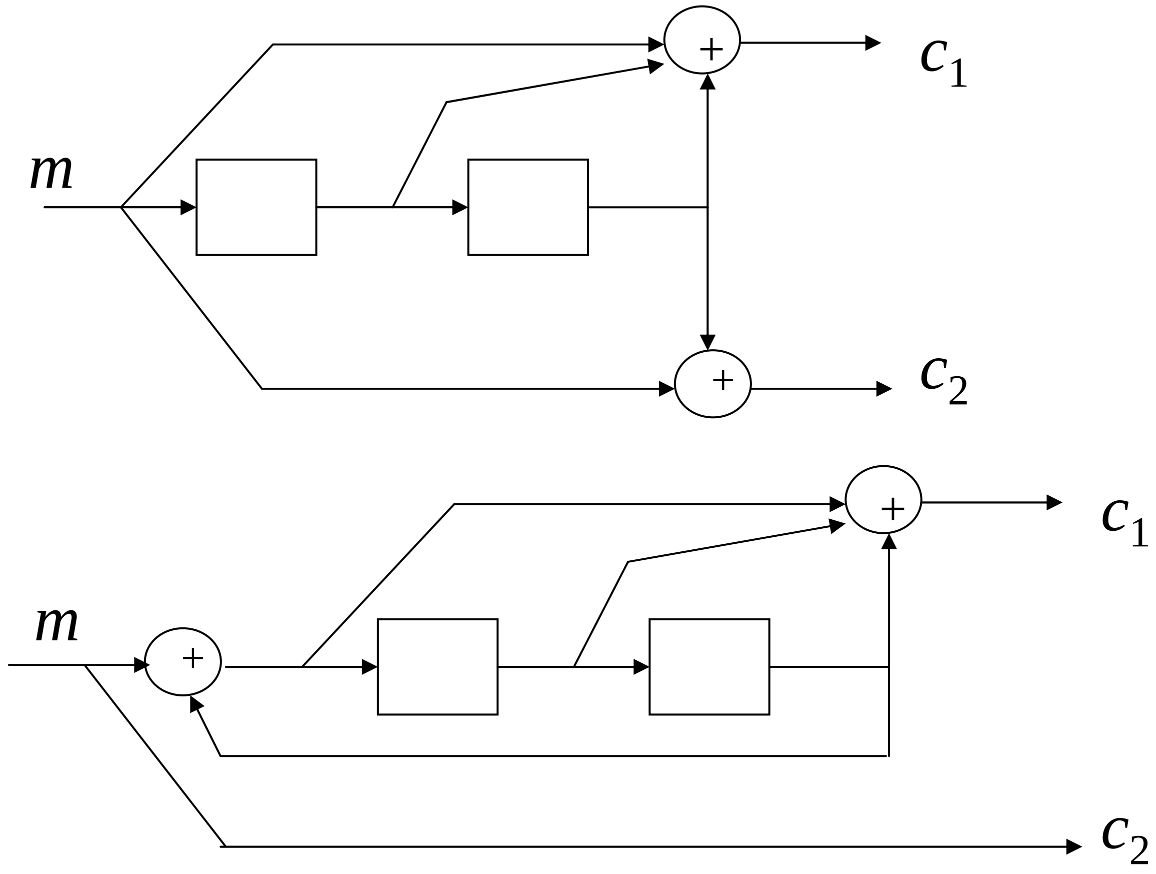
ELG 5372 Error Control Coding

Lecture 20: Convolutional Codes: Equivalent Codes and Basic Encoders

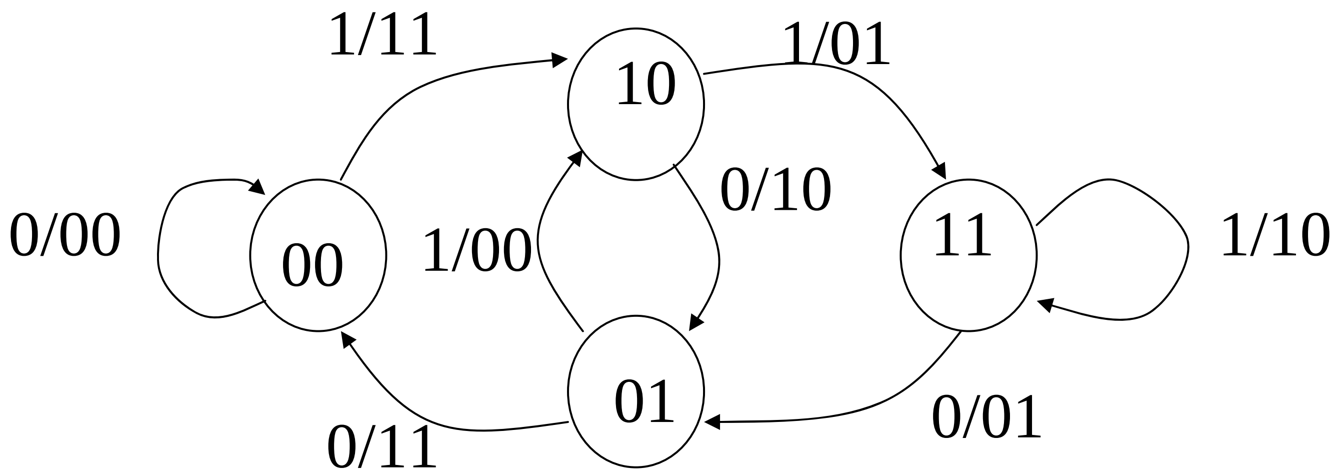
Equivalent Codes

- Two convolutional codes are said to be equivalent if the set of sequences that they produce are identical.
- Consider the following two encoders:
  - $G_a(D) = [1+D+D^2, 1+D^2]$
  - $G_b(D) = [1+D+D^2/(1+D^2), 1]$

