Boolean and Vector Space Retrieval Models

This material was prepared by Diana Inkpen, University of Ottawa, 2005, updated 2021. Some of these slides were originally prepared by Raymond Mooney, University of Texas Austin.

Retrieval Models

- A retrieval model specifies the details of:
  - Document representation
  - Query representation
  - Retrieval function
- Determines a notion of relevance.
- Notion of relevance can be binary or continuous (i.e., *ranked retrieval*).

Classes of Retrieval Models

- Boolean models (set theoretic)
  - Extended Boolean
- Vector space models (statistical/algebraic)
  - Generalized VS
  - Latent Semantic Indexing
- Probabilistic models

Other Model Dimensions

- Logical View of Documents
  - Index terms
  - Full text
  - Full text + Structure (e.g. hypertext)
- User Task
  - Retrieval
  - Browsing

Retrieval Tasks

- Ad hoc retrieval: Fixed document corpus, varied queries.
- Filtering: Fixed query, continuous document stream.
  - User Profile: A model of relative static preferences.
  - Binary decision of relevant/not-relevant.
- Routing: Same as filtering but continuously supply ranked lists rather than binary filtering.

Common Preprocessing Steps

- Strip unwanted characters/markup (e.g. HTML tags, punctuation, numbers, etc.).
- Break into tokens (keywords) on whitespace.
- Stem tokens to “root” words
  - computational → comput
- Remove common stopwords (e.g. a, the, it, etc.).
- Detect common phrases (possibly using a domain specific dictionary).
- Build inverted index (keyword → list of docs containing it).

Boolean Model

- A document is represented as a set of keywords.
- Queries are Boolean expressions of keywords, connected by AND, OR, and NOT, including the use of brackets to indicate scope.
  - [[Rio & Brazil] | [Hilo & Hawaii]] & hotel & !Hilton]
- Output: Document is relevant or not. No partial matches or ranking.

Boolean Retrieval Model

- Popular retrieval model because:
  - Easy to understand for simple queries.
  - Clean formalism.
- Boolean models can be extended to include ranking.
- Reasonably efficient implementations possible for normal queries.

Boolean Models – Problems

- Very rigid: AND means all; OR means any.
- Difficult to express complex user requests.
- Difficult to control the number of documents retrieved.
  - *All* matched documents will be returned.
- Difficult to rank output.
  - *All* matched documents logically satisfy the query.
- Difficult to perform relevance feedback.
  - If a document is identified by the user as relevant or irrelevant, how should the query be modified?

Statistical Models

- A document is typically represented by a *bag of words* (unordered words with frequencies).
- Bag = set that allows multiple occurrences of the same element.
- User specifies a set of desired terms with optional weights:
  - Weighted query terms:

    Q = < database 0.5; text 0.8; information 0.2 >
  - Unweighted query terms:

    Q = < database; text; information >
  - No Boolean conditions specified in the query.

Statistical Retrieval

- Retrieval based on *similarity* between query and documents.
- Output documents are ranked according to similarity to query.
- Similarity based on occurrence *frequencies* of keywords in query and document.
- Automatic relevance feedback can be supported:
  - Relevant documents "added" to query.
  - Irrelevant documents "subtracted" from query.

Issues for Vector Space Model

- How to determine important words in a document?
  - Word sense?
  - Word n-grams (and phrases, idioms,…) → terms
- How to determine the degree of importance of a term within a document and within the entire collection?
- How to determine the degree of similarity between a document and the query?
- In the case of the web, what is a collection and what are the effects of links, formatting information, etc.?

The Vector-Space Model

- Assume t distinct terms remain after preprocessing; call them index terms or the vocabulary.
- These "orthogonal" terms form a vector space.

