

ELG5377 Adaptive Signal  
Processing

# Simulation of LMS- Based Algorithms



# Introduction



- We have seen the development of the algorithms for LMS, NLMS and Affine Projection Adaptive Filters
- We wish to compare the three algorithms in terms of performance (convergence, misadjustment, mean square deviation).

# Equalization example



- Equalizer
  - $u(n) = d(n) - 0.2d(n-1) + v(n)$ 
    - Where
    - $d(n)$  is independent binary data (equiprobable +1 or -1)
    - $v(n)$  is White Gaussian noise with 0 mean and variance 0.1
    - We want to use a three tap transversal filter to equalize the input. The desired output is  $d(n)$ .



# Wiener Solution



- We can show that

$$\mathbf{R} = \begin{bmatrix} 1.14 & -0.2 & 0 \\ -0.2 & 1.14 & -0.2 \\ 0 & -0.2 & 1.14 \end{bmatrix}$$

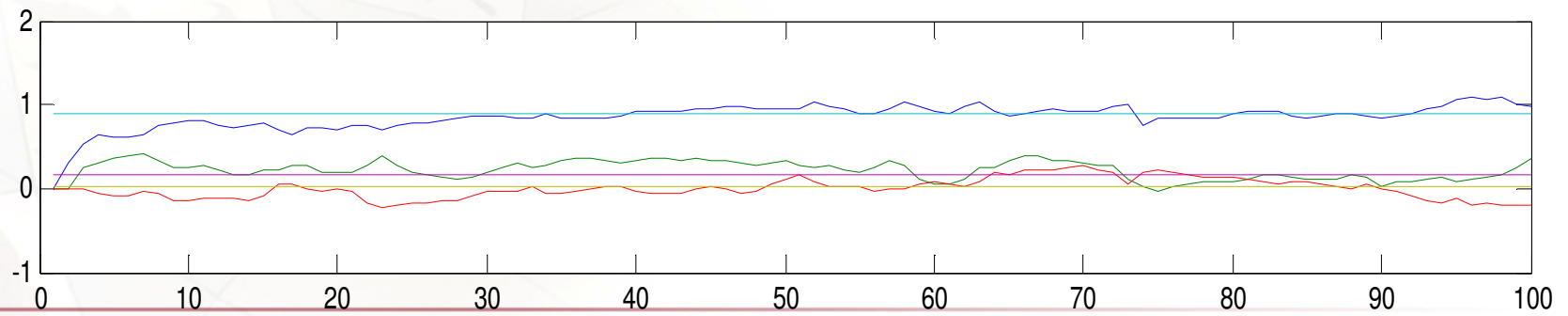
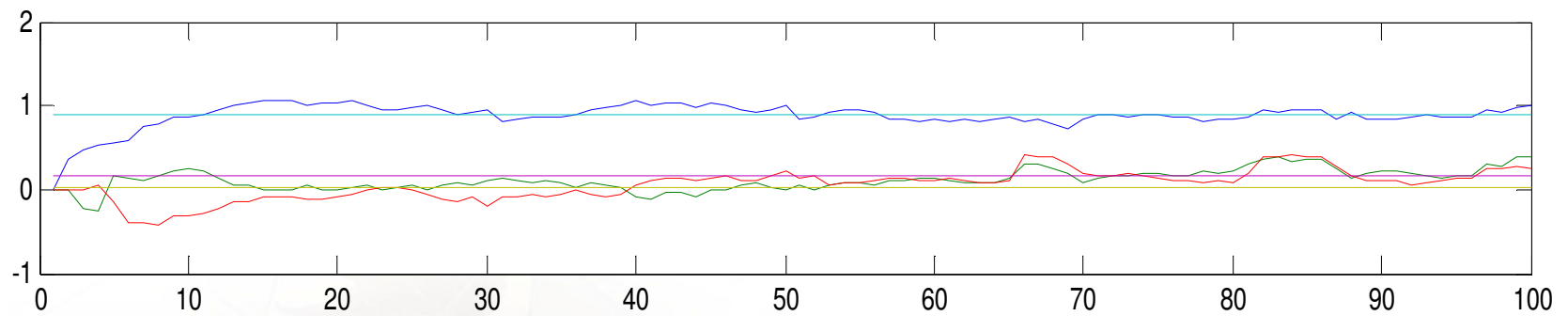
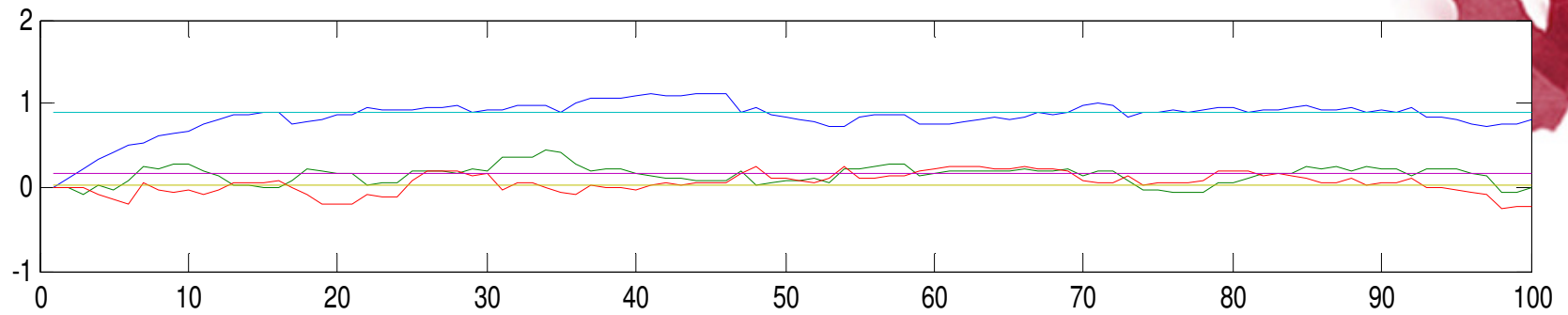
$$\mathbf{p} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p} = \begin{bmatrix} 0.906 \\ 0.164 \\ 0.029 \end{bmatrix}, \quad J_{\min} = 0.0994$$

