CSI 2165 Winter 2006

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Note: These lecture notes will be accompanied by additional explanations, demonstrations, and small-group exercises in class.

Course Content

- Introduction to Prolog and Logic Programming.
- Prolog basic constructs: Facts, rules, knowledge base, goals, unification, instantiation.
- Prolog syntax, characters, equality and arithmetic.
- Data Structures: Structures and trees, lists, strings.
- Control Structures:
  - Backtracking, recursion, cut and failure.
- Input and output, assertions, consulting.
- Applications: Databases, Artificial Intelligence
  - Games, natural language processing, meta-interpreters

Prolog

- Prolog = ***Pro***gramming in ***Log***ic.
- Prolog is based on first order logic.
- Prolog is **declarative** (as opposed to **imperative**):
  - You specify *what* the problem is rather than *how* to solve it.
- Prolog is very useful in some areas (AI, natural language processing), but less useful in others (graphics, numerical algorithms).

Propositional Logic

- **Propositions** are statements that can be assigned a truth value
  - Elephants are pink. **true or false?**
- **Operators** for assigning truth values to combinations of propositions (**sentences**)

Symbolic statement	*Translation*	*Informal characterization*
$p \wedge q$	p and q	$p \wedge q$ is true when both p and q are true
$p \vee q$	p or q	$p \vee q$ is true when either p or q or both p and q are true
$p \Rightarrow q$	p logically implies q	$p \Rightarrow q$ is true when p and q are both true, or p is false
$p \Leftrightarrow q$	p is logically equivalent to q	$p \Leftrightarrow q$ is true if p and q have the same truth value
$\neg p$	not p	$\neg p$ is true when p is false

Predicate Logic

- Involves **entities** and **relations** between entities.
- **Entities** are expressed using:
  - **Variables** : X, Y, Somebody, Anybody
  - **Constants** : fido, fiffy, bigger, dog, has, bone
- **Logical operators** - connectors between relations
  - and ($\wedge$), or($\vee$), not ($\neg$), logically implies ($\Rightarrow$), logically equivalent ($\Leftrightarrow$), for all ($\forall$), exists ($\exists$)
- **Relations** are expressed using:
  - **Predicates** - express a simple relation among entities, or a property of some entity
    - fido is a dog - dog(fido)
    - fiffy is a dog - dog(fiffy)
    - fido is bigger than fiffy - bigger(fido, fiffy)

Predicate Logic (cont.)

- **Formulas** - express a more complex relation among entities
  - if fido is bigger than fiffy, and fiffy has a bone, then fido can take the bone

    (bigger(fido, fiffy) $\wedge$ has(fiffy,bone)) $\Rightarrow$ can_take(fido,bone)
- **Sentences** - are formulas with no free variables
  - dog(X) contains a variable which is said to be **free** while the X in $\forall$ X.dog(X) is **bound**.

Logic → Prolog

- Involves **entities** and **relations** between entities.
- **Entities** are expressed using:
  - **Variables:** X, Y, Somebody, Anybody
  - **Constants:** fido, fiffy, bigger, dog, has, bone
- **Logical operators**: connectors between relations
  - and (,), or (;), not (\+), is logically implied by (:-)
- **Relations** are expressed using:
  - **Predicates** - relation among entities, or a property of an entity
    - fido is a dog - `dog(fido)`
    - fido is bigger than fiffy- `bigger(fido, fiffy)`

Logic → Prolog (cont.)

- **Rules** - complex relation among entities
  - if fido is bigger than fiffy, and fiffy has a bone, then fido can take the bone

```
can_take(fido, bone) :-
        bigger(fido, fiffy), has(fiffy, bone).
```

Or more general:

```
can_take(Dog1, bone) :-
        bigger(Dog1, Dog2),
        has(Dog2, bone).
```


Programming Language Comparison

Imperative programming languages

- 'procedural' -> they describe *how* a sequence of instructions compute the result to a certain problem.
- we concentrate on how to formulate a solution in terms of the primitive operations available
- what you see is what is being done

