ELG 5372 Error Control Coding

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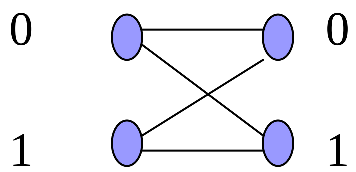
Lecture 2: Introduction to Coding 2

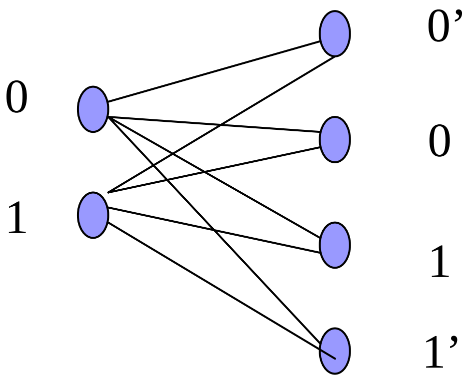
Decoding Techniques

- Hard Decision
  - Receiver detects data before decoding
- Soft Decision
  - Receiver quantizes received data and decoder uses likelihood information to decode.
  - Magnitude of decision variable usually indicates the likelihood that the detection is correct.
  - Maximum Likelihood Decoding (MLD).

Hard Decision vs Soft Decision



Hard Decision



Soft Decision (2 bit quantization)

Maximum Likelihood Decoding

- Transmit codeword **c**
- Receive word **r**.
- Decode **c'**.
- Decoding error occurs if **c'** ≠ **c**.

Maximum Likelihood Decoding (2)

$$P(E \mid \mathbf{r}) = P(\mathbf{c}' \neq \mathbf{c} \mid \mathbf{r})$$

$$P(E) = \sum_{\mathbf{r}} P(\mathbf{c}' \neq \mathbf{c} \mid \mathbf{r}) P(\mathbf{r})$$

The optimum decoding rule is one that minimizes $P(E)$. $P(\mathbf{r})$ is independent of the decoding rule, therefore we must minimize $P(\mathbf{c}' \neq \mathbf{c} | \mathbf{r})$, which is equivalent to maximizing $P(\mathbf{c}' = \mathbf{c} | \mathbf{r})$.

Maximum Likelihood Decoding (3)

$$P(\mathbf{c}\,|\,\mathbf{r}) = P(\mathbf{r}\,|\,\mathbf{c})P(\mathbf{c})\,/\,P(\mathbf{r})$$

Assuming all codewords are equally likely, maximizing $P(\mathbf{c}|\mathbf{r})$ is the same as maximizing $P(\mathbf{r}|\mathbf{c})$. Assuming hard decision with discrete memoryless channel (DMC):

$$P(\mathbf{r}\,|\,\mathbf{c}) = \prod_i P(r_i\,|\,c_i) = p^{d(\mathbf{r},\mathbf{c})}(1-p)^{n-d(\mathbf{r},\mathbf{c})} \qquad (1)$$

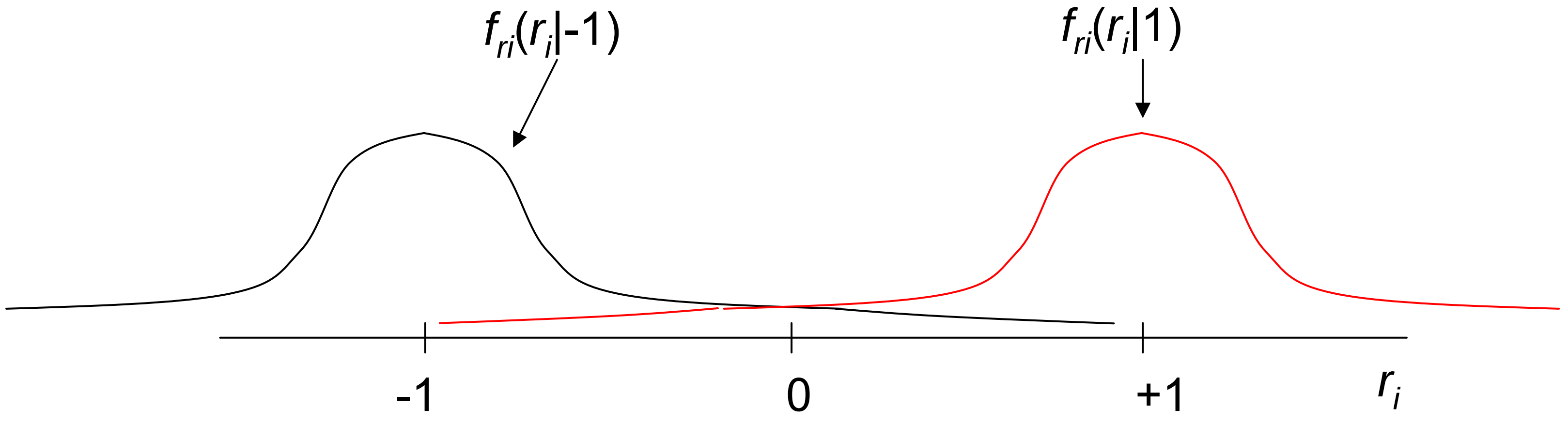
$$\log P(\mathbf{r}\,|\,\mathbf{c}) = d(\mathbf{r},\mathbf{c})\log p + (n - d(\mathbf{r},\mathbf{c}))\log(1-p) \qquad (2)$$