## Practical considerations

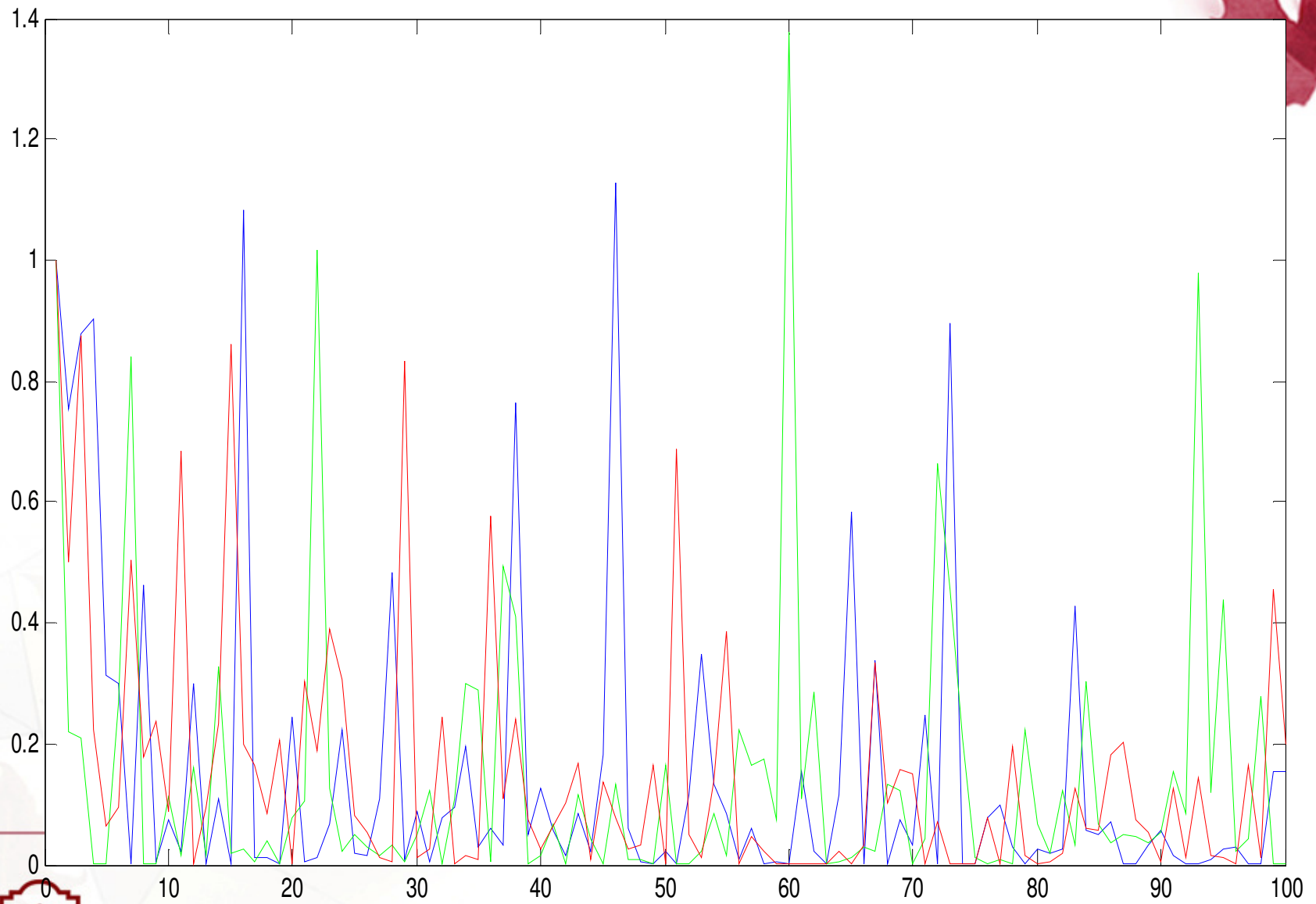


- $\lambda_{max} = 1.423, \mu_{max} \text{ (LMS)} = 1.41$
- In NLMS,  $\mu = \mu_{max} / \text{tr}(\mathbf{R})$ , therefore  $\mu_{max} = 1.41 * \text{tr}(\mathbf{R}) = 1.41 * 3.42 = 4.82$ .
- We will use  $\mu = 0.1 \mu_{max}$  in all cases.
- NLMS is special case of AP.
- For AP, we'll use  $N = 2$ . Since we are using two vectors, we'll divide  $\mu$  of NLMS by 2.

