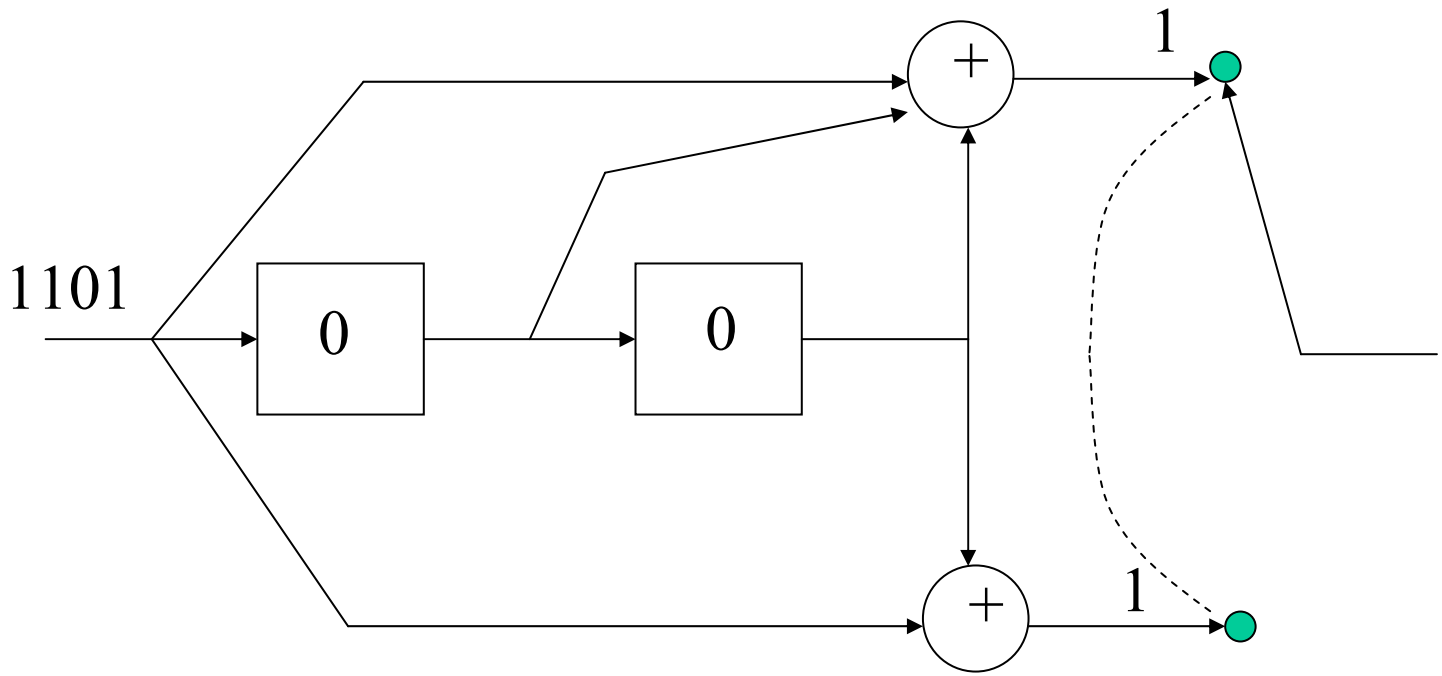
ELG 5372 Error Control Coding

Lecture 19: Introduction to Convolutional Codes

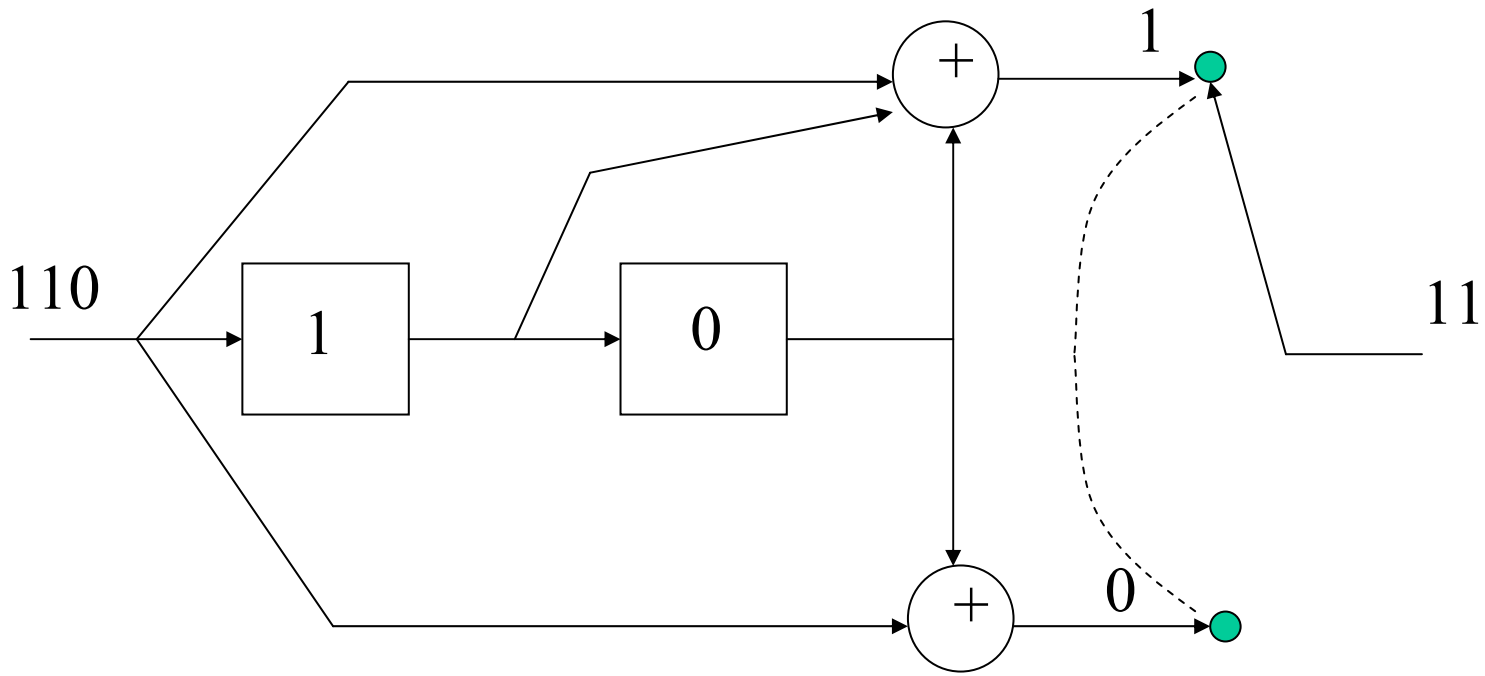
Introduction

- The encoder can be viewed as a filtering or convolution operation.
- The encoder is a set of linear time-invariant digital filters.
- The code sequence is the interleaved filter outputs.
- Whereas block codes take block of *k* message symbols and produce blocks of *n* code symbols, convolutional codes are viewed as stream codes in that they often operate on a stream of input symbols and produce a stream of output symbols.