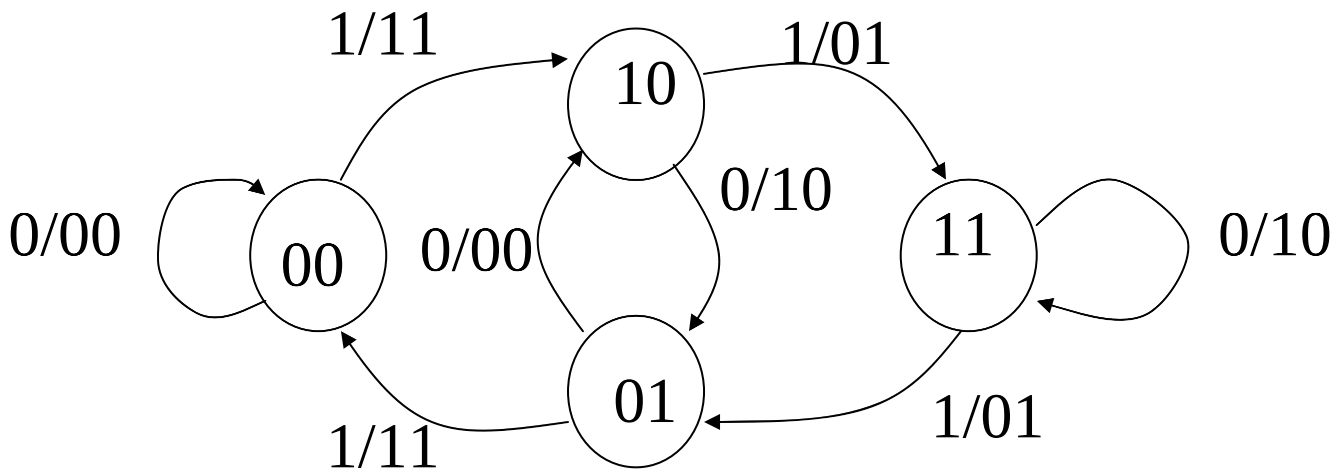
Encoders for $G_a(D)$ and $G_b(D)$



State Diagram for $G_a(D)$



State Diagram for $G_b(D)$



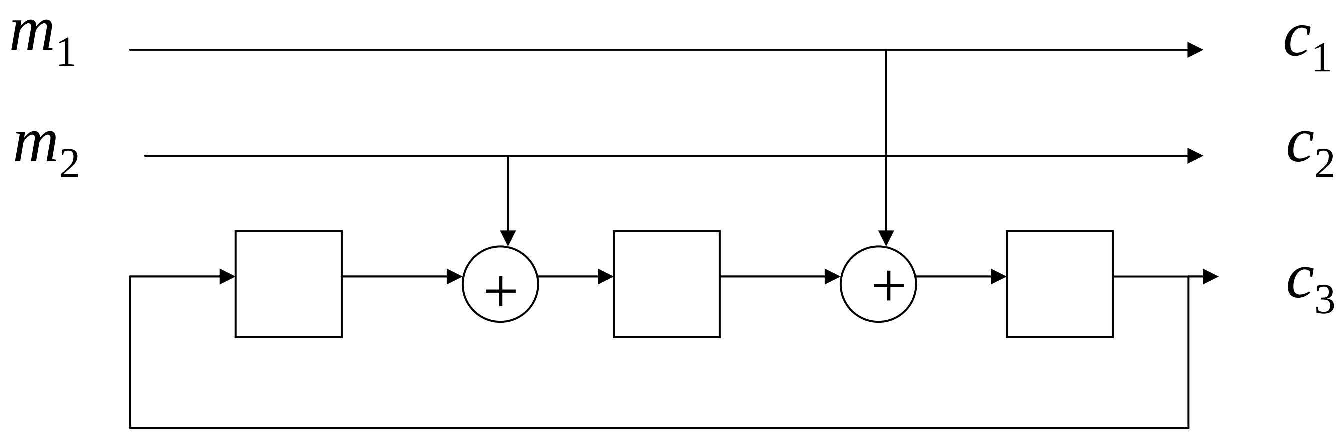
Comparison

- When comparing the state diagram of $G_a(D)$ to that of $G_b(D)$, we see that some state changes are affected, but the same outputs occur.
- For encoder *a*, if $\mathbf{m}_a = 1000$, output = 11,10, 11, 00
- For encoder *b*, if $\mathbf{m}_b = 1010$, output = 11, 10, 11, 00.
- This is because $G_b(D) = T(D)G_a(D)$, where $T(D)$ is a $k \times k$ invertible matrix.
- Let $\mathbf{c}_a = \mathbf{m}_a G_a(D)$ and $\mathbf{c}_b = \mathbf{m}_b G_b(D) = \mathbf{m}_b T(D) G_a(D) = \mathbf{c}_a$ if $\mathbf{m}_b T(D) = \mathbf{m}_a$. If $T(D)$ is invertible, then for every possible $\mathbf{m}_a$, there is a distinct $\mathbf{m}_b$ such that $\mathbf{m}_a = \mathbf{m}_b T(D)$.
- In this example, $T(D) = 1/(1+D^2)$

Example 2

Consider the code

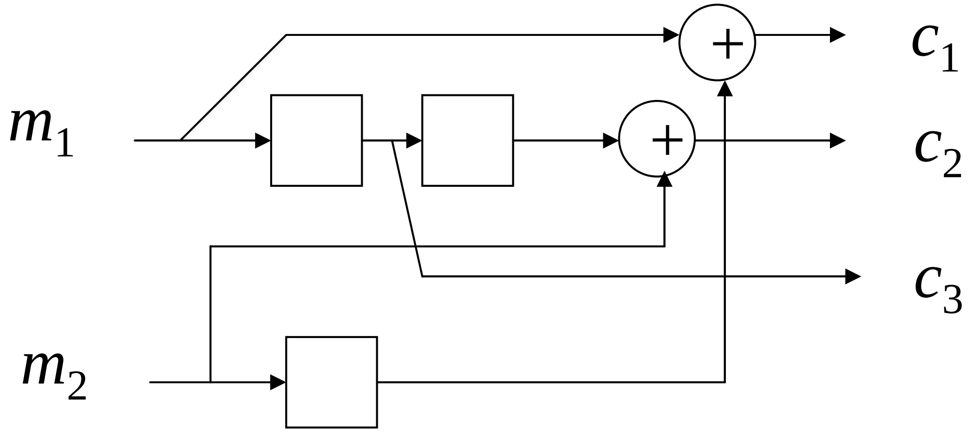
$$G_1(D) = \begin{bmatrix} 1 & 0 & D/(1+D^3) \\ 0 & 1 & D^2/(1+D^3) \end{bmatrix}$$



Example 2 cont'd

- Let $G_2(D)$ be given by:

$$G_2(D)=\begin{bmatrix}1 & D^2\\ D & 1\end{bmatrix}\begin{bmatrix}1 & 0 & D/(1+D^3)\\ 0 & 1 & D^2/(1+D^3)\end{bmatrix}=\begin{bmatrix}1 & D^2 & D\\ D & 1 & 0\end{bmatrix}$$



Both implementati0ns use 3 storage devices and 2 adders.