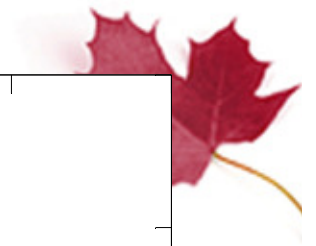




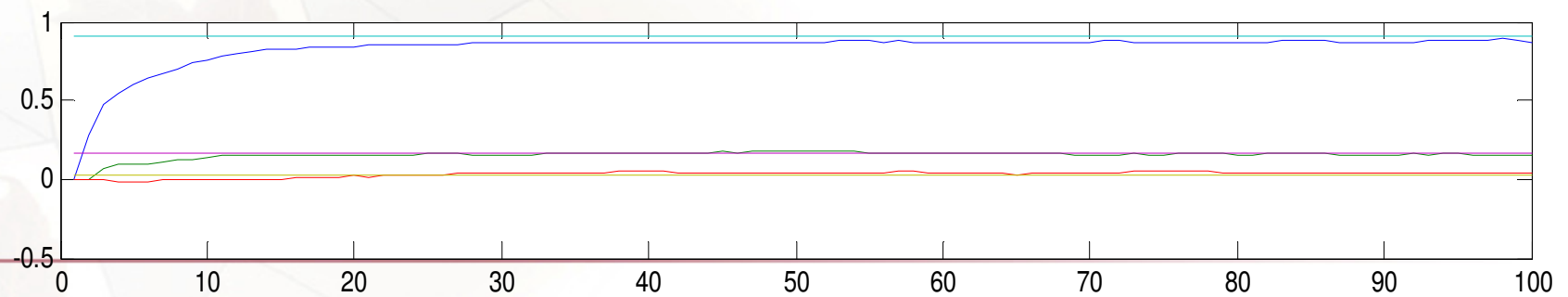
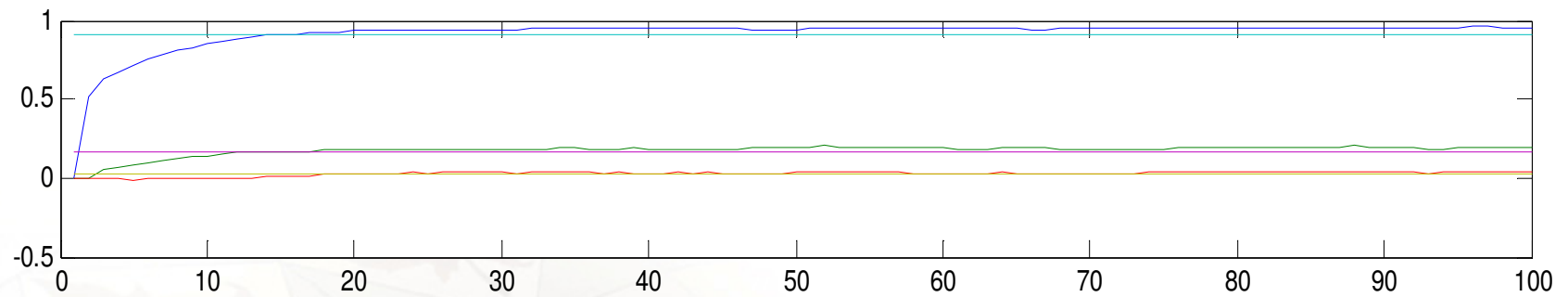
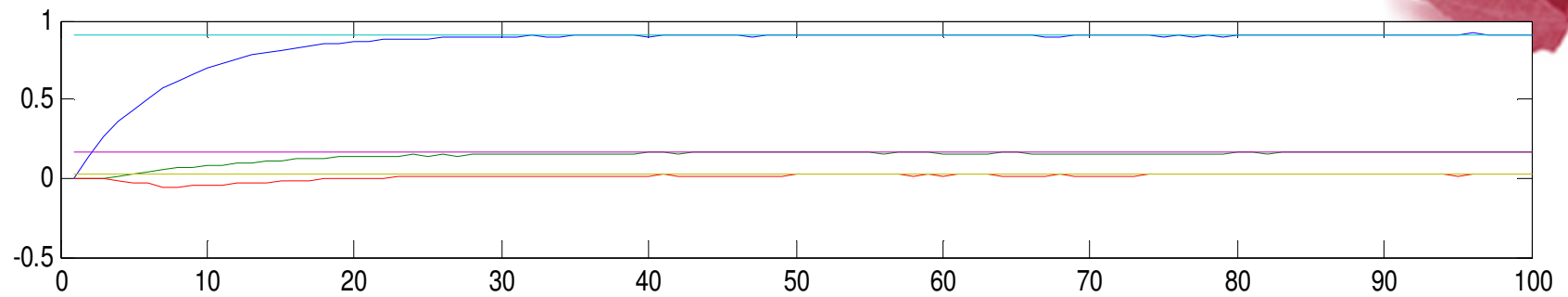