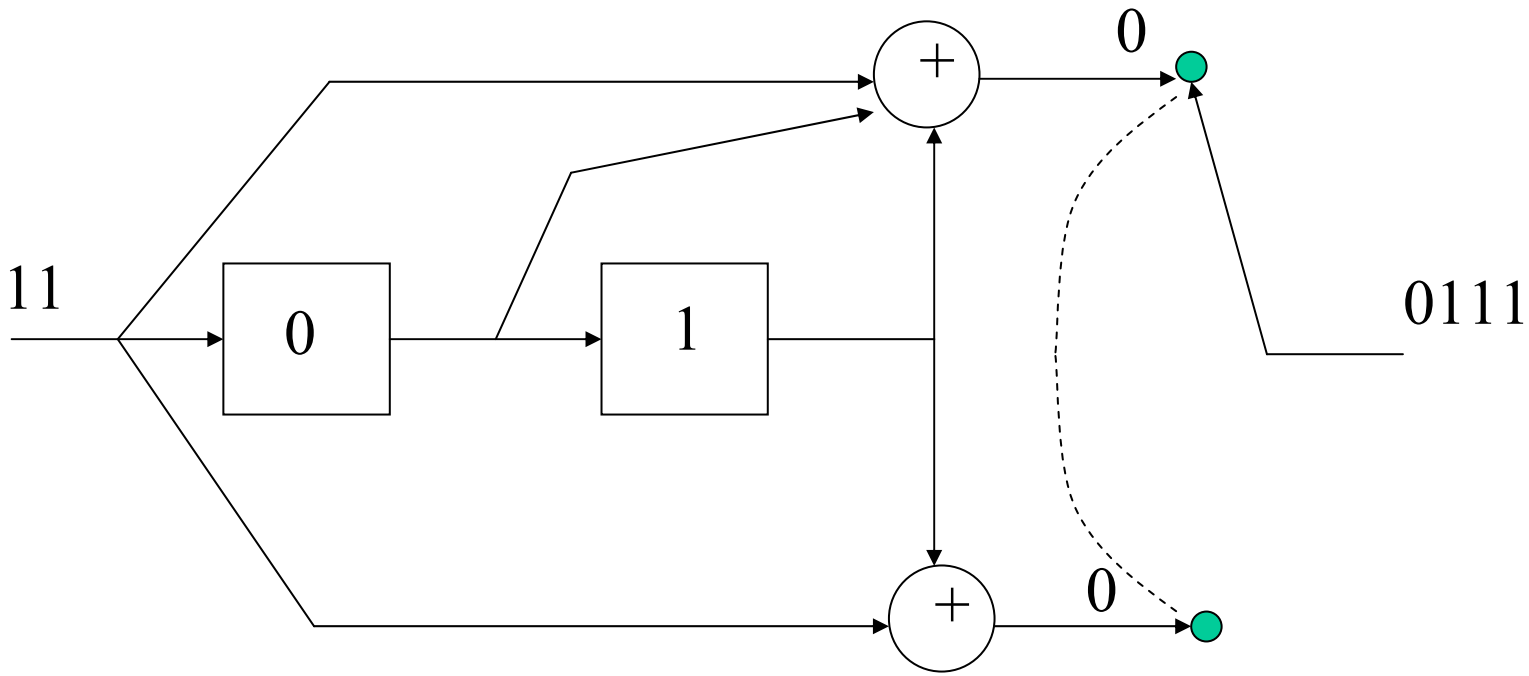
Example



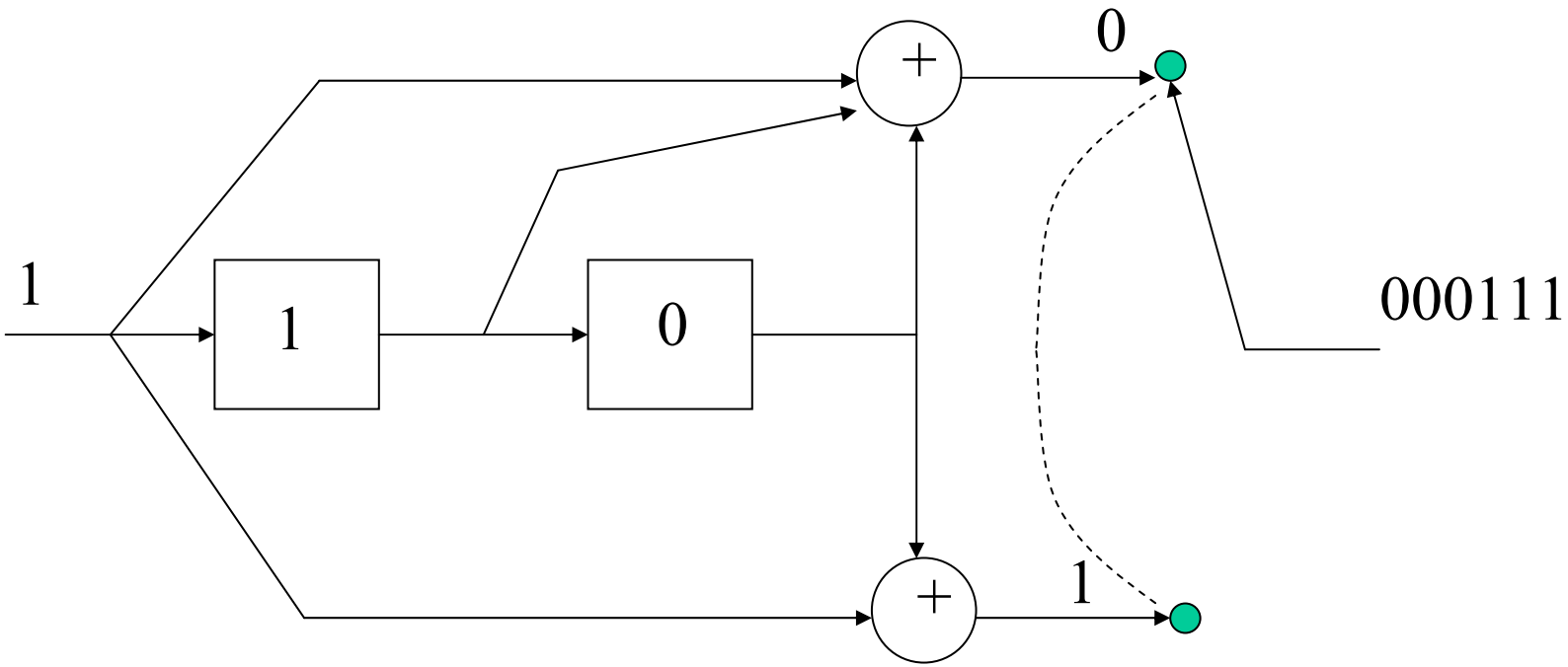
Example



