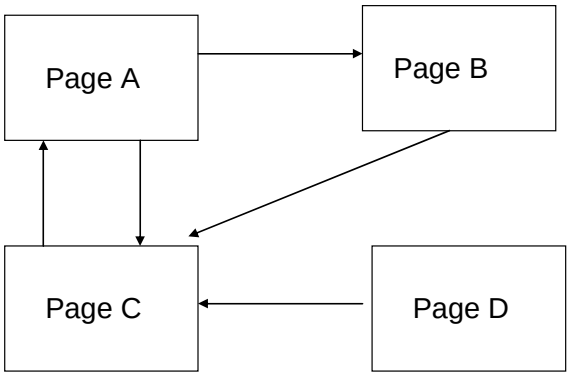
1. Consider the following subset of pages and their links:
Page A points to pages B and C
Page B points to page C
Page C points to page A
Page D points to page C



Apply the PageRank algorithm, in the following form :
For every page A: **P(A) = (1-d) + d * (P(T1)/C(T1) + … + P(Tn)/C(Tn))**
where T1 … Tn = pages that point to page A
d = damping factor. Use d = 0.85.
C(A) = number of links going out of page A

Skip the normalization step that divides each PageRank value by the sum of all the PageRanks in order to make the ranks of all the pages add up to 1 (for simplicity).

Solution:

P(A) = 0.15 + 0.85 * (P(C))
P(B) = 0.15 + 0.85 * (P(A)/2)
P(C) = 0.15 + 0.85 * (P(A)/2 + P(B) + P(D))
P(D) = 0.15

a) Initial values: could be any (one here)

P(A) = 1 P(B) = 1 P(C) = 1 P(D) = 1

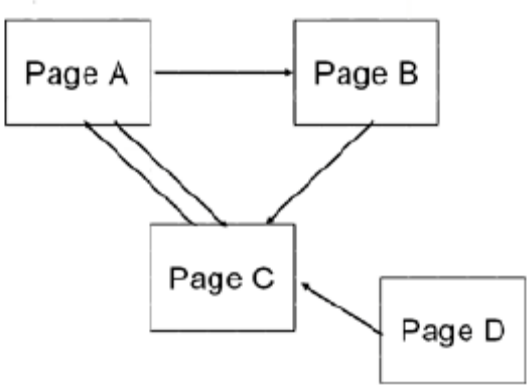
Iteration 1
P(A) = 0.15 + 0.85 * 1 = 1
P(B) = 0.15 + 0.85 * 0.5 = 0.575
P(C) = 0.15 + 0.85 * (0.5 + 1 + 1) = 2.275
P(D) = 0.15

Iteration 2
P(A) = 0.15 + 0.85 * 2.275 = 2.08375
P(B) = 0.15 + 0.85 * 0.5 = 0.575

P(C) = 0.15 + 0.85 * (0.5 + 0.575 + 0.15) = 1.19125
P(D) = 0.15

Iteration 3
P(A) = 0.15 + 0.85 * 1.19125 = 1.1625625
P(B) = 0.15 + 0.85 * 1.041875 = 1.03559375
P(C) = 0.15 + 0.85 * (1.041875 + 0.575 + 0.15) = 1.65184375
P(D) = 0.15

…….



PageRank for pages A, B, C, D at various stages of iteration

	A	B	C	D
Iteration	1	1	1	1
1	1,0000000000	0,5750000000	2,2750000000	0,1500000000
2	2,0837500000	0,5750000000	1,1912500000	0,1500000000
3	1,1625625000	1,0355937500	1,6518437500	0,1500000000
******	**********	**********	**********	**********
19	1,4900124031	0,7833688246	1,5766187723	0,1500000000
20	1,4901259564	0,7832552713	1,5766187723	0,1500000000
21	1,4901259564	0,7833035315	1,5765705121	0,1500000000
46	1,4901074052	0,7832956473	1,5765969475	0,1500000000
47	1,4901074054	0,7832956472	1,5765969474	0,1500000000
48	1,4901074053	0,7832956473	1,5765969474	0,1500000000
49	1,4901074053	0,7832956473	1,5765969474	0,1500000000
50	1,4901074053	0,7832956473	1,5765969474	0,1500000000
51	1,4901074053	0,7832956473	1,5765969474	0,1500000000
52	1,4901074053	0,7832956473	1,5765969474	0,1500000000
53	1,4901074053	0,7832956473	1,5765969474	0,1500000000
54	1,4901074053	0,7832956473	1,5765969474	0,1500000000
******	**********	**********	**********	**********

b) The same example, but with different initial values (zero instead of one)