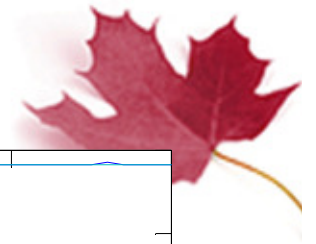
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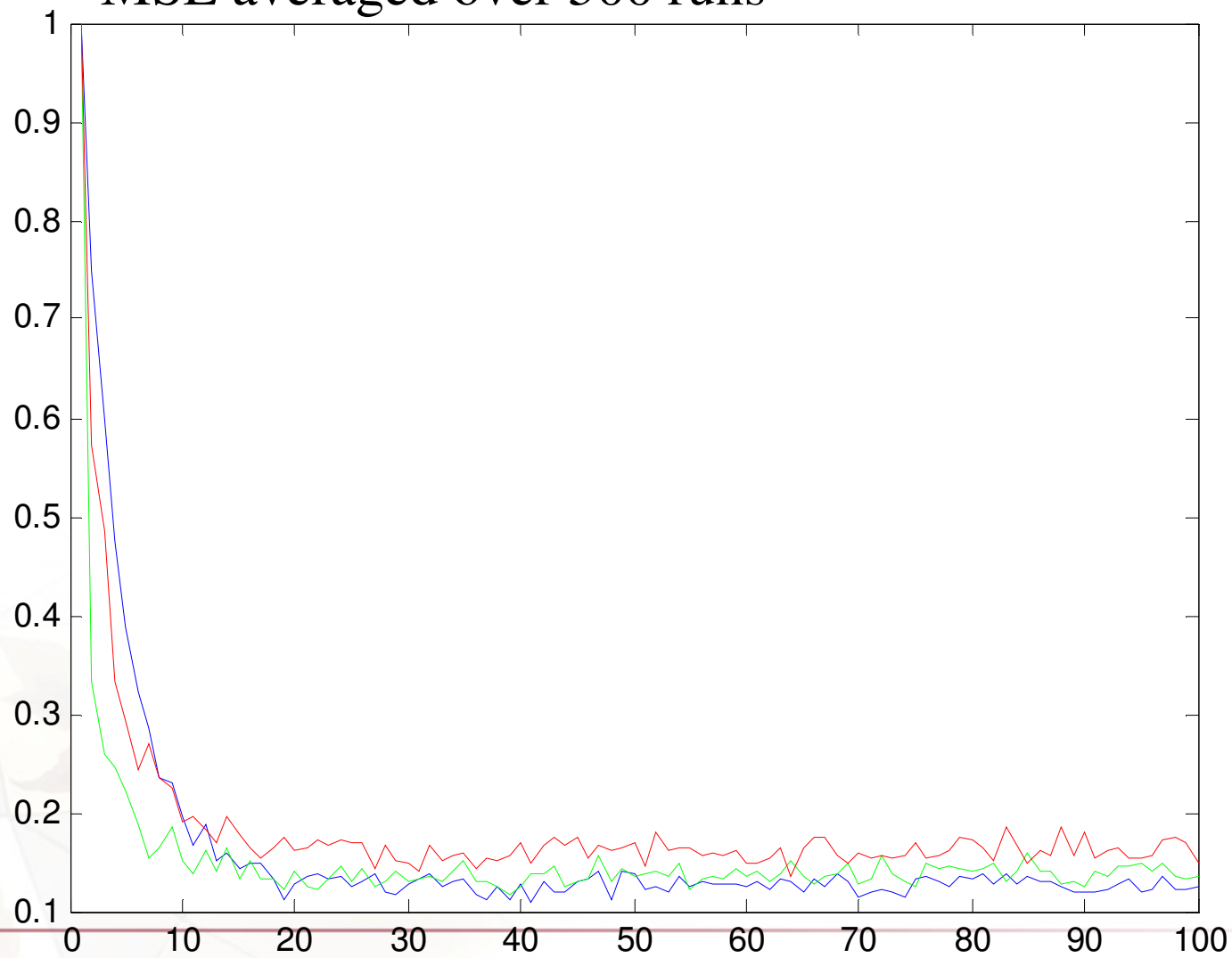