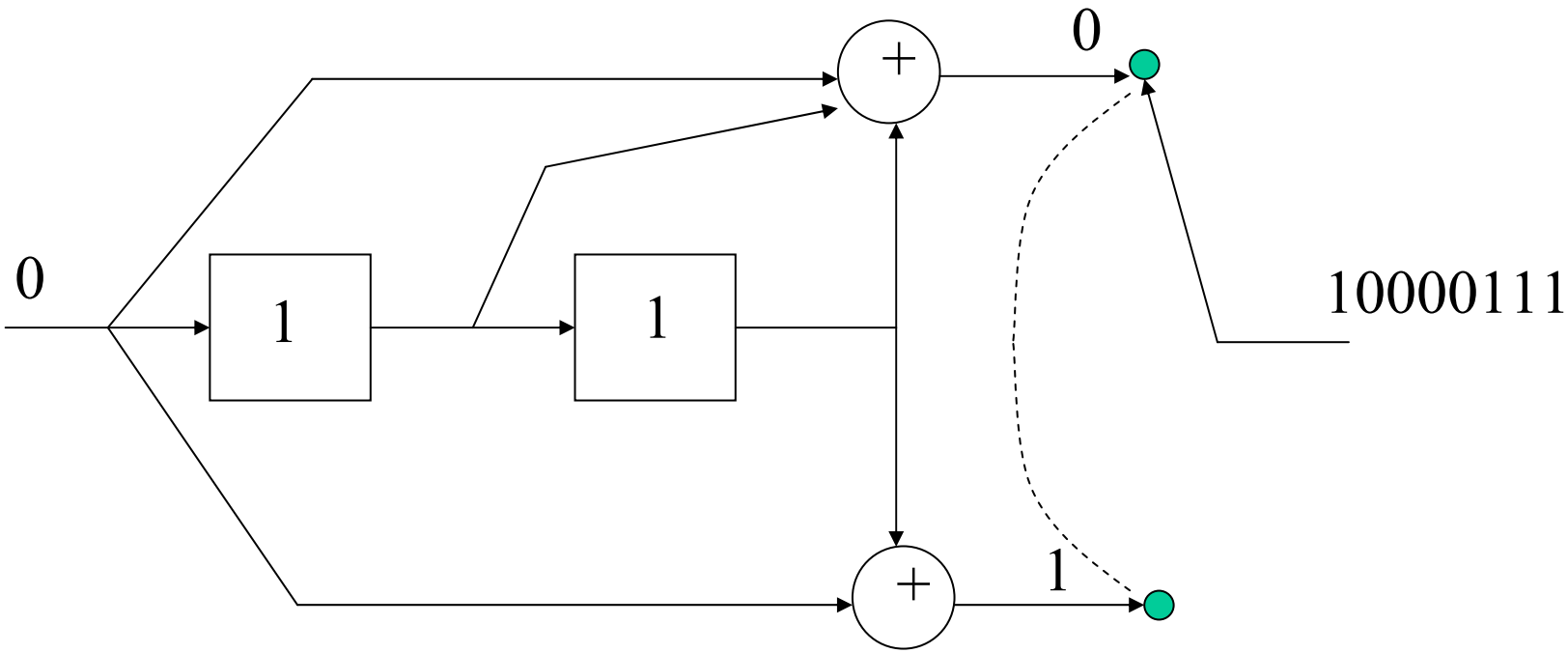
Example



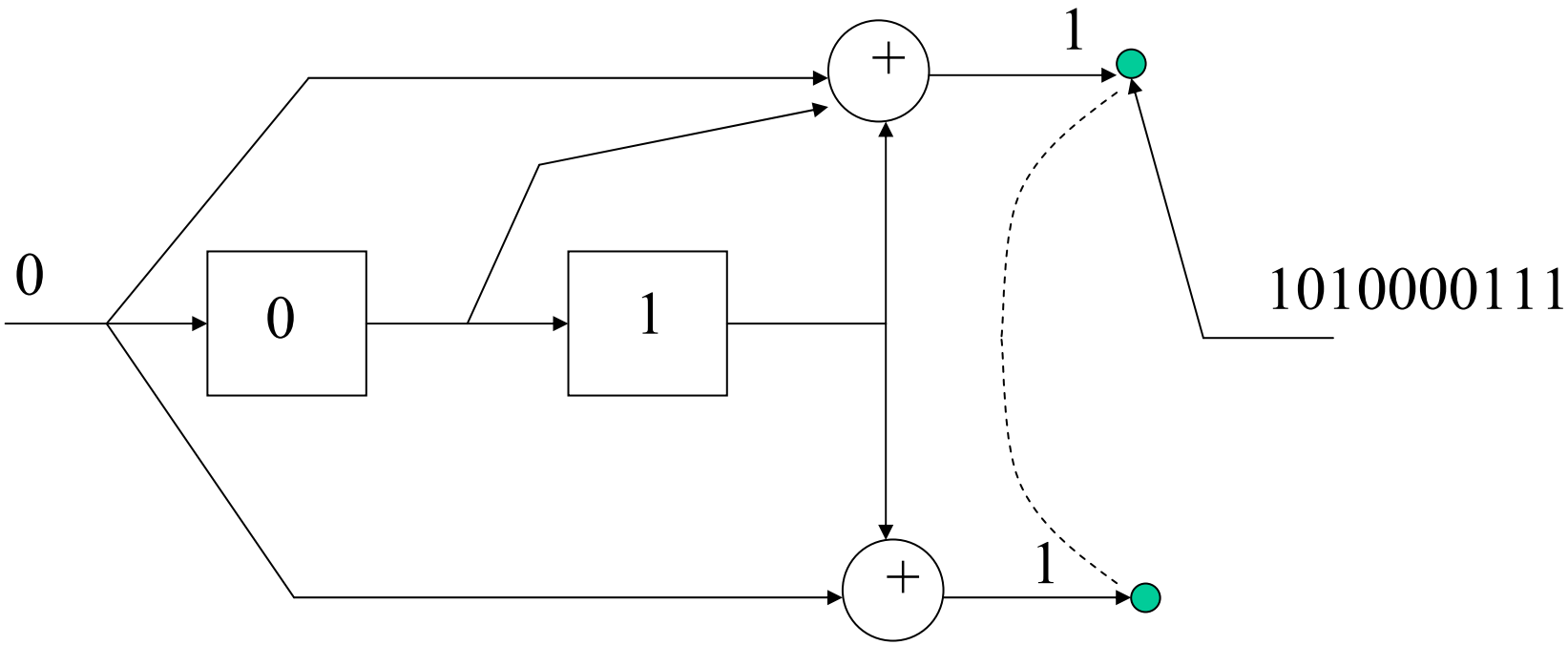
Example