Logic programming languages

- specify the problem in a declarative style (facts about objects, relations between objects), describe *what* is the objective and let the system prove it
- we concentrate on problem
- there are *underlying mechanisms* that help the program reach its goal

Logic Programming

- A program in logic is a definition (declaration) of the world - the entities and the relations between them.
- Logic programs establishing a theorem (goal) and asks the system to prove it.
- Satisfying the goal:
  - **yes**, prove the goal using the information from the knowledge base
  - **no**:
    - cannot prove the truth of the goal using the information from the knowledge base
    - the goal is false according to available information

Definitions

- Three basic constructs in Prolog
  - Facts, rules, and queries.
- Knowledge base (database)
  - A collection of facts and rules.
  - Prolog programs are knowledge bases.
- We use Prolog programs by posing queries.

Facts

- Facts are used to state things that are *unconditionally* true.
  We pay taxes. `we_pay_taxes.`
- The earth is round. The sky is blue.
  `round(earth).`
  `blue(sky).`
- Beethoven was a composer that lived between 1770 and 1827.
  `composer(beethoven,1770,1827).`
- Tom is the parent of Liz.
  `parent(liz, tom).`
- fido is bigger than fiffy. `bigger(fido,fiffy).`

  Exercise:
  John owns the book. John gives the book to Mary.

SWI-Prolog

- The SWI department of the University of Amsterdam.
- Free
- Small
- Available in the lab
- Download a copy to work at home
  http://www/swi-prolog.org
- Documentation

Queries on Facts

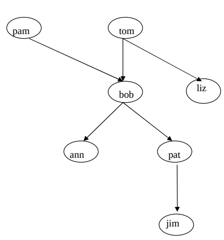
- John likes apples, csi2165 and Mary.
  likes(john,apples).
  likes(john,csi2165).
  likes(john,mary).
- Does John like apples?
  ?-likes(john,apples).
  yes
- What does John like?
  ?-likes(john,X).
  X = apples;
  X = csi2165;
  X = mary;
  no

SWI-Prolog Demo

‘;’ for more solutions. ‘Enter’ to stop.

Building a Knowledge Base (Lab 1)

Family Tree



Problem1. build a knowledge base to represent the parent relationships that can be deduced from the tree

parent(pam,bob).

Problem 2. build predicates that describe the following family relationships:

grandparent/2

mother/2 father/2

brother/2 sister/2 sibling/2

aunt/2 uncle/2

precursor/2

Rules

- If there is smoke there is fire. fire :- smoke.
- Liz is an offspring of Tom if Tom is a parent of Liz.
  offspring(liz, tom) :- parent(tom, liz).
- Y is an offspring of X if X is a parent of Y.
  offspring(Y, X) :- parent(X, Y).
- Two persons are sisters if they are females and have the same parents.
  siblings(P1, P2) :- parent(P, P1), parent(P, P2).

What is the problem with this rule?

Exercise:
- Family relations
grandparent(X,Y) :-

Queries on Rules

- Mary drinks beer. Terry drinks beer.
  drinks(mary, beer).
  drinks(terry, beer).
- John likes everybody who drinks beer.
  likes(john, X) :- drinks(X, beer).
- Does John like Mary?
  ?-likes(john, mary).
  yes
- Who does John like?
  ?-likes(john, X).
  X = mary;
  X = terry;
  no

Clauses

- In Prolog, rules (and facts) are called **clauses.**
- **A clause always ends with ‘.’**
- Clause: <head> :- <body>.
  – you can conclude that <head> is true, if you can prove that <body> is true
- Facts - clauses with an empty body: <head>.
  – you can conclude that <head> is true
- Rules - normal clauses (or or more clauses)
- Queries - clauses with an empty head: ?- <body>.
  – Try to prove that <body> is true

Rules

- Rules state information that is *conditionally* true of the domain of interest.
- The general form of these properties
  - p is true if (p_1 is true, and p_2 is true, … and p_n is true)
- Horn clause
  - p :- p_1, p_2, …, p_n.
- Interpretation (Prolog) :
  - in order to prove that p is true, the interpreter will prove that each of p_1, p_2, …, p_n is true
  - p - the **head** of the rule
  - p_1, p_2, …, p_n - the **body** of the rule (**subgoals**)

Rules and Conjunctions

- A man is happy if he is rich and famous.
- In Prolog:

```
happy(Person) :-
        man(Person),
        rich(Person),
        famous(Person).
```

- The ',' reads 'and' and is equivalent to $\wedge$ of predicate calculus.