## Tap weights averaged over 500 runs

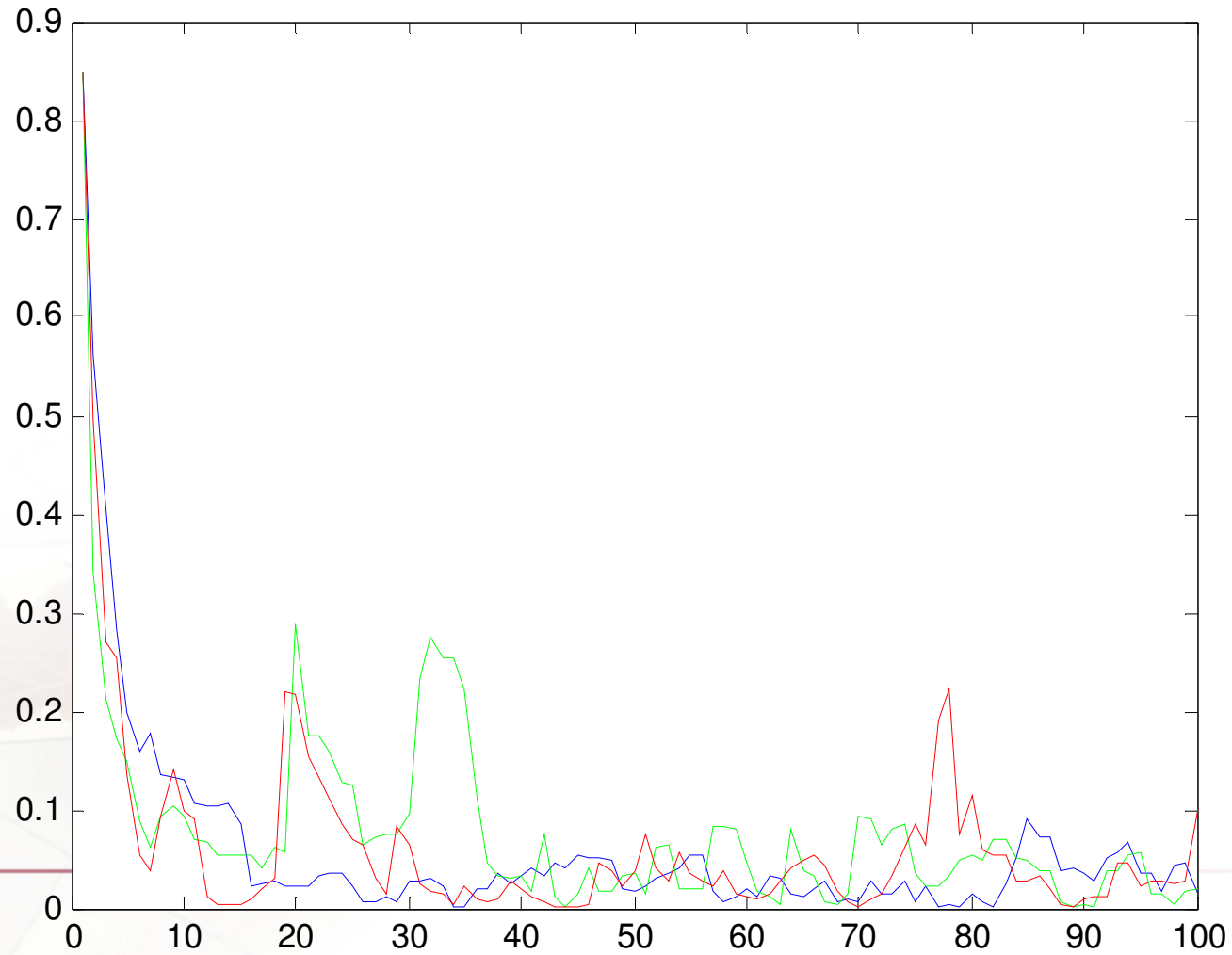




MSE averaged over 500 runs