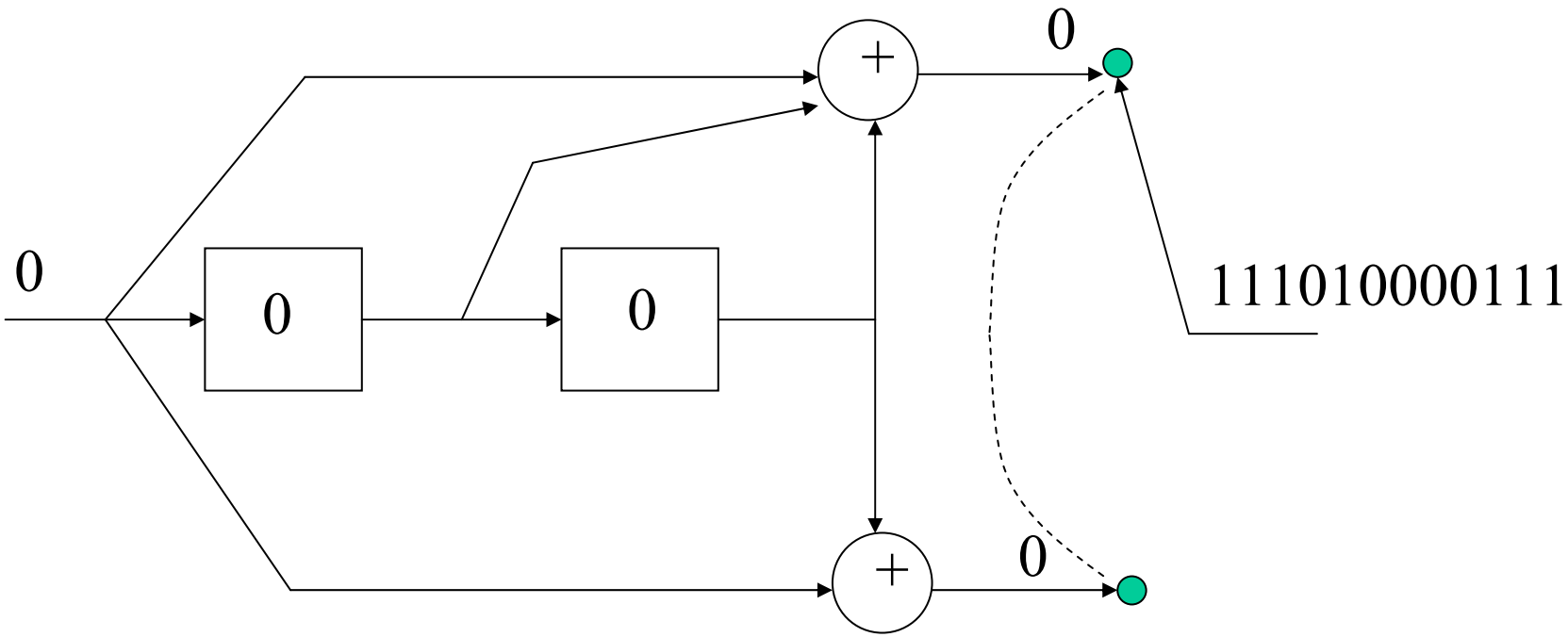
Example



Example



Example



$1011 \rightarrow 11, 10, 00, 01, 01, 11$

D transform

- The message **m** = (m_0, m_1, …, m_N) is can be represented by the D transform:

$$m(D) = \sum_{i=0}^{N} m_i D^i = m_0 + m_1 D + ... + m_N D^N$$

- In our example, $m(D) = 1+D^2+D^3$.
- The generators are also expressed in D transform notation:

$$g_1(D) = 1 + D + D^2$$

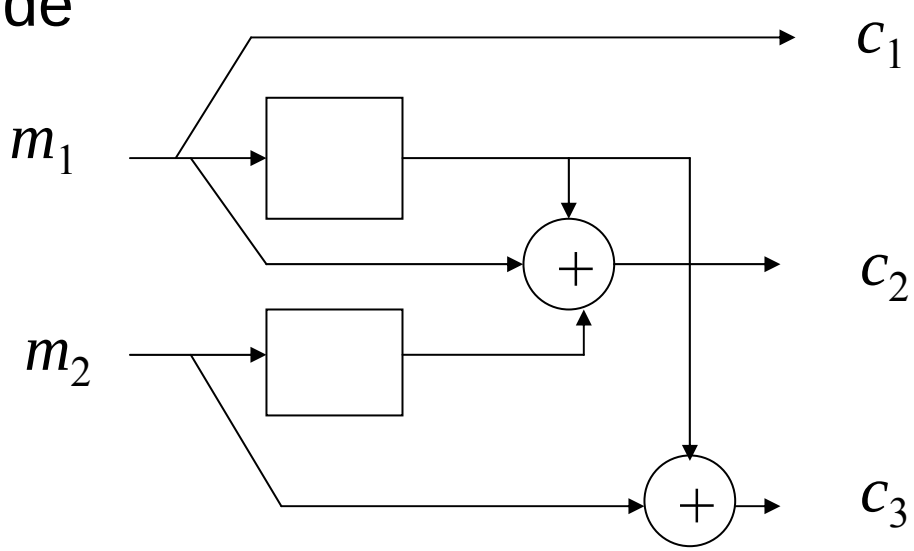
$$g_2(D) = 1 + D^2$$

D Transform of Encoder Output

- $c_i(D) = m(D)g_i(D)$.
- $c_1(D) = (1+D^2+D^3)(1+D+D^2) = 1+D+D^2+D^2+D^3+D^4+D^3+D^4+D^5 = 1+D+D^5 = 110001$.
- C2(D) = $(1+D^2+D^3)(1+D^2) = 1+D^2+D^2+D^4+D^3+D^5 = 1+D^3+D^4+D^5 = 100111$.
- Output = 11,10, 01, 01, 01, 11.