Rules and Disjunctions

- Someone is happy if he/she is healthy, wealthy or wise.
- In Prolog:

```
happy(Person) :- healthy(Person).
happy(Person) :- wealthy(Person).
happy(Person) :- wise(Person).
```

- More exactly:
  - Someone is happy if they are healthy OR
  - Someone is happy if they are wealthy OR
  - Someone is happy if they are wise.

Both Disjunctions and Conjunctions

- A woman is happy if she is healthy, wealthy or wise.
- In Prolog:

```
happy(Person) :- healthy(Person), woman(Person).
happy(Person) :- wealthy(Person), woman(Person).
happy(Person) :- wise(Person), woman(Person).
```


Variables

- Objects referred by a name starting with a capital letter.
- Scope rule:
  - Two uses of an identical name for a logical variable only refer to the same object if the uses are within a single clause.

```
happy(Person) :- healthy(Person). % same person
wise(Person) :- old(Person). /* may refer to other
    person than in above clause. */
```

Two commenting styles!

Queries

- The goal represented as a question.

```
?- round(earth). /* is it true that the earth is round? */

?- round(X). /* is it true that there are entities which are round?
   (what entities are round?) */

?- composer(beethoven, 1770, 1827). /* is it true that
   Beethoven was a composer who lived between 1770 and
   1827)? */

?- owns(john, book). /* is it true that john owns a book? */

?- owns(john, X). /* is it true that john owns something? */
```

Predicate

- composer(beethoven,1770,1827) $\rightarrow$ predicate
- composer $\rightarrow$ functor
- beethoven, 1770, 1827 $\rightarrow$ arguments
- number of arguments: 3 $\rightarrow$ arity.
- write as **composer/3**

Example

We have a Prolog program:

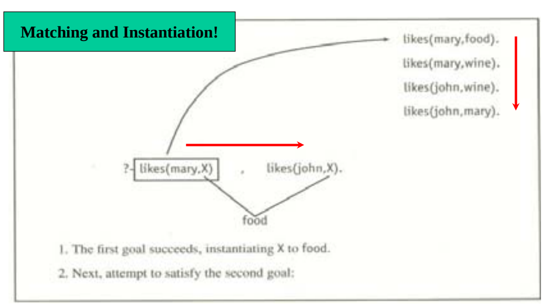
```
likes(mary, food).
likes(mary, wine).
likes(john, wine).
likes(john, mary).
```

Now we pose the query:

```
?- likes(mary, X), likes(john, X).
```

What answers do we get?

1/4



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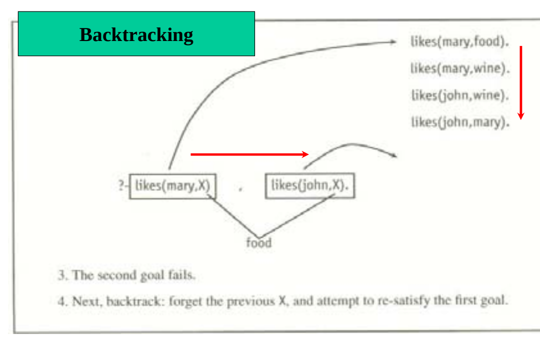

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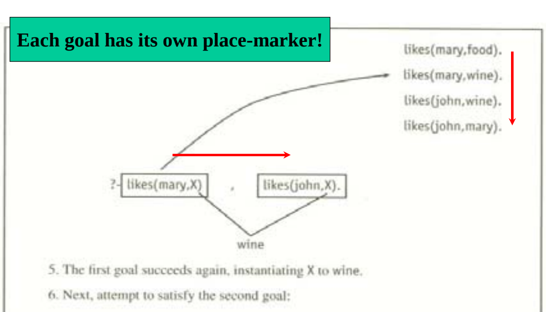