  Dimension = t = |vocabulary|

- Each term, i, in a document or query, j, is given a real-valued weight, w_{ij}.
- Both documents and queries are expressed as t-dimensional vectors:

  $d_j = (w_{1j}, w_{2j}, \ldots, w_{tj})$

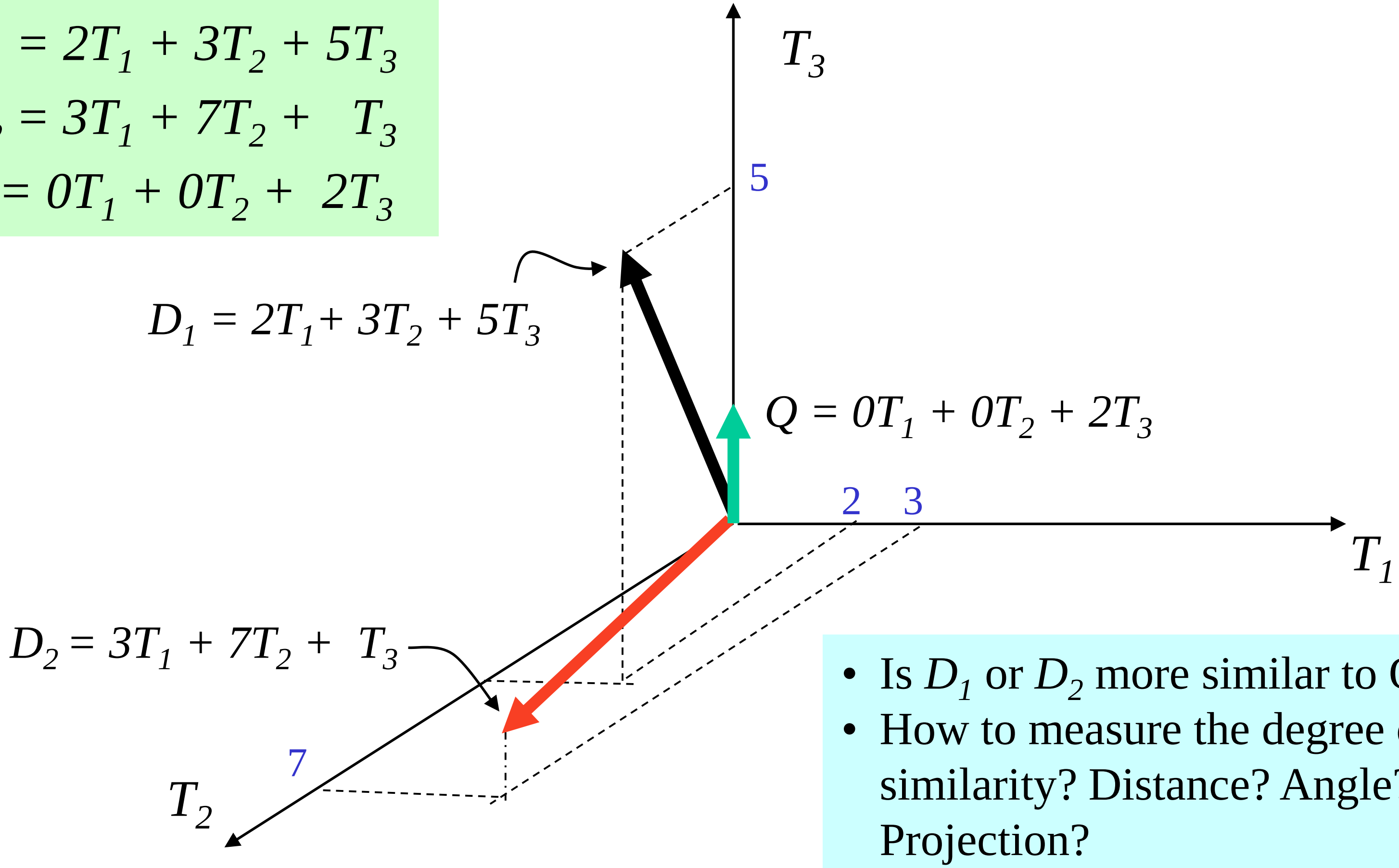
Graphic Representation

Example:

$D_1 = 2T_1 + 3T_2 + 5T_3$

$D_2 = 3T_1 + 7T_2 + T_3$

$Q = 0T_1 + 0T_2 + 2T_3$



- Is D_1 or D_2 more similar to Q?
- How to measure the degree of similarity? Distance? Angle? Projection?