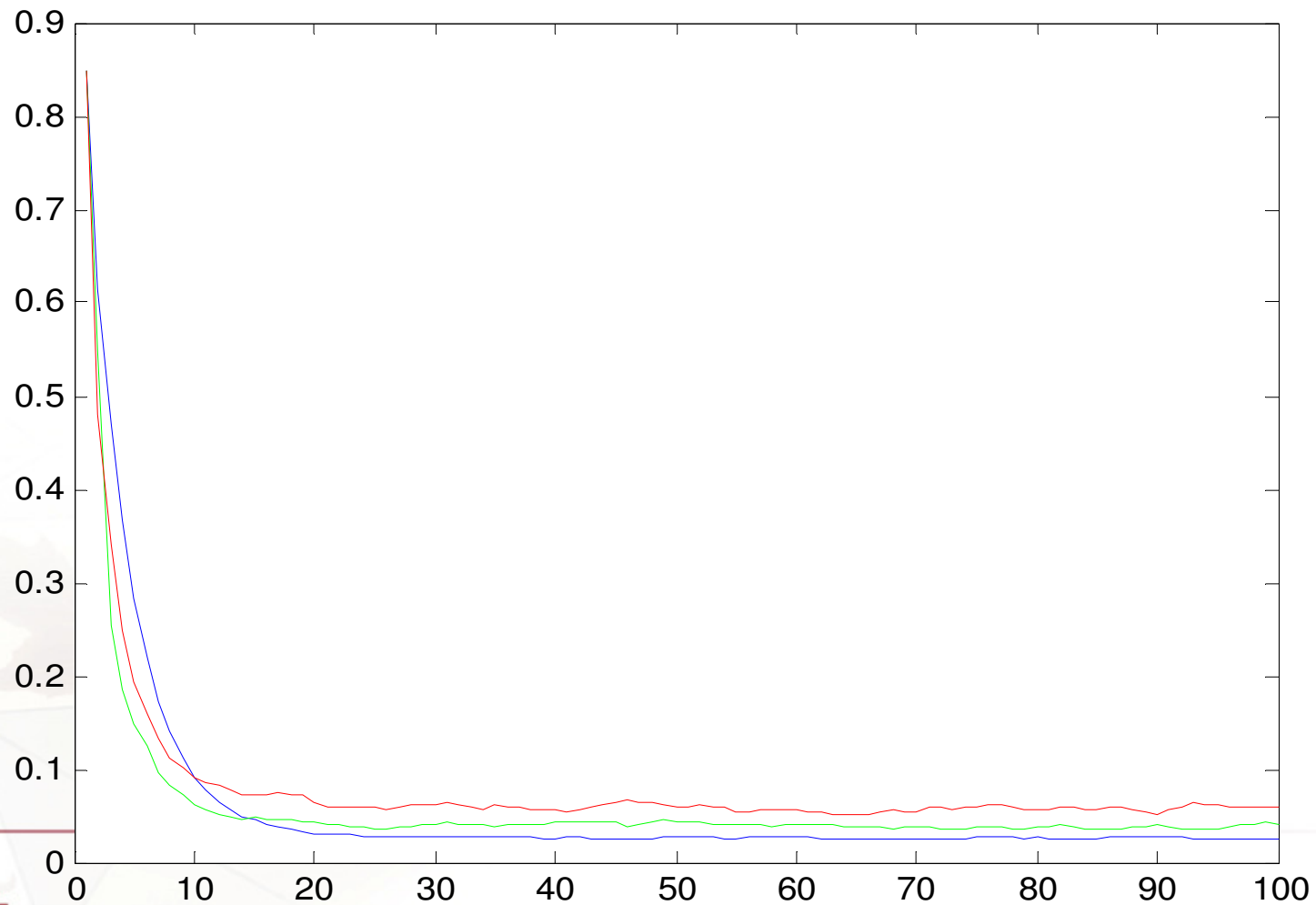


# MSDev



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# MSDev averaged over 500 runs