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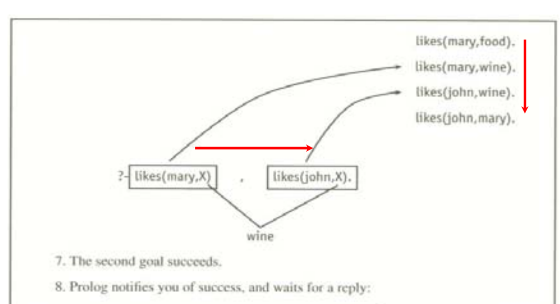


Declarative Semantics (what)

- **Declarative semantics** - telling Prolog what we know.
- If we don't know if something is true, we assume it is false - **closed world assumption**.
- Sometimes we tell it relations that we know are false. (sometimes it is easier to show that the opposite of a relation is false, than to show that the relation is true)

I know (it is true) that the max between two numbers X and Y is X, if X is bigger than Y. `max(X, Y, X) :- X > Y.`

I know that the max between two numbers X and Y is Y if Y is bigger or equal to X. `max(X, Y, Y) :- Y >= X.`

```
?- max(1, 2, X).
```


Declarative Semantics (cont.)

I know that 0 is a positive integer.

```
positive_integer(0).
```

I know that X is a positive integer if there is another positive integer Y such that X is Y+1.

```
positive_integer(X) :- positive_integer(Y), X is Y+1.
```

Recursive definition!

```
?- positive_integer(3).
?- positive_integer(X).
```


Procedural Semantics (how)

- **Procedural semantics** - how do I prove a goal?

```
max(X, Y, X) :- X > Y.
max(X, Y, Y) :- Y >= X.
?- max(1, 2, X).
```

If I can prove that X is bigger then Y, then I can prove that the max between X and Y is X.

or, if that doesn't work,

If I can prove that Y is bigger or equal to X, then I can prove that the max between X and Y is Y.

Procedural Semantics (cont.)

```
positive_integer(0).
positive_integer(X) :- positive_integer(Y), X is Y+1.
?- positive_integer(3).
```

If I can prove that X is 0, then I can prove that X is a positive integer   or,

If I can prove that Y is a positive integer, and if X is Y+1, then I can prove that X is a positive integer.

I can prove that Y is a positive integer if I can prove that Y is 0   or

If I can prove that Z is a positive integer, and if Y is Z+1, then I can prove ...

Terms

- Prolog programs are built from terms.
- Three types of terms
  - Constants
  - Variables
  - Structures
- Terms are composed of letters, digits and/or sign characters.

Another view: Objects

- Simple objects:
  - constants: for specific objects or specific relationships.
    - numbers (integers, floating point numbers)
    - atoms (bob, hello, *, '&?%', 'I`m not a variable')
  - variables:
    - anonymous variables
    - named variables
- Complex objects
  - lists
  - other structures

Variables

- Names that stand for objects that may already or may not yet be determined by a Prolog program
  - if the object a variable stands for is already determined, the variable is ***instantiated***
  - if the object a variable stands for is not yet determined, the variable is ***uninstantiated***
- a Prolog variable does **not** represent a location that contains a modifiable value; it behaves more like a mathematical variable (and has the same scope)
- **An instantiated variable in Prolog cannot change its value**

Variables (cont.)

- Constants in Prolog : numbers, strings that start with lowercase, anything between single quotes
- Variables in Prolog: names that start with an uppercase letter or with '_'
- Examples:

Variables	Constants
X,Y, Var, Const, _var, _const, _	x, y, var, const, some_Thing, 1, 4, 'String', "List of ASCII codes"

Anonymous variables

- a variable that stands in for some unknown object
- stands for some objects about which we don't care
- several anonymous variables in the same clause need not be given consistent interpretation
- written as _ in Prolog

```
?- composer(X, _, _).
X = beethoven;
X = mozart;
…
```

We are interested in the names of composers but not their birth and death years.