More on the Vector Space Model

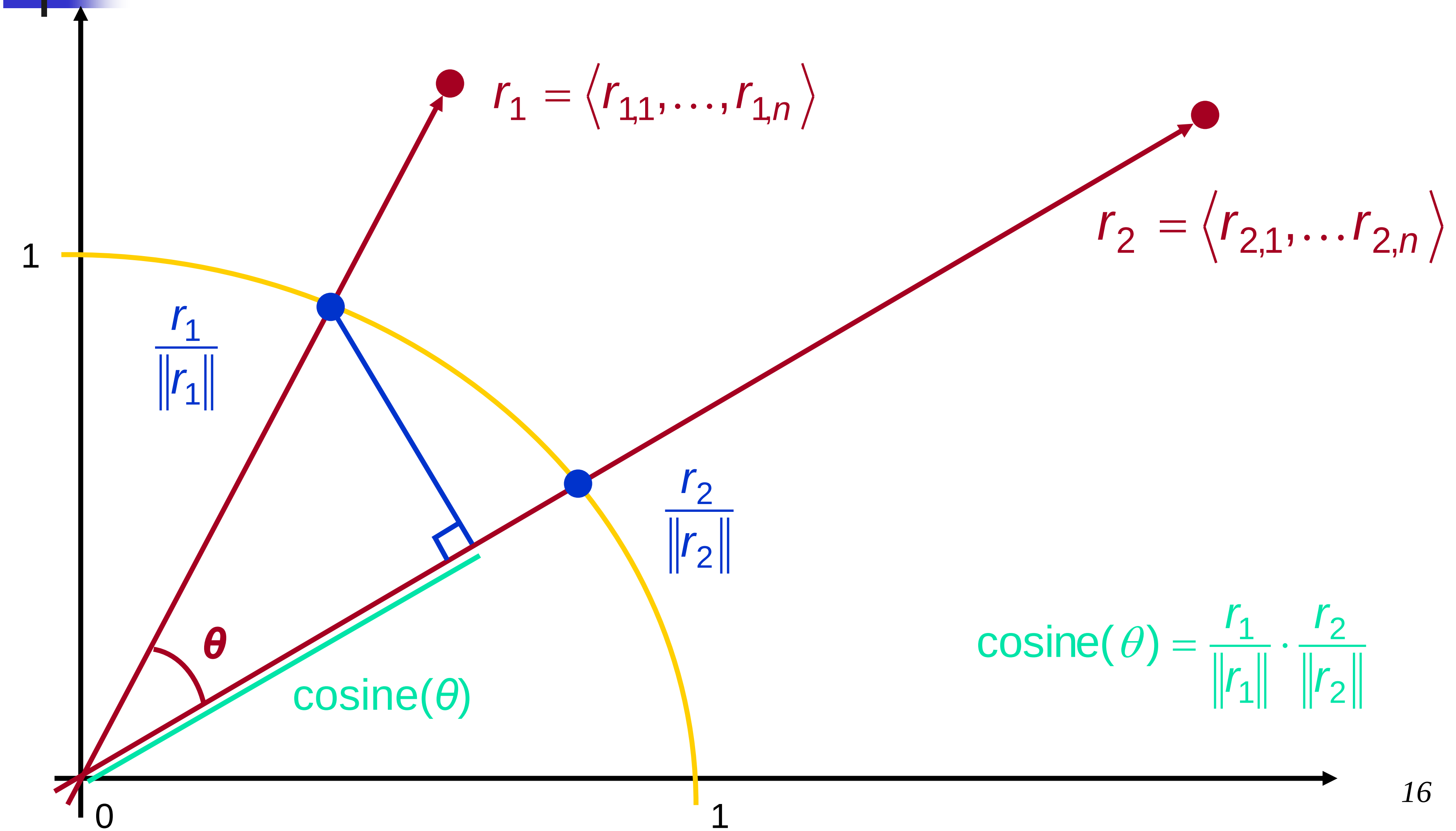
- create vectors, r_1 and r_2, that represent features of R_1 and R_2

$$r_1 = \langle r_{1,1}, \ldots, r_{1,n} \rangle \qquad r_2 = \langle r_{2,1}, \ldots r_{2,n} \rangle$$