Since $p < (1\text{-}p)$, $P(\mathbf{r}|\mathbf{c})$ is maximized by the codeword for which $d(\mathbf{r},\mathbf{c})$ is minimized. This is known as a minimum distance decoding rule.

Hamming Distance vs Euclidean Distance

- Hamming Distance = number of positions in which two vectors differ.
- Hard decision decoding decodes using minimum Hamming distance rule previously shown
- Euclidean distance between **r** and **c** is ||**r-c**||
- Soft decision decoding uses minimum Euclidean distance (approximately)

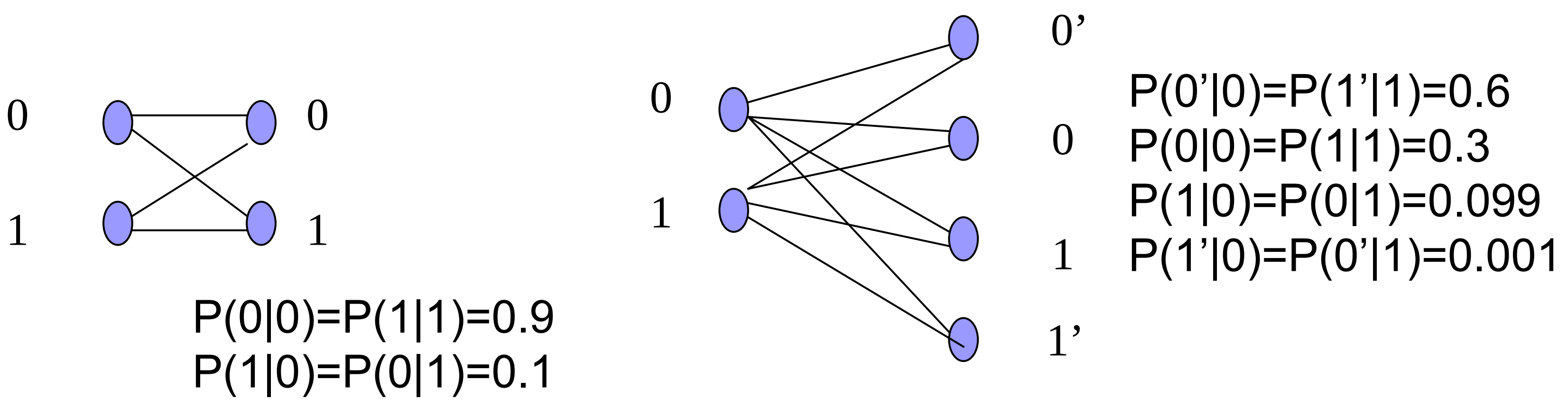
Decision Variables



$$P(c_i=1)/P(c_i=-1)=f_{ri}(r_i|c_i=1)/f_{ri}(r_i|c_i=-1)$$

Example: HD vs SD

- Consider Hamming (7,4) code used with the following channels



(a) HD (b) SD

Example: HD vs SD

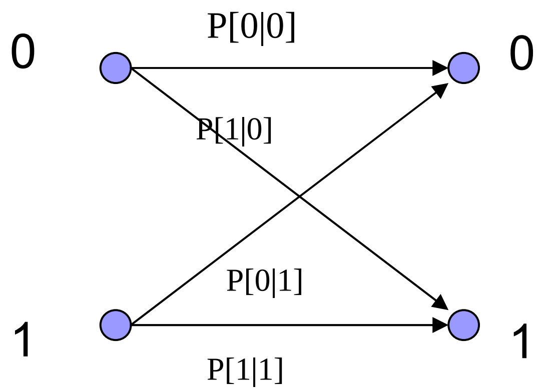
- Suppose we receive **r** = 1' 0 0' 0 0' 0' 0'
- For HD, there is no quantization, so **r** = 1000000 and will be decoded as 0000000 using minimum Hamming distance rule.
- In the SD case, using (1) with **c** = 0000000, we get P(**r**|**c**) = 0.000117
- However, for **c**=1101000, we get P(**r**|**c**) = 0.000762
- This means that for the given **r**, it is almost 7 times more probable that 1101000 was transmitted than 0000000.