Verify Type of a Term

- **var(**+*Term***)**
  Succeeds if *Term is* currently a free variable.
- **nonvar(**+*Term***)**
  - Succeeds if *Term is* currently not a free variable.
- **integer(**+*Term***)**
  - Succeeds if *Term* is bound to an integer.
- **float(**+*Term***)**
  - Succeeds if *Term* is bound to a floating point number.
- **number(**+*Term***)**
  - Succeeds if *Term* is bound to an integer or a floating point number.
- **atom(**+*Term***)**
  - Succeeds if *Term* is bound to an atom.
- **string(**+*Term***)**
  - Succeeds if *Term* is bound to a string.
- **atomic(**+*Term***)**
  - Succeeds if *Term* is bound to an atom, string, integer or float.
- **compound(**+*Term***)**
  - Succeeds if *Term* is bound to a compound term.

Some Built-in Predicates(Operators)

for constants:
- number/1
- integer/1
- float/1
- atom/1
- atomic/1

Examples:

number(15)	atom(my_atom)
number(0.001)	atom(*)
number(4.2E+01)	atom('This?')
integer(16)	atom(15)
integer(1.0)	atomic(a)
float(1)	atomic(4)
float(1.5E-1)	atomic(4.2E+01)
float(1.0)	

for variables:
- var/1
- nonvar/1
- is/2
- =/2
- …

Examples:

var(X)	X = abc, var(X)
var(x)	X = abc,nonvar(X)
var(5)	_ = abc, var(_)
nonvar(X)	_ = abc, nonvar(_)
nonvar(abc)	X is 5
Y = abc	X is 5+1
Z = 4.2E+01	X = 5+1
var(X), X = abc	X = 5

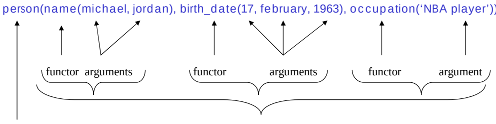
Try them out on your computer!

Structures

- Structures are objects that have several components, which in turn can be structures.
- Structures are treated in the program as single objects.
- **functor** is used to combine components into a single object.
- A **functor** must be an atom.
- Example:
  - date(1, may,1999)
  - course(csc2165, fall2005)

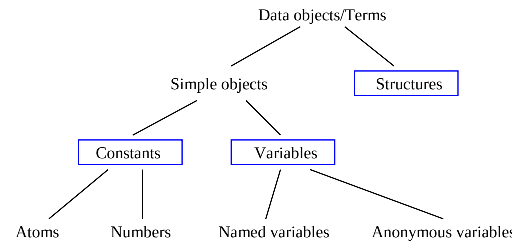
Structure: Example

- Description:
  - a person has:
    - name - first name, last name
    - birth date - day, month, year
    - occupation
- Prolog representation - example
  - person(name(michael, jordan), birth_date(17, february, 1963), occupation('NBA player'))



  - functor arguments

Data Objects in Prolog (Summary)



Structures - Exercise

- Description:
  - point in the 2D space
  - triangle
  - a country
    - has a name
    - is located in a continent at a certain position
    - has population
    - has capital city which has a certain population
    - has area

Structures - Exercise

- Knowledge base:

```
country(canada, location(america, north),
   population(30), capital('Ottawa',1),area(_)).

country(usa, location(america, north),
   population(200),capital('Washington DC', 17),
   area(_)).
```


A Particular Structure

How can we represent the courses a student takes?

courses(csi2111, csi2114, csi2165)
courses(csi2114, csi2115, csi2165, mat2343)
courses(adm2302, csi2111, csi2114, csi2115, csi2165)

Three different structures.

In general, how do we represent a **variable** number of arguments with a **single** structure?

A Particular Structure

Consider a single structure **courses/2:**
the first argument - a course
the second argument - a **courses/2** structure

```
courses(
   csi2111,
   courses(
      csi2114,
      courses(
         csi2165,
         nil)))
```

```
courses(
   csi2114,
   courses(
      csi2165,
      nil))
```

They are lists!

