Solution Assignment 1

1. **R**-1**R** = **I** (1)

Also, **RR**-1 = **I** (2)

Taking the hermitian of (2), we get **I**H = (**R**-1)H**R**H. (3)

But **I**H = **I** and **R**H = **R**, therefore (3) becomes (**R**-1)H**R**=**I** (4)

Obviously (1) and (4) are equal, therefore **R**-1 = (**R**-1)H.

1. **u**(n) = [*u*(n) *u*(n-1) *u*(n-2)]T. **R** = E[**u**(n)**u**H(n)], r(0)=E[(x(n)-0.5x(n-1)+v(n))(x(n)-0.5x(n-1)+v(n))\*] = E[|x(n)|2]+0.25E[|x(n-1)|2]+E[|v(n)|2], due to the independence of all terms and the fact that they all have zero mean. Furthermore E[|x(n-1)|2]=E[|x(n)|2] if we assume a stationary system (although it isn’t explicitly stated in the question). Therefore r(0) = 1.55. r(1) = -0.5 and r(2) = 0. The correlation matrix R is



(a), when d(n)=x(n), **p** = [1 0 0]T. Therefore **wo** = **R**-1**p** = [0.73 0.263 0.085]T. Jmin = d2-**wo**H**p** = 1-0.73=0.27.

(b) when d(n)=x(n-1), **p**=[-0.5 1 0]T. Therefore **wo** = [-0.102 0.683 0.22]T. Jmin = d2-**wo**H**p** = 1-(0.051+0.683)=1-0.734 = 0.266.

1. When we use a random filter, J = Jmin + **w**H**Rw**. **w** = **w**-**wo** (or vice versa, it doesn’t matter since this is a Hermitian form. It’s similar to 22=(-2)2 = 4). **w** = [1-0.73 0.6-0.263 -0.1-0.085]T = [0.27 0.337 -0.185]T. **w**H**Rw =** 0.313. Therefore J = 0.27+0.313 = 0.583.
2. To find the eigenvalues, we must find the characteristic polynomial of the matrix. The characteristic polynomial is det(**R**-**I**). The characteristic equation is 0.56-2.46+32-3 = 0. The roots of this equation are:  = 0.386, 0.8, 1.814. (Newton’s method can be used to find one root and then after factoring, the quadratic equation can be used to find the others). To find the eigenvectors, we recale that **Rq** = **q**. We will get a set of 3 equations and three unknowns. For example for  = 0.8, we get 1q1 +0.5q2 +0.2q3= 0.8q1, 0.5q1 +1q2+0.5q3 = 0.8q2 and 0.2q1+0.5q2+1q3 = 0.8q3. Eq1 and Eq3 both yield 0.2q1+0.5q2+0.2q3=0. Linearly combining equations 1 and 2 we get 1.05q2=0. Therefore q2=0. Then from Eq 2, we get 0.5q1=-0.5q3. Therefore q1=-q3. **q** = [1/sqrt(2) 0 -1/sqrt(2)]T for  =0.8. Similarly for for  =0.386, **q** = [0.464 -0.755 0.464]T and for  =1.814, **q** = [0.534 0.656 0.534]T.