Errors and Channel Models

- Memoryless channels: Noise affects each transmitted symbol independently.
  - Tx symbol has probability p of being received incorrectly and probability $1-p$ of being received correctly.
  - Transmission errors occur randomly in the received sequence.
  - Memoryless channels are often referred to as random-error channels.

Errors and Channel Models (2)

- Examples
  - AWGN: $r_i = s_i + n_i$, $E[n_i]=0$, $E[n_i^2]=\sigma_n^2$ and $E[n_i n_j]=0$ for $i \neq j$.
  - DMC:

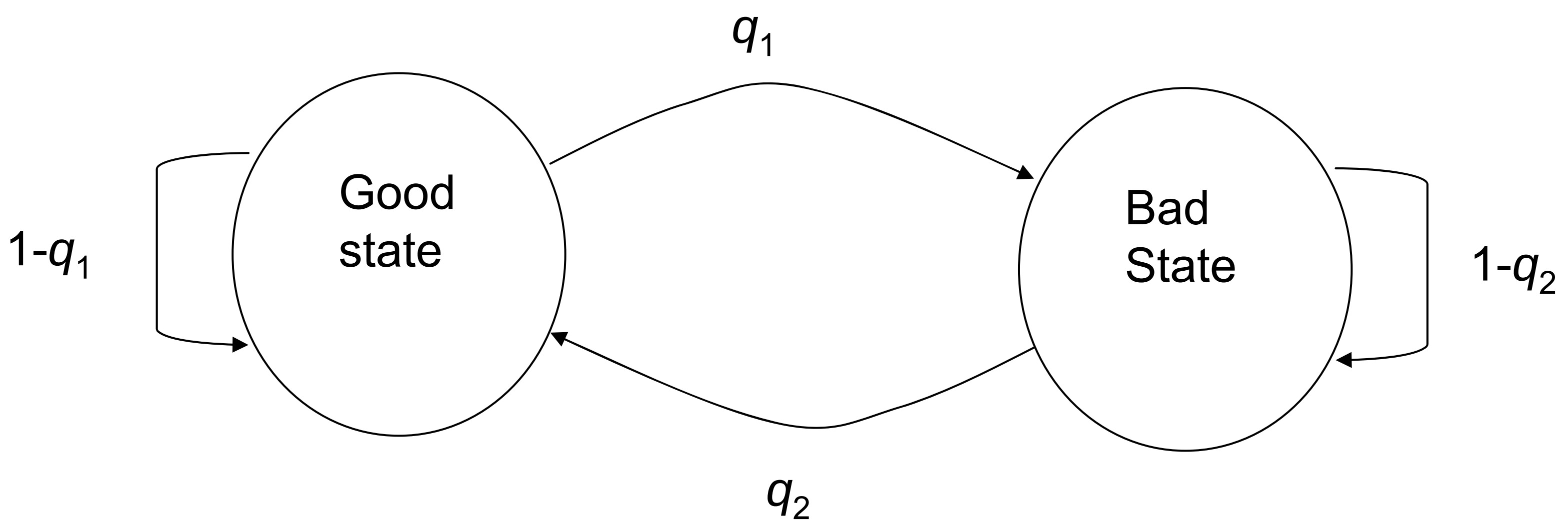


Errors and Channel Models (3)

- Channels with memory
  - Errors do not occur randomly.
  - Either noise is not independent from transmission to transmission (coloured noise)
  - Or slow time varying signal to noise ratio causes time dependent error rates (fading channels).

Errors and Channel Models (4)

- Gilbert and Fritchman model



Errors and Channel Models (5)

- Channels with memory lead to error bursts.
  - Burst-error correcting codes
  - Random error correcting codes with interleaving-deinterleaving to randomize errors.