That's useful but too messy, better use lists:

.(csi2111, .(csi2114, .(csi2165, [])))	[csi2111, csi2114,csi2165]
.(csi2114, .(csi2165, []))	[csi2114, csi2165]

Lists

- Functor name : .
- Arity : 2
  - first argument - can be *anything* - called the **head** of the list
  - second argument - must be a list - called the **tail** of the list
- Representing the lists:

  .(Head1,Tail) = [Head1 | Tail] =
  = [Head1, HeadOfTail | TailOfTail] = … =
  = [Head1, Head2, Head3, …, LastHead | []] =
  = **[Head1, Head2, …, LastHead]**

  ("…" here is not a Prolog notation)
- We use the square bracket notation in our program since it is more readable.

Lists

- Examples:

  [a,b,c] = .(a,.(b,.(c,[])) = [a | [b, c]] = [a, b | [c]] = [a, b, c | []]
  [1,2,3] = .(1, .(2, .(3, [ ]))) = [1 | [2 | [3]]]
- **Exercise:**

List	Head	Tail
[a,b,c]		
[]		
[[the,cat], sat]		
[the, [cat, sat]]		
[the, [dog, ate], bones]		
[X+Y, x+y]		

Try them out in SWI-Prolog

Matching, Unification, and Instantiation

- Prolog will try to find in the knowledge base a fact or a rule which can be used in order to prove a goal
- Proving :
  - **match** the goal on a fact or head of some rule. If matching succeeds, then:
  - **unify** the goal with the fact or the head of the rule. As a result of unification:
  - **instantiate** the variables (if there are any), such that the matching succeeds
- NB: variables in Prolog cannot change their value once they are instantiated !

Matching

- **Matching:** Prolog tries to find a fact or a head of some rule with which to match the current goal
- **Match:** the *functor and the arguments* of the current goal, with the functor and the arguments of the fact or head of rule
- Rules for matching :
  - **constants only match an identical constant**
  - **variables can match anything, including other variables**

Goal	Predicate	Matching
constant	constant	yes
constant	other_constant	no
Var	some_constant	yes
Var	Other_Var	yes
some_constant	Some_Var	yes

Instantiation and Unification

Instantiation

- the substitution of some object for a variable
- a variable is *instantiated* to some object

composer(X, 1770, 1827) succeeds with *X* instantiated to beethoven

Unification

- the instantiations done such that the two terms that match become **identical**
- two terms match if:
  - they are identical objects
  - their **constant** parts are identical and their **variables** can be instantiated to the same object

  composer(X,1770,1827) unifies with composer(beethoven,1770,1827)
  with the instantiation X = beethoven

Unification

- Done after a match between the current goal and a fact or the head of a rule is found
- It attaches values to variables (**instantiates** the variables), such that the goal and the predicate are a perfect match:
  - match:
    - goal - composer(beethoven, B, D) with fact - composer(beethoven,1770,1827)
  - unification:
    - B will be instantiated to 1770
    - D will be instantiated to 1827

  such that the goal will match the fact.

Unification (cont.)

- If the match is done on the head of some rule, then the instantiations done for the variables are also valid in the body of the rule:
  - match:
    - goal - contemporaries(beethoven, mozart) with head of contemporaries(X, Y) :- composer(X, B1, D1), composer(Y, B2, D2), X \== Y, ...
  - unification:
    - X will be instantiated to beethoven
    - Y will be instantiated to mozart and now the rule will look: contemporaries(beethoven, mozart) :- composer(beethoven,B1,D1), composer(mozart, B2, D2), beethoven \== mozart, ...

Instantiation and Unification - Exercise

unify	*with*	*result*
likes(jim, piano)	likes(jim, X)	
likes(jim, X)	likes(Y, piano)	
owns(X, Y)	owns(jim, calliope)	
owns(X, Y)	owns(Y, X)	
owns(jim, piano)	likes(jim, piano)	
owns(jim, piano)	owns(bill, piano)	
owns(jim, X, Y)	owns(jim, piano)	

Work them out and validate them with SWI-Prolog.