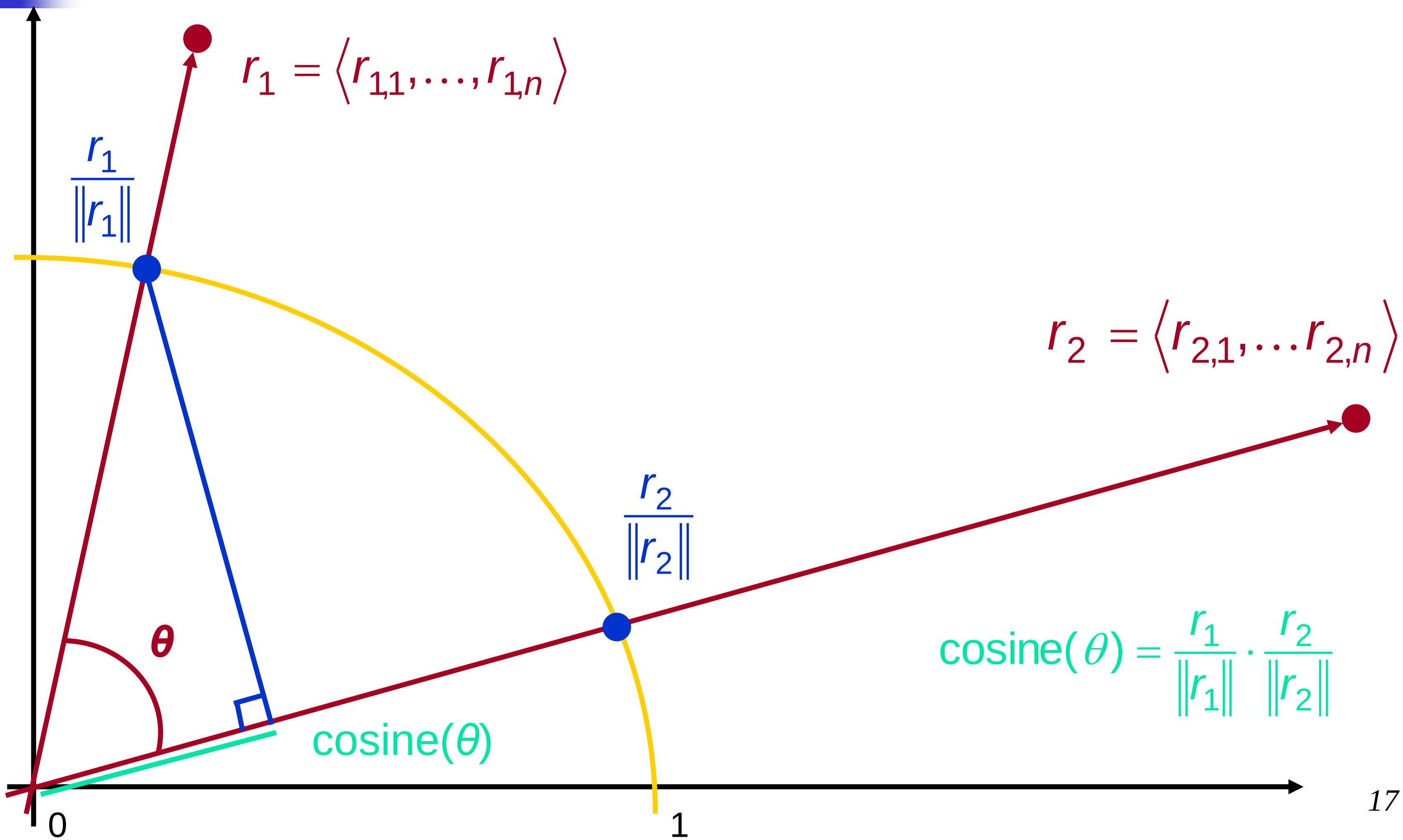
- measure the similarity of R_1 and R_2 by the cosine of the angle θ between r_1 and r_2

$$\text{cosine}(\theta) = \frac{\sum_{i=1}^{n} r_{1,i}\ r_{2,i}}{\sqrt{\sum_{i=1}^{n} (r_{1,i})^2 \cdot \sum_{i=1}^{n} (r_{2,i})^2}} = \frac{r_1 \cdot r_2}{\sqrt{r_1 \cdot r_1} \cdot \sqrt{r_2 \cdot r_2}} = \frac{r_1 \cdot r_2}{\|r_1\| \cdot \|r_2\|}$$