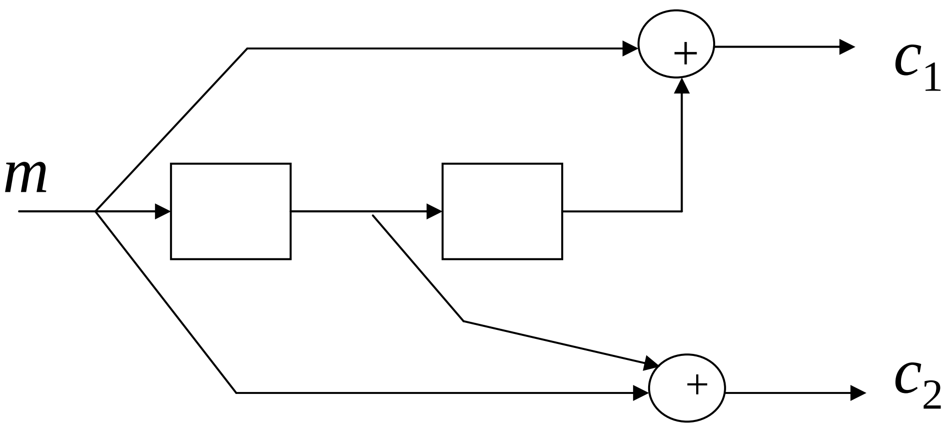
Catastrophic Encoders

Consider the rate ½ encoder

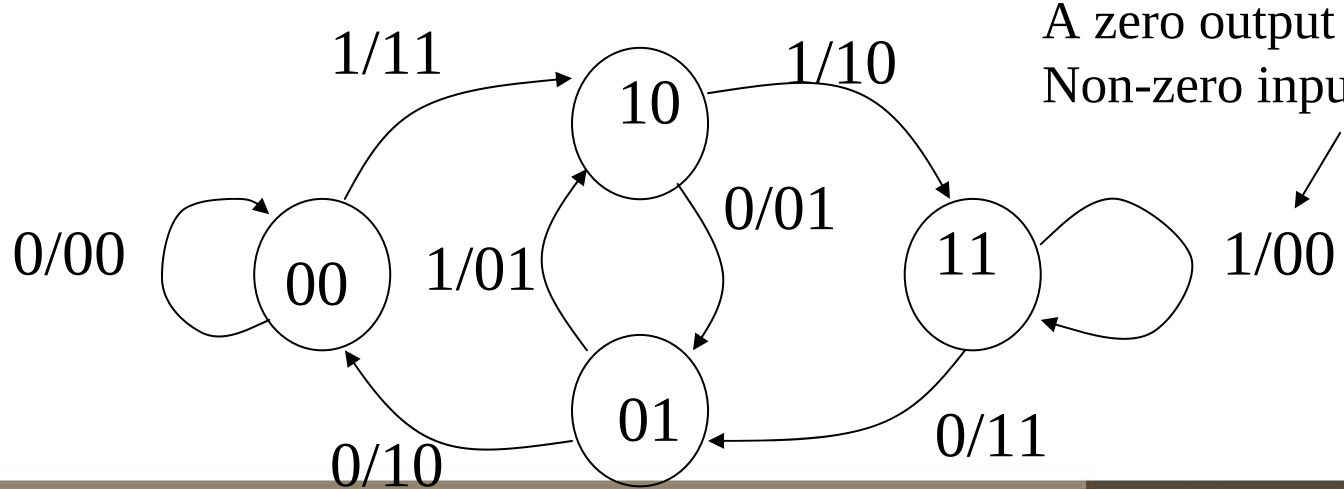
$$G(D) = [1 + D^2 \quad 1 + D]$$

Let m = 111111…., **m**(D) = 1+D+D^2+D^3+….=1/(1+D)
Then **c**(D) = [1+D, 1] = 1110000000….
Suppose the first three bits are received in error
r(D) = 0000000…, this will be decoded as
m’ = 0000…. Therefore a finite number of channel errors cause an infinite number of decoded bit errors. This is called a catastrophic code.