Initial values: could be any (zero here)

P(A) = 0 P(B) = 0 P(C) = 0 P(D) = 0

Iteration 1
P(A) = 0.15 + 0.85 * (P(C)) = 0.15
P(B) = 0.15 + 0.85 * (P(A)/2) = 0.15
P(C) = 0.15 + 0.85 * (P(A)/2 + P(B) + P(D)) = 0.15
P(D) = 0.15

Iteration 2
P(A) = 0.15 + 0.85 * 0.15 = 0.2775
P(B) = 0.15 + 0.85 * 0.075 = 0.21375
P(C) = 0.15 + 0.85 * (0.075 + 0.15 + 0.15) = 0.46875
P(D) = 0.15

Iteration 3
P(A) = 0.15 + 0.85 * 0.46875 = 0.5484375
P(B) = 0.15 + 0.85 * 0.13875 = 0.2679375
P(C) = 0.15 + 0.85 * (0.13875 + 0.21375 + 0.15) = 0.577125
P(D) = 0.15

…….

2. The same graph, different version of PageRank algorithm

Consider the following subset of pages and their links:
Page A points to pages B and C
Page B points to page C
Page C points to page A
Page D points to page C

Run the following PageRank algorithm. Use $\alpha = 0.15$. Simulate the algorithm for three iterations. Show the PageRank for each page twice for each iteration (the value before and after normalization). Order the pages in the sequence in the order: A B C D

Algorithm: Let S be the total set of pages.
Initialize $\forall p \in S: R(p) = 1/|S|$
Let $\forall p \in S: E(p) = \alpha/|S|$ ($\alpha = 0.15$)
Until ranks do not change (much) (*convergence*)
For each $p \in S$:

$$R'(p) = \sum_{q:q \to p} \frac{R(q)}{N_q} + E(p)$$

$$c = 1/\sum_{p \in S} R'(p)$$

For each $p \in S$: $R(p) = cR'(p)$ (*normalize*)

Solution:

Initial values:

A B C D

R = [0.25, 0.25, 0.25, 0.25]

E = [0.0375, 0.0375, 0.0375, 0.0375]

R'(A) = R(C) + E(A)
R'(B) = R(A)/2 + E(B)
R'(C) = R(A)/2 + R(B) + R(D) + E(C)
R'(D) = E(D)

c = 1 / (R'(A) + R'(B) + R'(C) + R'(D))
R(A) = c R'(A)
R(B) = c R'(B)
R(C) = c R'(C)
R(D) = c R'(D)

Iteration 1:

R'(A) = 0.25 + 0.0375 = 0.2875
R'(B) = 0.125 + 0.0375 = 0.1625
R'(C) = 0.125 + 0.25 + 0.25 + 0.0375 = 0.6625
R'(D) = 0.0375

c = 1 / (0.2875 + 0.1625 + 0.6625 + 0.0375) = 1 / 1.15

R(A) = c R'(A) = 0.2875 / 1.15 = 0.25
R(B) = c R'(B) = 0.1625 / 1.15 = 0.141
R(C) = c R'(C) = 0.6625 / 1.15 = 0.576
R(D) = c R'(D) = 0.0375 / 1.15 = 0.0326

R' = [0.2875, 0.1625, 0.6625, 0.0375]
Norm R = [0.25, 0.141, 0.576, 0.0326]

Iteration 2:

R'(A) = 0.576 + 0.0375 = 0.6135
R'(B) = 0.125 + 0.0375 = 0.1625
R'(C) = 0.125 + 0.141 + 0.0326 + 0.0375 = 0.3361
R'(D) = 0.0375

c = 1 / (0.6135, 0.1625, 0.3361, 0.0375) = 1 / 1.1496

R' = [0.6135, 0.1625, 0.3361, 0.0375]
Norm R = [0.533, 0.141, 0.292, 0.0326]

Iteration 3:

R’(A) = 0.292 + 0.0375 = 0.3295
R’(B) = 0.2665 + 0.0375 = 0.304
R’(C) = 0.2665 + 0.141 + 0.0326 + 0.0375 = 0.4776
R’(D) = 0.0375
c = 1 / (0.3295 + 0.304 + 0.4776 + 0.0375) = 1 / 1.1486

R' = [0.3295, 0.304, 0.4776, 0.0375]
Norm R = [0.286, 0.264, 0.415, 0.0326]

…….