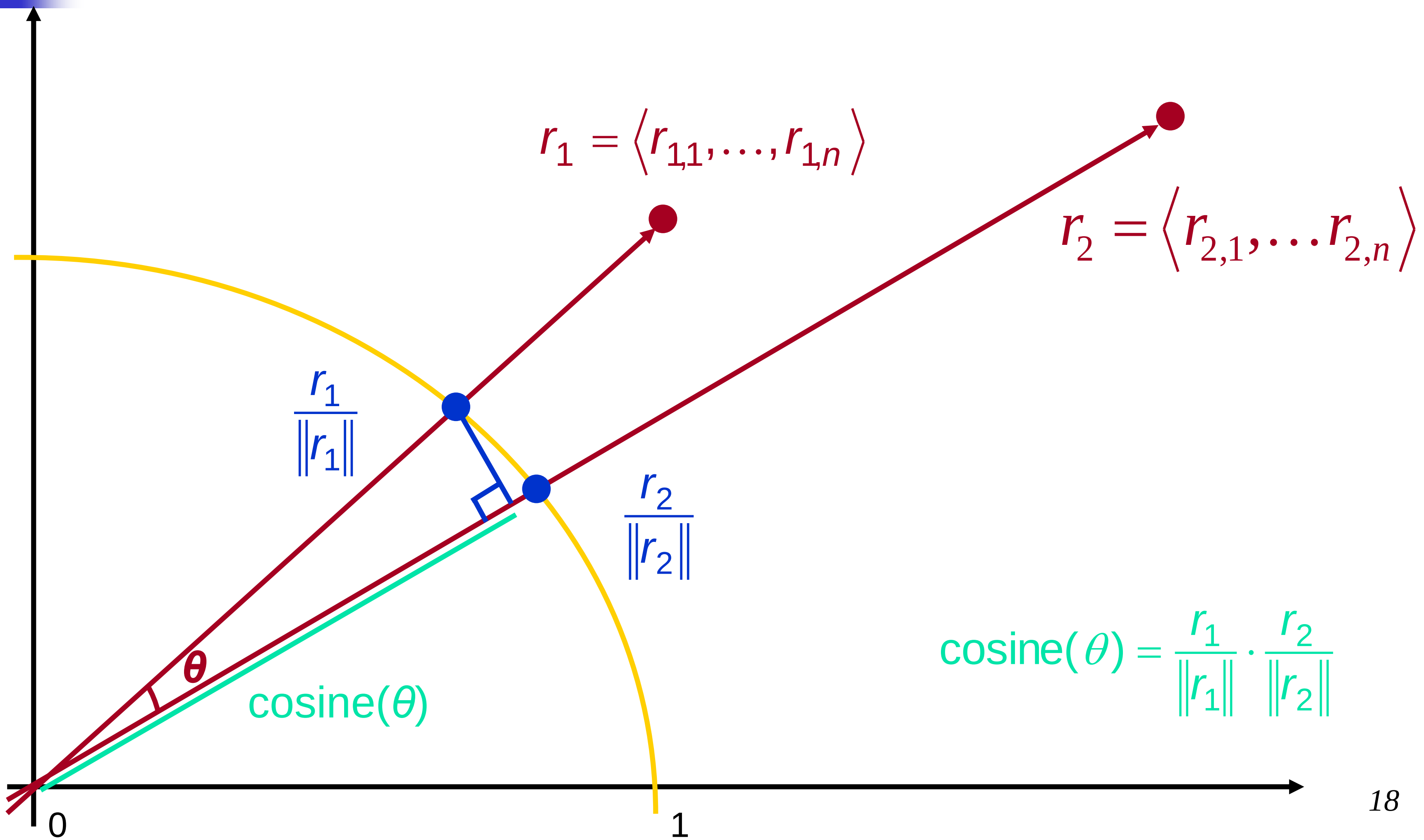
More on VSM (continued)



More on VSM (continued)



More on VSM (continued)



$$\text{cosine}(\theta) = \frac{r_1}{\|r_1\|} \cdot \frac{r_2}{\|r_2\|}$$

Document Collection

- A collection of *n* documents can be represented in the vector space model by a term-document matrix.
- An entry in the matrix corresponds to the "weight" of a term in the document; zero means the term has no significance in the document or it simply doesn't exist in the document.