Example 2

- The following code is a rate 2/3 convolutional code



$$\mathbf{G} = \begin{bmatrix} 1 & 1+D & D \\ 0 & D & 1 \end{bmatrix}$$

$$\mathbf{c} = [c_1(\mathrm{D}) \quad c_2(\mathrm{D}) \quad c_3(\mathrm{D})]$$

$$\mathbf{m} = [m_1(\mathrm{D}) \quad m_2(\mathrm{D})]$$

$$\mathbf{c} = \mathbf{mG}$$

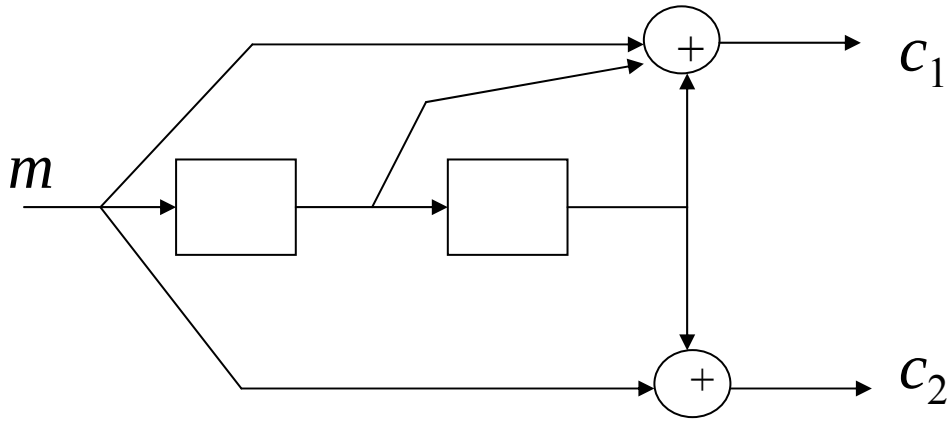
Example 2

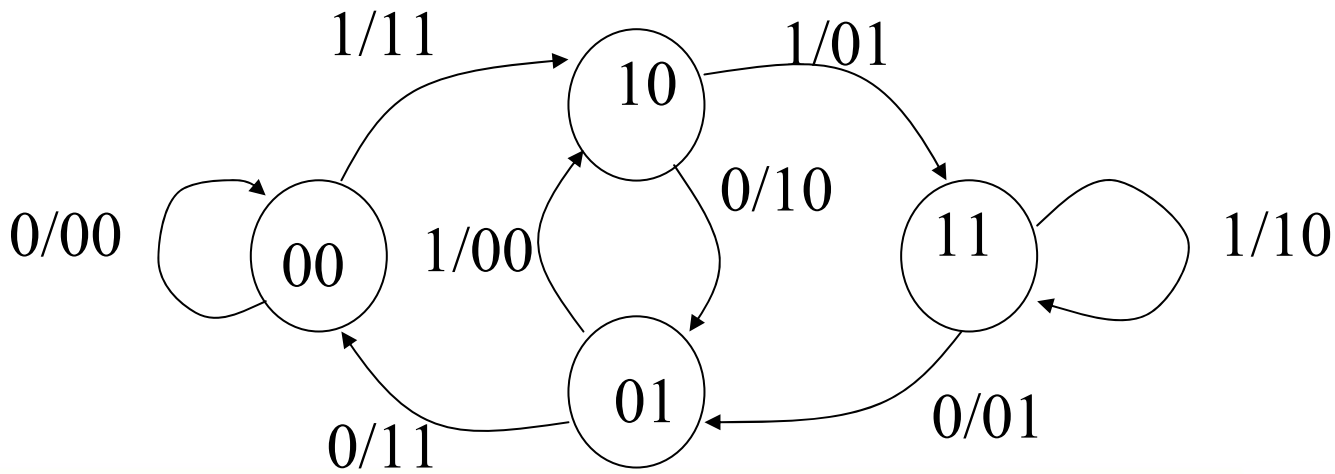
- $m_1(D) = 1+D+D^2$
- $m_2(D) = 1+D$.
- $c_1(D) = 1+D+D^2$
- $c_2(D) = (1+D+D^2)(1+D)+(1+D)D = 1+D+D^2+D^3$.
- $c_3(D) = (1+D+D^2)D +(1+D) = 1+D^2+D^3$.
- Output = 111, 110, 111, 011

Encoder State

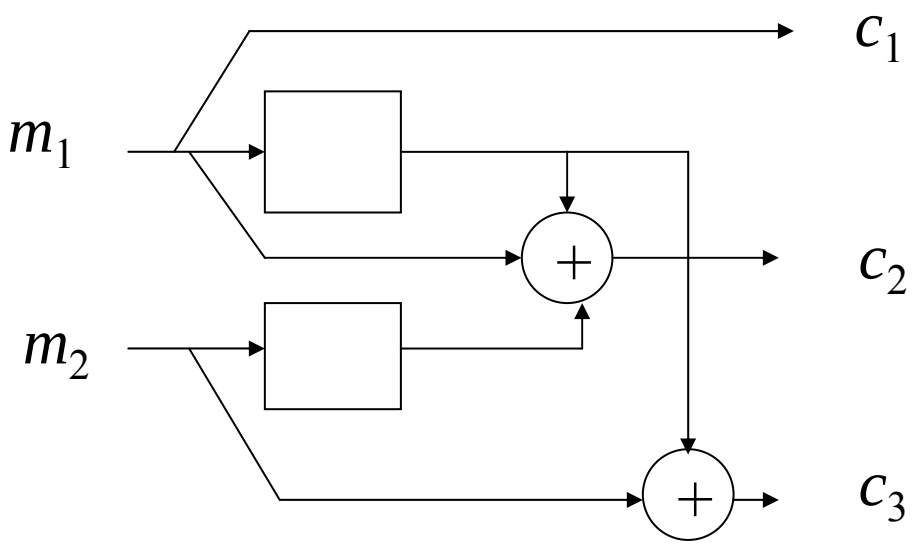
- A convolutional encoder is a state machine.
- For both encoding and decoding purposes, it is useful to draw the state diagram.
- A state diagram is a temporal diagram showing current state/next state information as well as the input and output information.
- Current state = current contents of memory
- Next State = contents of memory following clock pulse (depends on input).