Catastrophic Encoders cont'd

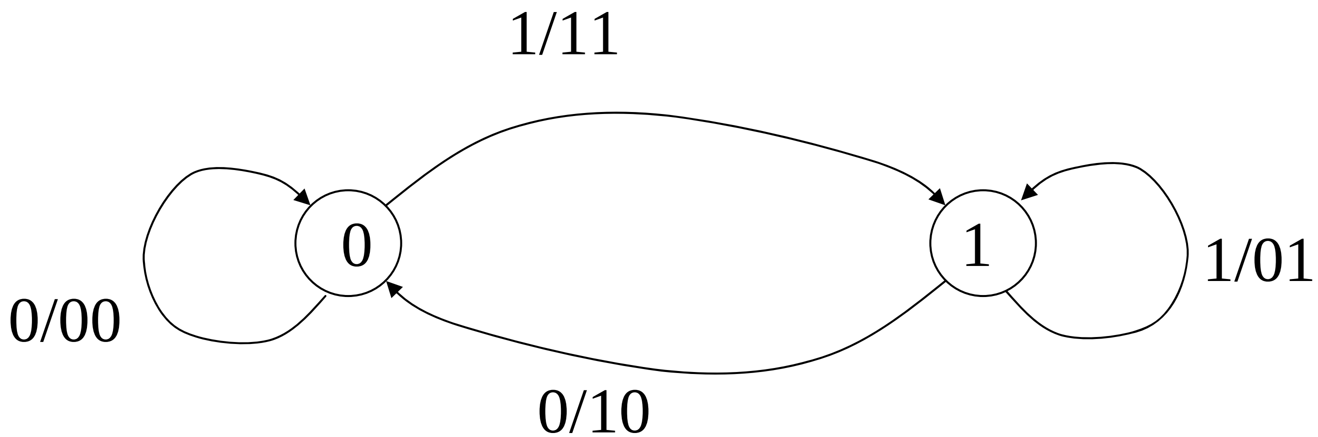


A code is catastrophic
If there exist a loop in the
State diagram that has
A zero output weight for a
Non-zero input weight.



Catastrophic Codes cont'd

The code $G_2(D) = [1+D, 1]$ is equivalent



This code, which is less complex to implement, is not catastrophic. The output sequence 111000000… is produced by input 10000….

Catastrophic Codes cont'd

- The main conclusion is that catastrophic codes may have equivalent codes that are not catastrophic.

Right Inverse

- Let k<n. A right inverse of a k×n matrix **G** is a n×k matrix $\mathbf{G}^{-1}$, such that $\mathbf{G}\mathbf{G}^{-1} = \mathbf{I}_k$.
- For $\mathbf{G} = [1+D^2, 1+D]$, $\mathbf{G}^{-1} = [1/(D+D^2)\ (1+D+D^2)/(D+D^2)]^T$.

Polynomial Encoder

- A transfer function matrix with only polynomial entries is said to be a polynomial encoder
  - It uses FIR filters.
- If the transfer function has rational entries, it is a rational encoder
  - It uses IIR Filters.

Catastrophic Codes revisited

- Let G(D) be the transfer function matrix of a convolutional code.
- $\mathbf{c}(D) = \mathbf{m}(D)\mathbf{G}(D)$ and $\mathbf{c}(D)\mathbf{G}^{-1}(D) = \mathbf{m}(D)$.
- If the code is catastrophic, there is a finite length c(D) that is produced by an infinite length m(D). This only occurs is $\mathbf{G}^{-1}(D)$ is rational.
- Therefore if $\mathbf{G}^{-1}(D)$ is a polynomial matrix, the code cannot be catastrophic.

Basic Encoders

- A transfer function matrix $\mathbf{G}(D)$ is basic if it is polynomial and it has a polynomial right inverse.
- It is shown on page 464 of the text that all rational encoders have a basic equivalent.