$$
\begin{pmatrix}
 & T_1 & T_2 & \dots & T_t \\
D_1 & w_{11} & w_{21} & \dots & w_{t1} \\
D_2 & w_{12} & w_{22} & \dots & w_{t2} \\
\vdots & \vdots & \vdots & & \vdots \\
\vdots & \vdots & \vdots & & \vdots \\
D_n & w_{1n} & w_{2n} & \dots & w_{tn}
\end{pmatrix}
$$

Term Weights: Term Frequency

- More frequent terms in a document are more important, i.e. more indicative of the topic.

  f_{ij} = frequency of term i in document j

- May want to normalize *term frequency* (*tf*) across the entire corpus:

  $$tf_{ij} = f_{ij} \;/\; max_i \{f_{ij}\}$$

Term Weights: Inverse Document Frequency

- Terms that appear in many *different* documents are *less* indicative of overall topic.

  df_i = document frequency of term i

  = number of documents containing term i

  idf_i = inverse document frequency of term i,

  $= \log_2 (N / df_i)$

  (N: total number of documents)

- An indication of a term's *discrimination* power.
- Log used to dampen the effect relative to *tf*.

TF-IDF Weighting

- A typical combined term importance indicator is *tf-idf weighting*:

$$w_{ij} = tf_{ij}\ idf_i = tf_{ij} \log_2 (N/\, df_i)$$

- A term occurring frequently in the document but rarely in the rest of the collection is given high weight.
- Many other ways of determining term weights have been proposed.
- Experimentally, *tf-idf* has been found to work well.

Computing TF-IDF -- An Example

Given a document containing terms with given frequencies:

A(3), B(2), C(1)

Assume collection contains 10,000 documents and document frequencies of these terms are:

A(50), B(1300), C(250)

Then:

A: tf = 3/3; idf = log(10000/50) = 5.3; tf-idf = 5.3

B: tf = 2/3; idf = log(10000/1300) = 2.0; tf-idf = 1.3

C: tf = 1/3; idf = log(10000/250) = 3.7; tf-idf = 1.2

Query Vector

- Query vector is typically treated as a document and also tf-idf weighted.
- Alternative is for the user to supply weights for the given query terms.

Similarity Measure

- A similarity measure is a function that computes the *degree of similarity* between two vectors.

- Using a similarity measure between the query and each document:
  - It is possible to rank the retrieved documents in the order of presumed relevance.
  - It is possible to enforce a certain threshold so that the size of the retrieved set can be controlled.

Similarity Measure - Inner Product

- Similarity between vectors for the document $\boldsymbol{d}_i$ and query $\boldsymbol{q}$ can be computed as the vector inner product:

$$\text{sim}(\boldsymbol{d}_j,\boldsymbol{q}) = \boldsymbol{d}_{\mathrm{j}}\bullet\boldsymbol{q} = \sum_{i=1}^{t} w_{ij} \cdot w_{iq}$$

where w_{ij} is the weight of term i in document j and w_{iq} is the weight of term i in the query

- For binary vectors, the inner product is the number of matched query terms in the document (size of intersection).
- For weighted term vectors, it is the sum of the products of the weights of the matched terms.

Properties of Inner Product

- The inner product is unbounded.
- Favors long documents with a large number of unique terms.
- Measures how many terms matched but not how many terms are *not* matched.

Inner Product -- Examples

Binary: retrieval database architecture computer text management information

- $D = 1, \quad 1, \quad 1, \quad 0, \quad 1, \quad 1, \quad 0$
- $Q = 1, \quad 0, \quad 1, \quad 0, \quad 0, \quad 1, \quad 1$

Size of vector = size of vocabulary = 7
0 means corresponding term not found in document or query

sim(D, Q) = 3

Weighted:

$D_1 = 2T_1 + 3T_2 + 5T_3 \qquad D_2 = 3T_1 + 7T_2 + 1T_3$

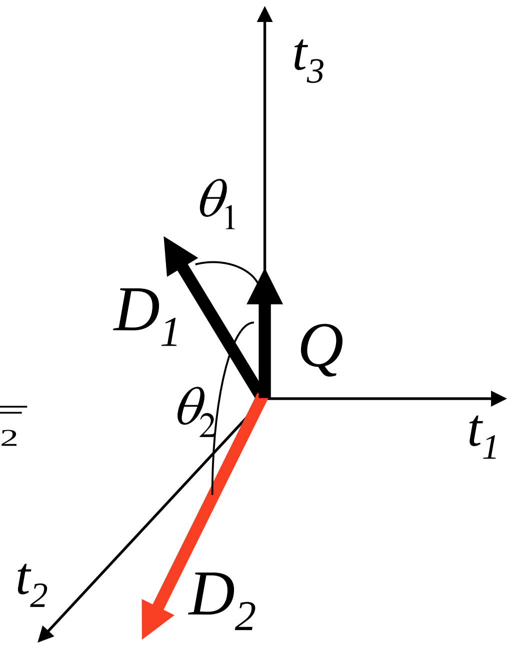
$Q = 0T_1 + 0T_2 + 2T_3$

$\text{sim}(D_1, Q) = 2*0 + 3*0 + 5*2 = 10$

$\text{sim}(D_2, Q) = 3*0 + 7*0 + 1*2 = 2$

Cosine Similarity Measure

- Cosine similarity measures the cosine of the angle between two vectors.
- Inner product normalized by the vector lengths.



$$\text{CosSim}(\boldsymbol{d}_j, \boldsymbol{q}) = \frac{\vec{d}_j \cdot \vec{q}}{\left|\vec{d}_j\right| \cdot \left|\vec{q}\right|} = \frac{\sum_{i=1}^{t} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{t} w_{ij}^2 \cdot \sum_{i=1}^{t} w_{iq}^2}}$$

$D_1 = 2T_1 + 3T_2 + 5T_3$ CosSim(D_1 , Q) = $10 / \sqrt{(4+9+25)(0+0+4)} = 0.81$
$D_2 = 3T_1 + 7T_2 + 1T_3$ CosSim(D_2 , Q) = $2 / \sqrt{(9+49+1)(0+0+4)} = 0.13$
$Q = 0T_1 + 0T_2 + 2T_3$

D_1 is 6 times better than D_2 using cosine similarity but only 5 times better using inner product.

Naïve Implementation

Convert all documents in collection D to tf-idf weighted vectors, $\boldsymbol{d_j}$, for keyword vocabulary V.

Convert query to a tf-idf-weighted vector $\boldsymbol{q}$.

For each $\boldsymbol{d_j}$ in D do

Compute score $s_j = \text{cosSim}(\boldsymbol{d_j}, \boldsymbol{q})$

Sort documents by decreasing score.

Present top ranked documents to the user.

Time complexity: $O(|V|\cdot|D|)$ Bad for large V & D !

$|V| = 10{,}000$; $|D| = 100{,}000$; $|V|\cdot|D| = 1{,}000{,}000{,}000$

Comments on Vector Space Models

- Simple, mathematically based approach.
- Considers both local (*tf*) and global (*idf*) word occurrence frequencies.
- Provides partial matching and ranked results.
- Tends to work quite well in practice despite obvious weaknesses.
- Allows efficient implementation for large document collections.

Problems with Vector Space Model

- Missing semantic information (e.g. word sense).
- Missing syntactic information (e.g. phrase structure, word order, proximity information).
- Assumption of term independence (e.g. ignores synonomy).
- Lacks the control of a Boolean model (e.g., *requiring* a term to appear in a document).
  - Given a two-term query "A B", may prefer a document containing A frequently but not B, over a document that contains both A and B, but both less frequently.

Exercise

The corpus C consists in the following three documents:

- d1: “new york times”
- d2: “new york post”
- d3: “los angeles times”

1. Assuming that the term frequencies are normalized by the maximum frequency in a given document, calculate the tf-idf scores for all the terms in C. Assume the words in the vectors are ordered alphabetically.

2. Given the following query: “new new times”, calculate the tf-idf vector for the query, and compute the score of each document in C relative to this query, using the cosine similarity measure. Assume that term frequencies are normalized by the maximum frequency in a given query.