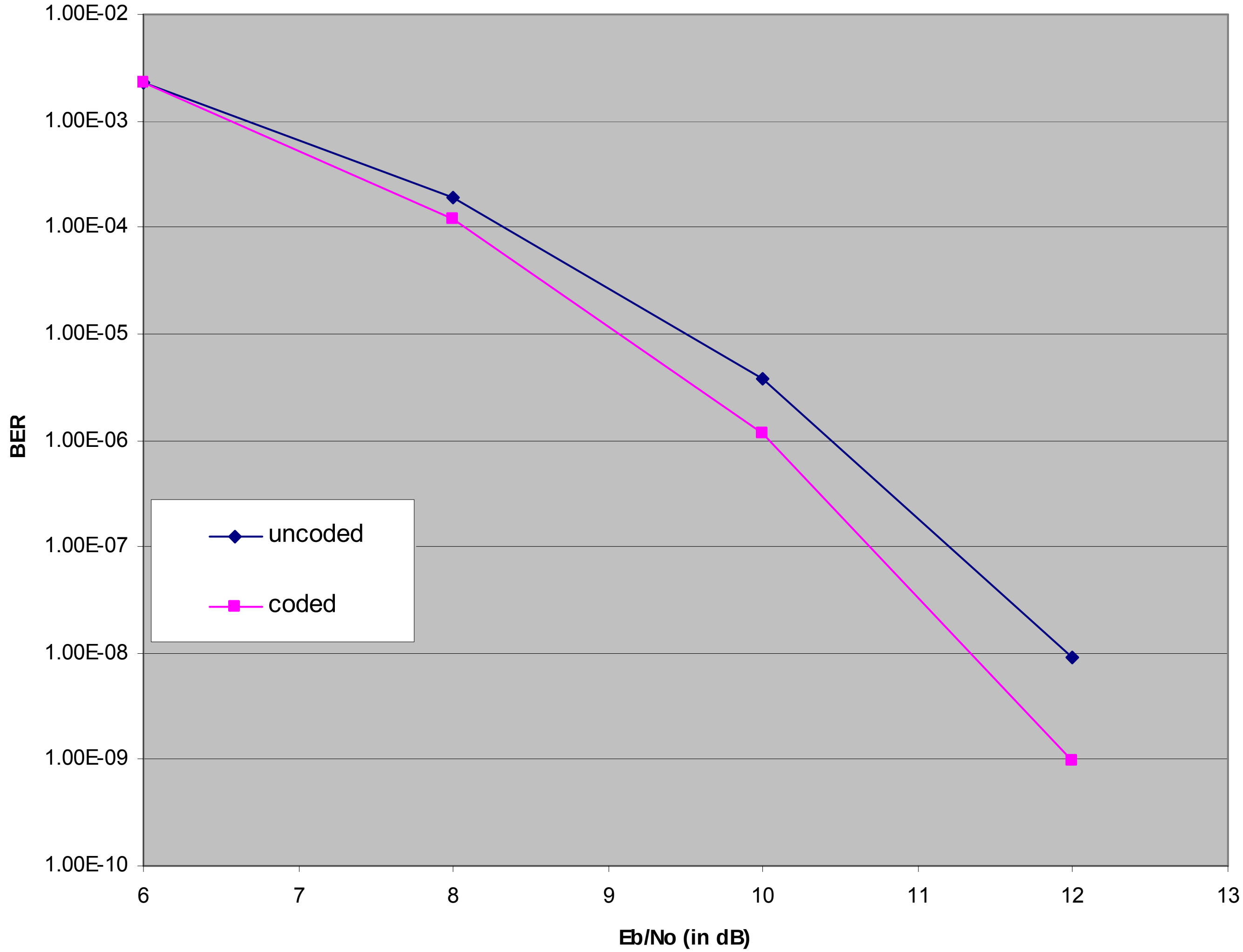
Performance Measures

- Probability of decoding error $P(E)$.
  - Probability that codeword at output of decoder is not the transmitted one.
  - Also referred to as word error rate (WER) or Block error rate (BLER).
- Bit error rate (BER) P_b
  - Probability that message bit at output of decoder is incorrect.
- Coding Gain (measured in dB)
  - Savings in transmitted power to achieve a specific BER using coding compared to uncoded case

Performance Measures 2

- Asymptotic coding gain
  - Coding gain when $E_b/N_o \rightarrow \infty$

Performance Measures 3



Coded Modulation

- Use of ECC creates bandwidth expansion due to redundant symbols.
- Combining ECC and modulation allows the redundancy to be contained in the modulation
  - 1 0 1 1
  - $s_1(t)+s_0(t-T)+s_1(t-2T)+s_1(t-3T)$
  - $s_1(t)+s_2(t-T)+s_1(t-2T)+s_3(t-3T)$
- Memory is created without adding redundant bits by using a higher order modulation scheme and using a bit in two successive symbols.

Trellis Coded Modulation

- State machine adds redundant bits and creates memory
- State change is encoded by selecting a symbol from a larger than needed constellation, thus no bandwidth expansion occurs and significant coding gains are achieved.