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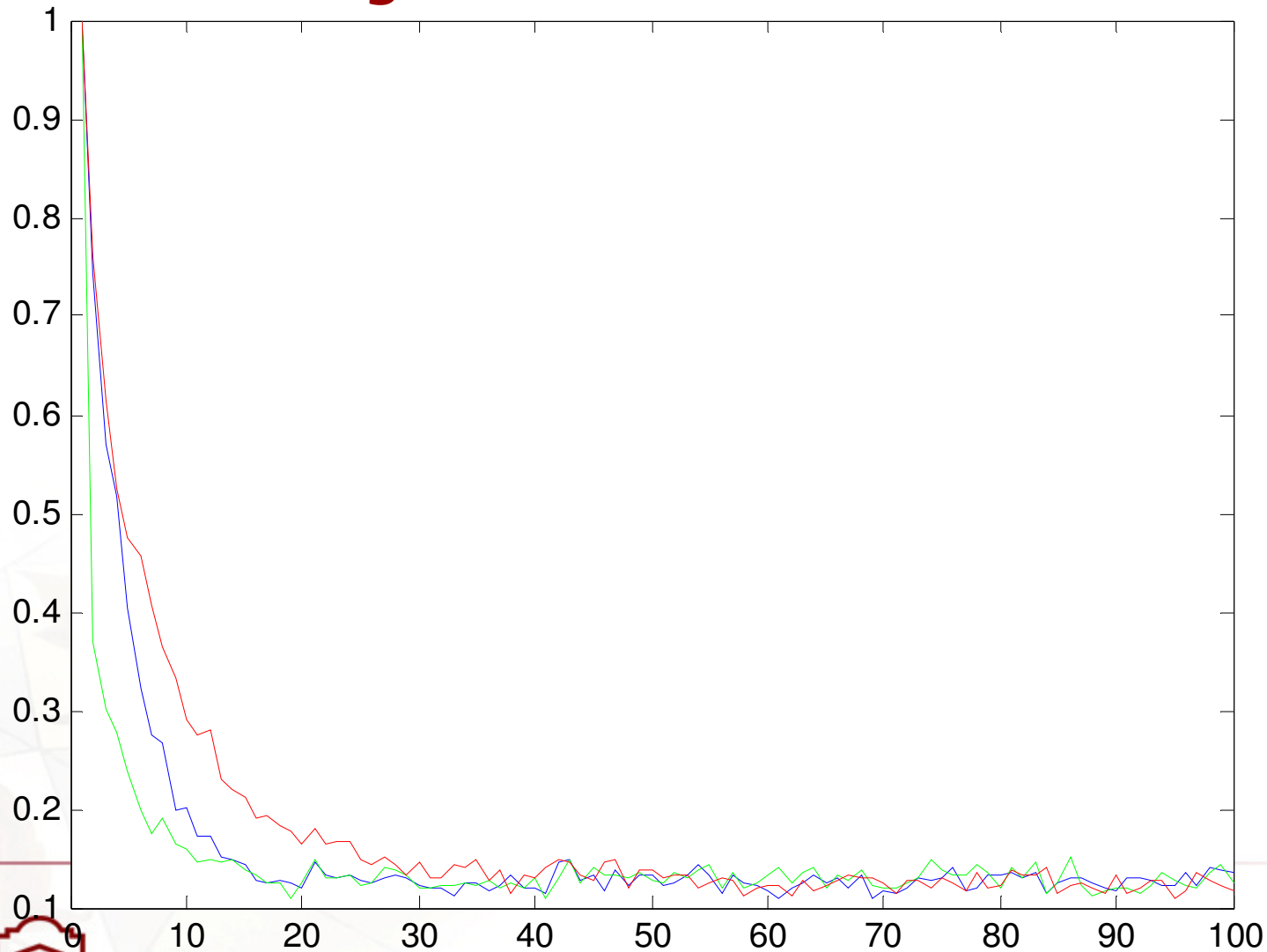
# Comparison



- The misadjustment of LMS, NLMS and AP were found by simulation
  - $\text{LMS} = 0.3$
  - $\text{NLMS} = 0.36$
  - $\text{AP} = 0.6$
- We can adjust the values of the step sizes to try to obtain the same value of misadjustment.
- Since misadjustment is roughly proportional to the step size, we will decrease the step size in NLMS by 1.2 and by 2 in AP.



## MSE averaged over 500 runs



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## MSDev averaged over 500 runs