Example 1



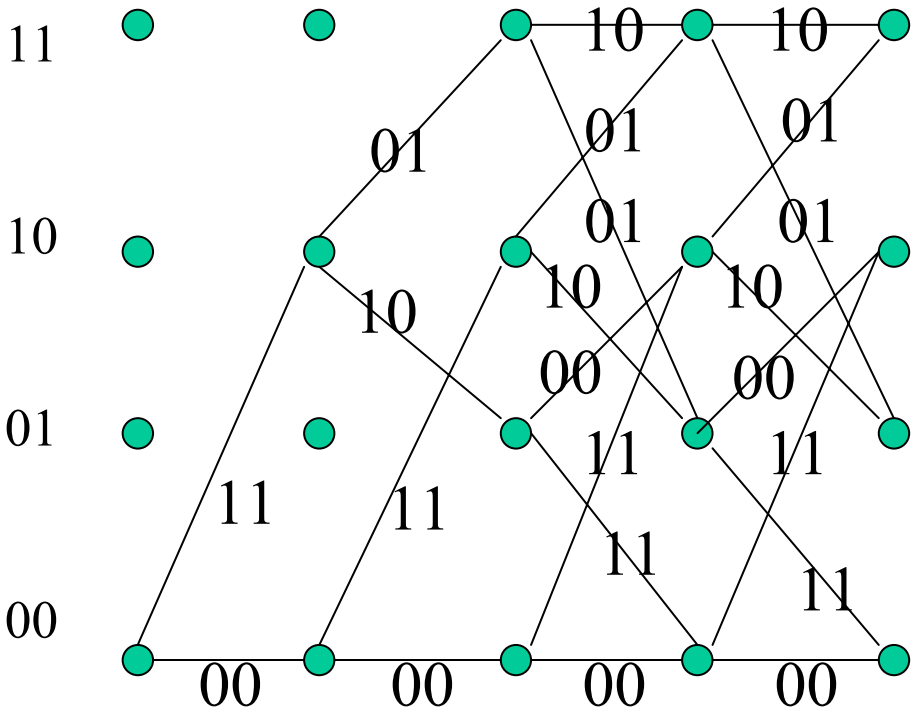


Example 2



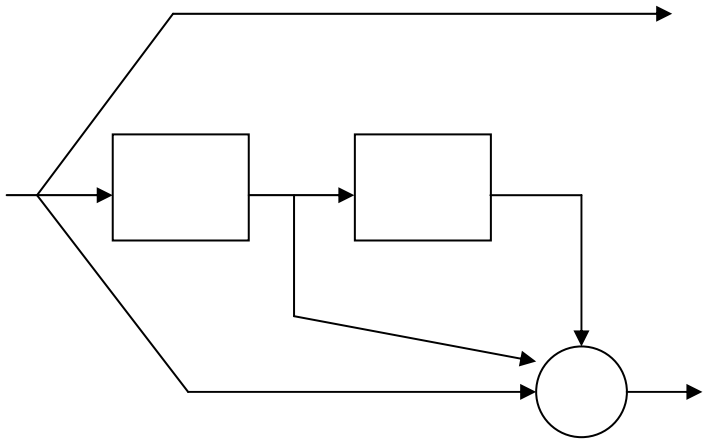
Trellis Diagram

- The Trellis diagram shows the state transitions over a number of time intervals.



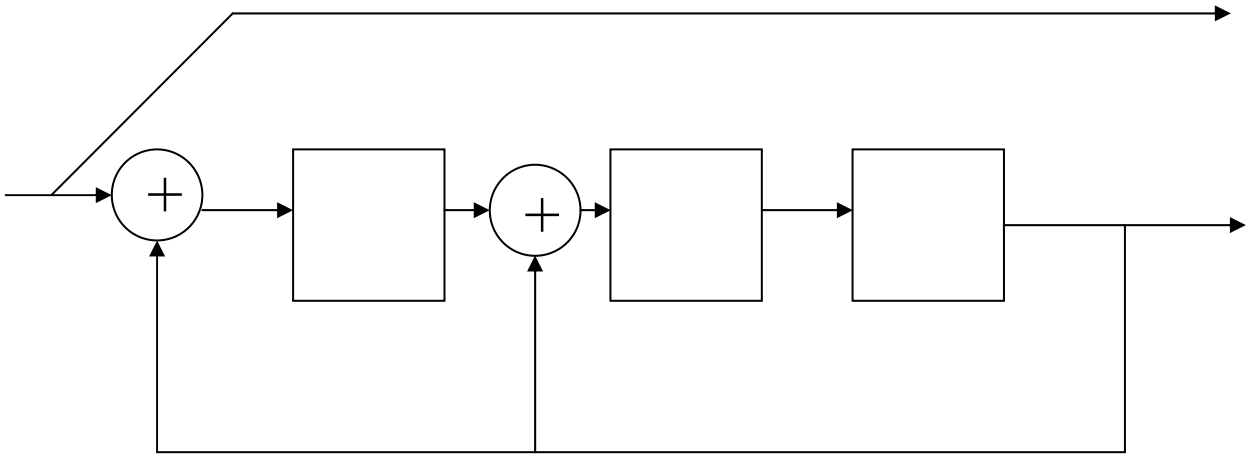
Systematic Convolutional Codes

- For all i, there exists one generator $g_{ij}(D) = 1$.



Recursive Convolutional Code

- Some generators have feedback



$g_1(D) = 1$
$g_2(D) = D^3/(1+D^2+D^3)$

$m(D) = 1$
$c_1(D) = 1$
$c_2(D) = D^3/(1+D^2+D^3) =$
$D^3+D^5+D^6+D^7+\ldots$