Structures Matching and Unification

- Matching on structures:
  - match the *functor* of the two structures
  - match each argument of the two structures (if some argument is complex, match it according to the same rules)
- Example:

a(b,c)	a(b, c)	→	match
a(b,C)	a(b, x)	→	C = x
a(X)	a(B, c)	→	don`t match

Structure Matching and Unification

- Exercise

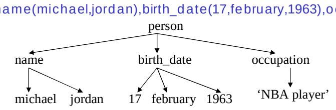
Structure 1	=	Structure 2:	Instantiations:
a(b, X)	=	a(Y, c)	
a(b, X)	=	a(X, Y)	
a(b, X)	=	a(b, c(d))	
a(b(X), Y)	=	a(Y, c)	
a(b(c(X)), Y)	=	a(b(Y), c(Z))	
a(b(c(X)), Y)	=	a(b(Y), Z)	
[X,Y]	=	[john, skates]	
[cat]	=	[H | T]	
[[the,Y] | Z]	=	[[X, hare], [is, here]]	
[H | T]	=	a(b, c(d))	
[n(X,Y),a(1)]	=	[Name, Age]	
X	=	a(b, c(d))	
a(b, c)	=	X(b, c)	

Work them out on your computer!

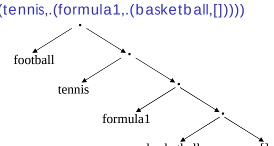
Structures - Another View

- We can view structures as **trees**:

person(name(michael,jordan),birth_date(17,february,1963),occupation('NBA player'))



.(football,.(tennis,.(formula1,.(basketball,[]))))



Unification Operators

= \= == \== *is*

Three Kinds of Equality

- When are two terms said to be equal?
- We introduce 3 types of equality now (more later)
  - X = Y: this is true if X and Y match.
  - X is E: this is true if X matches the *value* of the arithmetic expression E.
  - T1 == T2: this is true if terms T1 and T2 are *identical*
    - Have exactly the same structure and all the corresponding components are the same. The name of the variables also have to be the same.
    - It is called: *literal equality.*
    - If X == Y, then X = Y. the former is a stricter form of equality.

Unification Operator: =

- = → **unifies with**: X = Y
  - succeeds as long as X and Y can be unified
  - X may or may not be instantiated
  - Y may or may not be instantiated
  - X and Y become bound together (they now refer to the same object)
- ? - p1(a, [A, [B, C]],25) = p1(C, [B, [D, E]], 25).
  A = B = D, C = E = a, yes
- ? - a(b, X, c) = a(b, Y, c).
  X = Y, yes

Unification Operators: \=

- \= → **does not unify with**: X \= Y
  - succeeds as long as X and Y cannot be unified
  - both X and Y must be instantiated (why?)
  - X and Y may have uninstantiated elements inside them
- ? - [A, [B, C]] \= [A, B, C].
  yes
- ? - a(b, X, c) \= a(b, Y, c).
  no

Back to slide 14: siblings/2

Unification Operator: ==

- == → **is already instantiated to**: X == Y
  - succeeds as long as X and Y are already instantiated to the same object
  - in particular, any variable inside X and Y must be the same
- ? - a(b,X,c) == a(b,Y,c).
  no
- ? - a(b,X,c) == a(b,X,c).
  yes

Unification Operators: \==

- \== → **not already instantiated to:** X \== Y
  - succeeds as long as X and Y are *not* already instantiated to the same object
- ? - A \== hello.
  yes
- ? - a(b,X,c) \== a(b,Y,c).
  yes

Question: would it make any difference if we replace \= with \== in the siblings/2 on slide 14?

Arithmetic Operator: *is*

- is → **arithmetic evaluation** : X is Expr
  - succeeds a long as X and the arithmetic evaluation of Expr can be unified
  - X may or may not be instantiated
  - Expr must not contain any uninstantiated variables
  - X is instantiated to the arithmetic evaluation of Expr
- ? - 5 is ((3 * 7) + 1) / 4.
  yes
- ? - X is ((3 * 4) +10) mod 6.
  X = 4

is is different from =

?- X is 3 + 1.
X = 4

? X = 3 + 1.
X = 3 + 1

Summary of Part I

- Introduction to Prolog and Logic Programming.
- Prolog basic constructs: facts, rules, queries.
- Unification, variables.
- Prolog syntax, equality, and arithmetic.